A CONCISE HISTORY OF ASTRONOMY

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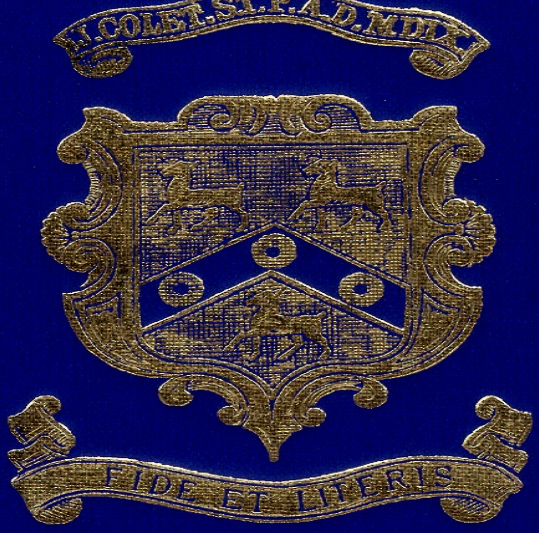
PETER DOIG



A CONCISE HISTORY OF ASTRONOMY

by

PETER DOIG
F.R.A.S.
Editor, Journal of the British Astronomical Association

With a Foreword by
SIR HAROLD SPENCER JONES, F.R.S.
Astronomer Royal

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FOREWORD

ASTRONOMY is the oldest of the sciences. Before the dawn of recorded history men observed the early rising and setting of the Sun, its annual northward and southward movements in the sky, the phases of the Moon, the motions of the other wanderers relative to the background of the fixed stars, the diurnal movements of the stars, and the changing face of the sky throughout the year. From the very early times men sought to explain these phenomena and to be able to predict the positions of the wanderers. The first ideas about the structure of the Universe gradually took shape, with the Earth fixed and immovable at its centre and everything else rotating around it—a small compact universe, with the stars on an outer sphere just beyond the orbit of Saturn, the most distant of the then known planets.

To-day the Earth is merely one of a family of planets revolving round the Sun, which is an average sort of star in the outlying regions of a vast Galaxy, containing about one hundred thousand million stars. It is estimated that there are about a hundred million separate galaxies, more or less comparable with our own, in the region of space at present accessible to observation. At the greatest observable distance there is no evidence of any thinning out in galactic distribution; the Universe must extend far beyond our present limit of exploration.

How the change of view has been brought about and how more and more knowledge has been acquired about the Universe in which we live and about its constituent parts—the stars, the planets, the nebulous clouds of gaseous matter and particles of dust, the tenuous matter widely dispersed through the vast interstellar spaces—is well told by Mr. Doig in this volume. There has been a much felt need for a new history of astronomy. During the present century the advances have been so rapid that the history of the subject needed to be brought up to date.

It is a fascinating study to follow the development of ideas and theories as more knowledge was gained; to see how, at

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times, a first simple theory, propounded to account for certain phenomena, was progressively elaborated as more and more facts were revealed by observation, until at length the theory proved to be incapable of accounting for the observed data, and had to be completely abandoned, to be replaced by a new and more satisfactory theory. That is the way in which progress in science is achieved; theories must be continually tested by observation and modified or discarded when agreement with observation is not obtained.

The development of ideas has necessarily been dependent upon improvements in instruments and in techniques and methods of observation. Mr. Doig has included in his history an account of the progressive steps by which the evolution of the modern telescope from the first primitive instruments has been brought about.

The human side of the story has not been neglected. To know something of the lives of the men who have made notable contributions to astronomical knowledge, of their struggles and difficulties, of the development of their ideas add to the interest of the story. As far as limitations of space allow, Mr. Doig has done this.

The latter portion of the book contains essentially a brief survey of the present state of the subject. The enquiring reader will probably wish to follow particular developments in greater detail. Mr. Doig has given a selected list of books which will serve to introduce the reader to further courses of study.

This concise history is a welcome addition to astronomical literature and should prove of interest to a wide circle of readers.

H. SPENCER JONES,
Astronomer Royal.

Royal Greenwich Observatory,
Herstmonceux.
7th October, 1949.

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AUTHOR'S PREFACE

THE purpose of this volume is to provide in a concise manner but as comprehensively as possible, an up-to-date account of value to the general reader seeking a knowledge of the development of the science of Astronomy, which will also be a handy reference book useful to students as a record of the main events and of the chief work of individual astronomers.

It may be noted that no general history of the kind has appeared in English for forty years, a period marked by progress perhaps greater than in any similar extent of time in the past and by many revolutionary discoveries, some of which have completely reversed ideas current at its beginning.

An endeavour has been made to give an outline of the development of the science preserving a proper balance between the relative values of the discoveries and work during the various epochs and between the achievements of the different men.

It is a truism that the attainment of a sound comprehension of any science is much assisted by an acquaintance with its history. It is trusted that this book will perform for students of the stars a useful function of the kind.

Inside the limits of space it has naturally not been practicable in some instances to do much more than stimulate interest without providing sufficient information for the enquiring reader. To allow for this a list of books for reference, containing some dealing fully with a particular period, is

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given in an appendix. But those who wish to study Astronomical history intensively will soon learn that consultation of original sources, wherever possible, becomes more and more desirable as the subject is further pursued.

To reduce interference with continuity in reading, some notes of reference to sources have been appended at the ends of chapters; those few notes that are of an explanatory nature are given at the foot of the pages where they occur.

The grateful thanks of the author are rendered to Sir H. Spencer Jones, Astronomer Royal, for his kind provision of the Foreword.

P. D.

September, 1949.

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CHAPTER I

THE OLDEST ASTRONOMY

ASTRONOMY is probably the most ancient of all the physical sciences. The striking spectacles presented to mankind by the varied appearances of a sky covered with thousands of shining objects of different degrees of brightness, all apparently revolving around the Earth; the changing phases of the Moon; the bright morning and evening stars; comets and shooting stars; and the surprising phenomena of eclipses of Sun and Moon, have from the very earliest times raised feelings of admiration and often of superstitious alarm.

Even the most primitive of peoples must have found it desirable to take note of the configurations and movements of the stars and the Sun and Moon, the two latter objects being obviously very closely connected with the subsistence and wellbeing of the human race. And the stars were found to be guides for the traveller by land or sea, while the fixing of the changes of the seasons, so essential to the husbandman, was at a later date found to be possible by means of the times of their rising and setting throughout the year.

It has been well remarked that the principle of cause and effect is not more striking in any natural sphere than in Astronomy. Mankind had to relate all activities to the daily alterations of light and darkness, and to the daily and seasonal alterations of warmth and cold, and soon saw that these were due to the varying power of the Sun; while marked variation of night illumination, and the tides in the sea, followed the changes of the Moon.

ASTROLOGICAL IDEAS

Some encouragement for the study of Astronomy resulted also from an idea, which seems strange to most modern minds, that the configurations and movements of the celestial bodies affected human destinies and were "signs in the heavens" from which future events could be foretold. If a comet

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appeared or an eclipse occurred at the time of the death of a great king or national leader, or when, simultaneously with a remarkable celestial event, some plague, flood or drought afflicted the people, these sky and earth happenings were apt to be regarded as causes and effects.

And it was thought that as the annual course of the Sun in the sky produced so great results on the Earth's animal and vegetable products, the planets in their movements among the stars and their resulting configurations with each other in the heavens, and other sky occurrences, might well influence the events of human lives. As a consequence the appearance of comets, new stars, meteor showers, eclipses, remarkable conjunctions of the planets which take place when they are near each other in passing on their paths in the sky, and celestial phenomena generally, were noted by the more contemplative members of human society, and carefully recorded along with events.

Nevertheless the statement which has been sometimes made that the origin of Astronomy is to be found in astrology is far from correct. Historical evidence is lacking for the assertion ; all well ascertained facts are in contradiction to it, and show that what really directed the early stages of Astronomy were the problems of the determination of the seasons and the calendar. It is still one of the most difficult questions in the history of ancient Astronomy to uncover the real roots of astrology and establish their relation to Astronomy. Very little has yet been done in this direction, mainly because of the prejudice in favour of accepting without question the priority of astrology.[1]

It may as well at this point to explain what is comprised in ancient astrology. Nowadays astrology is generally taken to mean the prediction of the fate of a person, supposed to be determined by the configuration of the planets, Sun and Moon, at the moment of his birth. This form is comparatively recent, and was preceded by a more general kind of prediction, frequently called "judicial" astrology in contrast to the "horoscopic" astrology for the individual. In judicial astrology the celestial phenomena were used to predict the immediate future of a country or its government. From planetary configurations, eclipses, lunar haloes, etc., the

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conditions of a coming harvest, flood, storm and so on were predicted; but nothing like a "horoscope" of an individual was involved. In other words, it was an elaborate "omen" prognostication. Horoscopes were not possible until the invention and use of methods for determining the positions in the sky of celestial bodies. However, even if the earlier astronomers and astrologers derived "laws" of cause and effect which seem absurd to a modern mind, we should try to realize the different mental attitude to natural phenomena of these first workers, and be thankful for their strange notions which, although crude, were instrumental in laying the foundations of observation and deduction. That these early astrologers may have had some idea of scientific method is suggested by the report that the Babylonians tried to check up on "horoscopes" with the recorded events in the lives of individuals over a long period of time!

It is true to say that the astronomers of later days were much indebted in their investigations to the observations made and recorded by these primitive astrologer-astronomers, but the astrological predictions themselves depended on purely astronomical work of still earlier observers. These records have been useful from the time of the first star catalogue compilers (Hipparchus, about 150 B.C.) and Ptolemy (about 140 A.D.), up to the present century in such investigations as those of Cowell on the lunar orbit, from records of Solar eclipses in 1062 B.C. and 762 B.C. found on Chaldaean baked bricks, and the tracing back of the recorded appearance of Halley's comet to 240 B.C. from Chinese observations.

THE EARLIEST DISCOVERIES

The order of discovery of some of the fundamental facts of the science may be briefly sketched as follows. The first phenomena to be noted would be the regularly recurring dawn, sunrise, daylight, sunset, twilight and night. Next to the measurement of a day thus provided, the month would be instituted as related to the variation of light with the Moon's phases. In the temperate regions, where probably the first astronomical observations were systematically made, the changing length of the day, or the direction of the Sun at

rising or setting, or the lengths of shadows cast at midday, would show that the Sun's daily path in the sky altered throughout the year, a time interval which was already marked by the changing vegetation. According to Sir W. C. Dampier "attempts were made to determine the number of months in the cycle of the seasons in Babylonia about 4000 B.C. and in China soon after. About 2000 B.C. the Babylonian year settled down to one of 360 days or twelve months, the necessary adjustments being made from time to time by the interposition of extra months."[2]

But the discovery of a more precise length of the seasons and of the year was the work of a more intelligent and specialized class of men than the primitive husbandman or herdsman. This class was the priesthood or its equivalent, considerable authority being thus acquired over the ordinary tribesman. Before long it would be found that there is not an exact number of months in the year or days in the month, and it would become necessary to make careful night observations of the Moon and stars to obtain greater accuracy.

The suitability of the stars for measurement of dates would not be immediately obvious to these earliest observers. A considerable period might well elapse before it would be noticed that at a particular season of the year the same stars are seen at corresponding hours of the night, this circumstance being less conspicuous than the regular variation of the Sun's altitude in the sky as the year progresses. It is, however, thought probable that the striking naked-eye cluster, the Pleiades, which must have been one of the earliest noted star groups, was the object which provided the first fairly close determination of the length of the year as approximately 365 days.[3] The rising of this cluster in the evening was a mark of the coming of winter to primitive man; and the husbandman judged the time of reaping by its rising, and of ploughing by its setting in very ancient times; Sirius, Arcturus, the Hyades, and Orion were similarly useful to him.

THE ZODIAC

It was eventually noted that the Sun and Moon travel over very similar paths among the stars during their circuit

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of the sky. This led to the formation of the Zodiac and its constellations, the centre of this zone, a belt about 16° broad, being the annual path of the Sun, or Ecliptic. The division into twelve parts, each corresponding to a month of the Sun's movement, was made; and their connection with the solar course during the year was found by observations of *heliacal risings* or *settings*. These were the times of the year when certain bright stars would first be seen to rise before the Sun, or when they were last seen to set after sunset. In the case of Sirius, the brightest fixed star, these would happen when the Sun was about ten degrees below the horizon. For less bright stars the angle would be a larger one.

But it seems likely that before this division had been effected, it would be found that the Moon in going like the Sun round the heavens always in the same direction from west to east (*i.e.*, opposite to the diurnal motion which she shares with the other bodies), kept in general to the same track in the sky. After a time, however, it would be noted by careful observers that this path was not constant, but deviated from the centre line of the Zodiac, getting away from that line up to a maximum deviation on either side, but slowly returning to it. In the course of a number of years it would thus become evident that the Moon's path among the stars does not lie always in the same line on the celestial sphere, but in a zone or band about twenty moon breadths (10°) wide, occupying the middle of the Zodiacal zone itself.

Among the brighter stars, Mercury, Venus, Mars, Jupiter and Saturn (the first two of which are never seen very far from the Sun in the sky) would soon be noted to be moving in the Zodiac with varying periods. The name of planet (from the Greek, *planetes*, a wanderer) was later given to them because of their changing positions among the Zodiacal stars.

THE FORMATION OF THE CONSTELLATIONS

But long before the Zodiacal belt was divided into "Signs," which according to some authorities took place about 700 B.C. when the intersection of the Ecliptic with the celestial Equator was in the constellation of Aries (the Ram), a number of asterisms, or configuration of stars in the sky, had been

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arranged. The brighter stars of these configurations, thus identified, proved very useful in indicating the seasons of the year by their times of rising or setting and also in locating the positions on the celestial vault of such moving objects as planets, comets and shooting stars, and in helping the traveller by land or sea to determine direction.

These named constellations date back to a very early period except for some (chiefly situated in southern skies) which have been added in modern times. In many, the stars form a well-marked group, clearly separated from other groups, and the names given to these formations are supposed to have been suggested by a resemblance to the shapes of certain familiar objects. The resemblance is usually very slight, and this may justify the suspicion that often some fancied figure was first thought of, and then the stars chosen to represent it in a very rough fashion.

The similarity of the constellations as early recognized in different countries is remarkably great; and this points to a common origin for them. The late Dr. A. C. D. Crommelin considered that there is reason to believe that the stars may have been grouped to some extent by the Egyptians as early as 4000 B.C.; and he remarked on their use of the then Pole Star for orienting the Great Pyramid (see p. 10). And if their ancient records can be relied upon the Chinese can be taken to have mapped out the sky into many divisions of stars by 2500 B.C.; on a system very different, however, from that devised by the early astronomers elsewhere.

But it has been shown that the constellations are, most of them, perhaps even older than that date. Forty-eight of them have come down from extremely ancient times, but these do not cover the whole extent of the sky. The part not occupied by any of them evidently did not rise above the horizon where the early astronomers to whom we owe their naming lived; and the stars concerned were therefore not included in their constellation schemes. The centre of this part (near the bright star Achernar) must have been near the South Pole of the heavens of the time, and its angular radius from the pole gives us roughly the latitude of their homes. The date appears to have been about 2800 B.C., when, owing to the precession of the equinoxes, the South celestial pole

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was in the position indicated. The latitude seems to have been about 38° North. These are the findings of E. W. Maunder,[4] but by the same method the somewhat later date of 2460 B.C. and a latitude of 36° North have been deduced by Dr. Crommelin, and R. A. Proctor found 2200 B.C. Maunder also suggested that the presence of the Lion and the Bear among the stellar configurations and the absence of the Elephant, Tiger, Camel and Crocodile, seem to exclude India towards the East and the countries towards the West, the latitude and longitude indicated being those of Asia Minor or Armenia. The suggestion that the blank area in the sky referred to gave an approximate date for the formation of the constellations appears to have been first put forward in 1807 by Carl Schwartz, for some time Swedish Consul at Baku.

MOVEMENTS OF SUN, MOON AND PLANETS

The fact that some of the celestial objects observed are closer to the earth than others must have shown itself to careful observers at a comparatively early date by the occurrence of eclipses; and by occultations, which are caused by the passage of the Moon between us and a planet or a fixed star. From this the Moon was noted to be nearer to us than the other heavenly bodies.

As an indication of the nearness of the planets and the Sun and Moon the rapidity of their motions with respect to the fixed stars was taken. The three slower moving planets, Saturn, Jupiter and Mars, were thus considered to be beyond the Sun, and were called the *superior planets*. The other two planets, Venus and Mercury, which appear to accompany the Sun in its annual path in the sky, never very far from it, were termed the *inferior planets*.

Eclipses of the Sun and of the Moon must have roused great interest, and often alarm, from the earliest times. That the former always happened at New Moon and the latter at Full Moon would soon be noticed, and cycles of their occurrence would be noted after the accumulation of numerous records of dates of observation.

When we try to estimate the value of the work of astronomers of more than 3000 years ago, the mental effort necessary by them is not easy for us to appreciate; it is very difficult

to put our present knowledge on one side and grasp the originality of thought required in making the first steps. The sky appeared to be a hollow vault or hemisphere overhead on which the stars were fastened, the movement among them of the Moon and planets being noted. It was a considerable step forward to realize that this vault also carried the Sun and very difficult to realize that the stars are really shining during the day-time as they do at night, even with the assistance provided for such an idea by the notable obliteration of all but the brighter stars by the light of the Full Moon. To acquire the idea that the great luminary, the Sun, is the same body at sunrise as it was at sunset in the west the evening before, having returned to the east without being visible in the interval, was a mental feat of no small magnitude. And perhaps one of the greatest advances made in early astronomical theory was the placing of the Sun, Moon, and planets at different distances from the Earth instead of all at the same distance affixed to the celestial sphere.

The further great steps involved in the attribution of a spherical shape to the Earth, and in explaining, by its rotation, the diurnal motions of celestial bodies in rising in the east and traversing the sky to setting in the west, were of much later date. The very early observers thought of the Earth as a circular plane over which the celestial vault extended, the hemispherical outline defining the extremities of the Earth in all directions. The Sun, and the Moon, and stars appeared to move on, or with, the inner surface of this vault; and as the ocean was supposed to flow in a stream round the outer margin of the plane of the Earth, the heavenly bodies were believed to emerge from the ocean when they rose, and to sink into it when they set, with the exception of those "circumpolar" objects which were near enough to the celestial pole to avoid this immersion. This infantine astronomy did not explain how the Sun found his way from the west after his course daily through the sky, back to the east and the region of dawn. Other views seem to have been, however, that the Sun did not pass under the Earth but that it travelled after its setting round north to the east, night being caused by an elevation of the northern part of the Earth which shut out the light of the Sun.

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Nevertheless, in spite of these crude notions (which are perhaps no cruder than those of many ordinary present-day citizens) there was undoubtedly a larger proportion of early mankind aware of the apparent movements of the heavenly bodies than there is nowadays in civilized countries.

It has indeed been remarked that as civilization progresses this proportion becomes less and less. This is because artificial methods of indicating time come more generally into use, and because fewer persons in proportion are engaged out of doors at night under conditions which make the movements of the heavens worth observing. In this connection it is interesting to note that during the "blackout" of the recent war the increase in numbers of people taking an interest in Astronomy (and joining Astronomical Associations) was very marked.

ORIENTATION OF ANCIENT MONUMENTS

The orientation of monuments of great age like Stonehenge and the avenues of Standing Stones at Carnac in Brittany, as well as of many other large megalithic monuments, is believed by most archaeologists to show a knowledge of the Sun's position in the heavens with regard to the seasons, which could only have been acquired by careful astronomical observation. At Stonehenge the Sun was seen at the summer solstice to rise over a stone known as the "Friar's Heel," which stands on the common axis of the two circles and two horse-shoes of standing stones, when viewed through the aperture of the arch formed by the two upright stones and the capping stone of the "Great Trilithon." At Carnac there are three huge avenues of standing stones. One of these avenues faces the east (*i.e.*, the point of sunrise at the equinoxes) while the other two face the north-east (the point of sunrise at midsummer—the summer solstice). Approximate dates for the setting up of such stones have been worked out allowing for the alteration in the points of the sunrise caused by change in the Obliquity of the Ecliptic (the angle between the plane of the Earth's equator and the plane of the Earth's orbit round the Sun). For Stonehenge the date is found to be about 1800 B.C., although archaeological research indicates

that this monument is not all of one period but has probably been gradually built up over hundreds of years after that date. That time is of course contemporary with more advanced astronomical observation in countries of a higher degree of civilization such as Babylonia and Egypt. In Egypt the orientation of the Great Pyramid, and the angle, 26° to the horizontal, of its downward inclined interior gallery, which seems to have been directed to the pole star of the time (about 2900 B.C.), α Draconis, showed skilled application of some astronomical knowledge.[5] If it is assumed that the Egyptians did not know how to correct for refraction (see p. 39), and this is almost certainly the case, then it looks as if the centre of the Great Pyramid may have been meant to be exactly on the thirtieth parallel of latitude; while the sides are so accurately in alignment for North and South and East and West that the average error is only about a twentieth of a degree. And some temples in Egypt were certainly constructed so as to point in the direction of the Sun when rising, either at the solstice or at the equinoxes. The Karnak temples in Egypt were so built that when the Sun was setting at the time of the summer solstice its light shone right through the entire length to illuminate a sanctuary wall, while the Temple of Heaven in Pekin was oriented to midwinter sunrise.

At this point it seems well to remark that the history of ancient science and even the general histories of early civilizations such as China, Egypt, Mesopotamia, Greece and other countries are in many respects not yet too well founded. In calling for more intensive research work into the original writings, much of which have still to be thoroughly studied, an eminent authority has remarked that in many instances scientific history is "completely antiquated and at best enjoyable for its literary style; 'the careful study of the original works of the ancients, however, will reveal to everyone and at any time the development of their achievements."[6]

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References

[1] Neugebauer, *Publication of the Astronomical Society of the Pacific*, vol. 58, pp. 18, 19 (1946).

[2] W. C. Dampier, "A History of Science," p. 3 (1946).

[3] Whewell, "History of the Inductive Sciences," pp. 95-96 (1857).

[4] E. W. Maunder, "Astronomy Without a Telescope," p. 5 (1902).

[5] Edwards, "The Pyramids of Egypt," Chap. VI. (1947). Similarly inclined galleries, all pointing accurately northwards, were features of the Pyramids built during the next thousand years ; after that time the entrance was placed elsewhere in an effort to puzzle tomb robbers.

[6] Neugebauer, *loc cit*, p. 130.

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CHAPTER II

CHINA. EGYPT. MESOPOTAMIA. INDIA

CHINESE ASTRONOMY

A GREAT part of what is known of the work of early astronomers in China was obtained by Jesuit priests who lived in Pekin during the seventeenth century. It appears from their accounts that celestial phenomena were observed and recorded in that country more than four thousand years ago. But it has been suspected by some historians of the science that the Jesuits may have been, in some instances at any rate, deliberately misled by false records, made by their Chinese friends calculating backwards (perhaps with the training given them by the Jesuits); or through falsification of the real occurrences, presented in apparently authentic documents the accuracy of which could not be challenged by men to whom the whole system of Chinese written characters was completely new.

Be that as it may, there are a number of more or less authentic items which are of interest to the student of ancient astronomy. One of the earliest of these refers to the making of a sphere, to represent the motions of the celestial bodies, by Yü Shih at a date shortly after the reign of the emperor Fuh Hsi about 2950 B.C., and a similar sphere is recorded as having been constructed about six hundred years later. A great observatory is said to have been built in the reign of Huang Ti (about 2650 B.C.) and a reformation of the calendar is also attributed to that time, the official astronomers having been ordered to observe the movements of the Sun, Moon and planets for the purpose.

It is also stated that the Chinese records tell of a conjunction of Mercury, Venus, Mars, Jupiter and Saturn during the reign of the emperor Chaun Hsü (2513 to 2436 B.C.); and modern calculations have shown that on February 28th, 2446 B.C., these planets were all visible close together in the sky. Another calculation gives the date for the conjunction as February 29th, 2449 B.C. There seems, therefore, to be some chance

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that this particular record is correct. On the other hand, H. Chatley states that it is "generally considered to be a late invention. Like most conjunctions which include Mercury, it could hardly have been visible on account of proximity to the Sun."[1]

By some historians it has been thought that about this time, or perhaps even some time before it, the Chinese may have discovered the cycle commonly attributed to the Greek astronomer Meton (about 433 B.C.). In this cycle of 19 years of $365\frac{1}{4}$ days, to which 235 lunar synodic months of 29·53 days, the average time between successive New or Full Moons, are very nearly equal, the phases of the Moon recur on the same days of the synodic month; it is still used in the ecclesiastical calendar for finding the date of Easter.

The emperor Yao (about 2360 B.C.) is stated to have instructed his astronomers to make observations by which the positions of the solstices and the equinoxes could be determined. In 2285 B.C. he was presented with a report giving the names of the stars in the parts of the sky where the Sun was at these times in the year, and he then gave them further orders. If this is accurate history it means that the Chinese astronomers of forty-two centuries ago could determine (probably by heliacal risings and settings) the position of the Sun in the sky throughout the year, and that they were aware that the stars are shining in the daytime though not visible because of the light of the Sun. It is also said that the length of the year was established at this time at the closely correct value of $365\frac{1}{4}$ days. This is supported by the fact that the Chinese are known to have divided the circle into 365 divisions or degrees at a rather early date. It is also stated in the records that between 2255 B.C. and 2200 B.C., in the reign of Yao's successor Yü Shun, observations were made of the positions of Mercury, Venus, Mars, Jupiter, Saturn and the Sun and Moon by means of instruments, one of which was a quadrant with a one inch diameter tube seven or eight feet long attached to it.

The year 2159 B.C. is given as the one in which the royal astronomers Hi and Ho (or the two astronomical families of these names responsible to the emperor), failed to predict an eclipse of the Sun. The confusion and terror caused by the

absence of the warning which should have been provided by the official prediction was so great that those responsible were executed.

If this is a true story it follows that a method of calculating the occurrence of eclipses was known as early as in the twenty-second century B.C. The account seems to be of doubtful authenticity, however. The late Dr. J. K. Fotheringham, a high authority on ancient Astronomy, considered that the fault of Hi and Ho was not failure to predict an eclipse but great carelessness in production of the official calendar probably "involving a false reckoning of the day of conjunction of the Sun and Moon. No astronomer in primitive China can have been expected to predict eclipses. Eclipses were not regarded as happening according to natural law. They were supposed to be disturbances of natural law, produced through the kinship of Earth and Heaven by the errors of the people, and more particularly of the emperor."[2]

It is not to be doubted, nevertheless, that some careful observational work must have been carried out by the Chinese several thousand years ago. This work, like that done in other ancient civilizations, consisted only in the art of observing the returns and measuring the places of the heavenly bodies with reference to the stars or to the Earth's horizon. The next step was made by the Greeks who added a complex geometrical plan of movements which was accepted as the true explanation until the Copernican system was advanced hundreds of years later.

One result claimed for early Chinese observational work was a very accurate determination of the Obliquity of the Ecliptic in the reign of Chou Kung about 1100 B.C. The angle found was within a few minutes of arc of the truth. This close agreement is particularly remarkable when it is borne in mind that the instrument used is said to have been a vertical gnomon* of only 8 feet in height which is small for an ancient instrument of the sort.

Chinese observations of comets (372 of which have been found in the records between 611 B.C. and 1621 A.D.) have

*A vertical shaft or column erected on a horizontal plane; the measurement of the length of the shadows cast by it provided the means of calculating angles of elevation of the Sun.

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been useful in modern times for identification of comets which return to the Sun's vicinity in a periodic orbit. In the case of Halley's Comet it is thus known that it was certainly observed in 240 B.C., and probably also in 467 B.C. (its average period is about 76 years); and it was seen by the Chinese astronomers at a number of its returns since these dates, when no European record is available. One comet, seen by them in 134 B.C., appears to have been the reason for the formation of the first star catalogue by the Greek astronomer Hipparchus. Until recently it was thought that the phenomenon leading to Hipparchus's great work was a Nova or Temporary Star, but research by Dr. J. K. Fotheringham has shown that it was a comet. There seems, however, to have been an even earlier catalogue, made by the Chinese astronomer Shih Shen in 400-300 B.C., containing 800 stars, with positions related to the principal stars of the 28 divisions into which the sky had been divided from a very remote date.

The early Chinese astronomers were evidently aware of the fact that the tails of comets point away from their heads in a direction opposite to that of the Sun. In the collection of Chinese observations made by Ma Tuan Lin the following is found: "In general, in a comet east of the Sun, the tail, reckoning from the nucleus, is directed to the east; but if the comet appears to the west of the Sun, the tail is turned towards the west." As these Chinese records usually give the constellations of the sky through which the comet was seen to pass, they are of particular value for the purposes of identification referred to.

Another celestial phenomenon, about which information is found in the Chinese annals, is that of showers of shooting stars; the records date back to several hundred years B.C. Sunspots visible to the naked eye were also recorded. (See p. 139).

The accounts of Novae or Temporary stars are also of some potential value. These objects are referred to as "K'o-hsing" or "guest-stars." They are distinguished from the moving comets by their stationary position with reference to surrounding stars, and by the records of their appearance and disappearance. Their brightnesses compared with the planets or fixed stars and a progressive account of their fading,

are sometimes given. Within the last year or two, investigations based on Chinese observations of a Nova seen in 1054 A.D. have shown that it was one of the brighter type of Temporary star ;* and that a well-known nebula in the constellation Taurus is its relic, left after an outburst the effects of which were seen 893 years ago, which is observed to be still expanding. The details of the Chinese (and Japanese) observations are of great interest ;[3] and the investigation leads to the wish that similar ancient records could be found available for work which might help in establishment of connection between Temporary Stars of the past with some of the nebulous objects in the sky.

The foregoing brief account of ancient Chinese Astronomy contain items such as are found in works on the history of Astronomy ; and among them some are of very doubtful authenticity. It may be as well, therefore, to give a few points which recent research seem to indicate as being fairly well established. These are : Careful study of the older records shows that a considerable part date back to 400 B.C., some to about 800 B.C., a little to probably 1000 B.C., and a few items to 1500 B.C. or even earlier. As far back as 1000 B.C. the Chinese believed in a reciprocal action between Heaven and Earth and that the behaviour of man caused, and was caused by, the phenomena of the sky, this leading to a system of Judicial Astrology (see p. 2). The whole of the visible sky was mapped into 284 small constellations each with about 5 stars. A broad equatorial band wide enough to include the paths of the Sun and Moon was also divided into 28 unequal parts or "mansions." The first measurements in degrees ($365\frac{1}{4}$ to the circle) were probably made about 350 B.C. by Shih Shen ; he also found relative positions for about 800 stars in about 120 groups, and may thus be said to have forestalled the Greek astronomer Hipparchus (see p.34). Precession of the equinoxes was not known until about 450 A.D. ; about 400 B.C. the position of the winter solstice was observed to be near β Capricorni. Solar eclipses were not forecast before about A.D. 25. Prior to 100 B.C. planetary

*There are two types of Novae : ordinary, about 50,000 times as bright as the Sun ; and Supernovae, which may outshine the Sun about 10,000,000 or 100,000,000 times. (See pp. 278-292).

motions on the sky were very imperfectly known; Jupiter and Saturn had been given periods of 12 and 28 years respectively. A fairly full account of comets, eclipses, shooting stars and meteorites (fallen meteors) was kept from about 700 B.C., but there is nothing very reliable in the records before that date.[4]

EGYPTIAN ASTRONOMY

According to an eminent authority it is much easier to show that certain familiar current ideas about the origin of Astronomy are unsound than to give a satisfactory account of our knowledge of Egyptian Astronomy.[5] In contrast with the records of Chinese and Mesopotamian Astronomy in which many accounts of observations have been found, no Egyptian record of an observation has so far been discovered in spite of the enormous amount of archaeological work. Many observations for ceremonial date fixing and orientation of monuments were no doubt made, but not recorded.

Most of our knowledge has been derived from inscriptions and representations on the ceilings of tombs and mortuary chapels, and some inscriptions on coffin lids which apparently represent the rising and setting of star groups. The Egyptians divided a zone of the sky into 36 small star groups or "dekans" which by their heliacal risings marked the beginning of each 10-day period. They also had certain larger groups elsewhere in the sky, for example, one they called the Thigh or Ox-leg (our Plough).

The astronomers among the ancient Egyptians seem to have been satisfied with rough qualitative descriptions of phenomena rather than close measurements. Their mathematics was never at a very high level; and when the complicated nature of phenomena required more advanced methods of calculation than those possessed by the Egyptian, simplifications were adopted, a procedure which of course could not give more than qualitative results.

It appears to be very probable, however, that they had a value for the Obliquity of the Ecliptic before the Chinese measurement of 1100 B.C. described previously in this chapter, and they knew that the length of the year is about 365¼ days.

They were the first to adopt the year as the standard time measurement instead of the month, and they employed 365 days for it. They have also been credited with a belief that Venus and Mercury revolve round the Sun.

Their priesthood were the observers ; they alone possessed the acquired knowledge of the subject. This privileged class performed the important and valuable function of predictors for the Nile floods which irrigated and fertilized large parts of the country. These floods occurred at about the summer solstice and the orientation of the temple referred to in Chapter I would provide the necessary information.

The priests of Thebes claimed to be the originators of exact astronomical observation, attributing this partly to the clear skies of the country.[6] At a very early date they determined the periods of the planets round their circuits of the sky, but their assertion of a general priority in exact observation does not seem to have been proved. In support, however, it may be said that some authorities believe that the water-clock, gnomon and sundial were first introduced in Egypt (see p. 21). Portable instruments for observing transits of stars across the meridian, for orientation of monuments and probably for time measurement, certainly existed at and before the time of Tutankhamon (about 1355 B.C.).

That Egyptian Astronomy was of great antiquity, however, may be admitted in view of the orientation to the Sun of monuments such as the Pyramids and the temples. But the claim that certain temples are oriented to rising points of particular stars does not appear to be so certain, as some star or other may often be chosen which could have apparently done for the purpose at a remote date. On the other hand, it has been found by Sir Norman Lockyer that a group of temples sited close together appear to have been constructed at different times with varying orientations which would have suited the star α Columbae at three dates—2525, 1250 and 900 B.C., the changes in date being due to the alteration in the point of rising of the star caused by the precession of the equinoxes (see p. 34). The brightness of the star in question is not great—only about the third magnitude—and it hardly seems conspicuous enough to have been selected for the purpose, although it may have been the only one rising

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in the proper quarter at the particular time of the year. Doubt has been thrown on such methods of orientation or date-fixing, by Neugebauer who remarks[7] that no stars except Sirius and Canopus, the two brightest fixed stars, can properly be seen until they are well above the horizon.

It is also thought by some that at the temple at Luxor, originally built at a very early date with an addition to it built much later, there is a corresponding difference in the orientation of the old and new parts.

The Egyptian priests tried to keep strictly to themselves all the astronomical knowledge acquired, and this may partly account for the comparative lack of any astronomical information from the work of Egyptologists. One discovery is attributed to them—that of an arbitrary and empirical method of predicting eclipses—which could be of great value for impressing the other inhabitants of the country. It has even been thought that the earliest Egyptian calendar based on the heliacal rising of Sothis (the Dog Star, Sirius) shows the existance of activities nearly as far back as 4000 B.C.; but this and the idea that Astronomy in Egypt was influenced by that of Babylonia are thought by some to be not based on any solid foundation.[8]

MESOPOTAMIAN ASTRONOMY

The astronomy of the ancient Babylonians provides a contrast to that of the Egyptians. From the collected observations of a long period of time the Babylonians developed methods for computation of lunar and planetary movements in the sky which may be classed among the greatest achievements of ancient science, comparable with the work of the Greeks. Systematic observational work, dating from about 700 B.C. onwards, gave two important results : correct average values for the chief periods of the phenomena, which could be still further corrected from older observations, and ability to make good short-range predictions from these values when applied to a recorded individual observation of a particular astronomical event. After some further development a new idea appears to have been conceived which led to systematic long-range prediction of greater accuracy than before. This

consisted in treating any complicated periodic phenomenon as the result of a number of simpler periodic effects. For example, correct times of New Moons would be easy to find if the Sun and Moon each moved with constant speeds in the sky; this could be assumed to be the case in the first place, and average values of speed used which gave average positions for the New Moon. The actual movement departs on one side or the other from the average and these periodic departures could be treated as new phenomena. Additional deviations are caused by the inclination of the Moon's orbit and a separate treatment can be applied for them based on the same idea.

Thus by starting with average positions the corrections necessary for the periodic deviations from the average could be applied with closer approximatures to the truth than ever before obtained. Evidence seems to indicate that these new methods were possibly the work of a single person in about the third or fourth century B.C.

It may be remarked that the planetary theory developed was not of the same degree of refinement as the lunar one, the reason for this being evidently the great importance of the latter in the Babylonian calendar.

These methods were possible because of the advance in Babylonian mathematics, which were much superior to those of Egypt. It is to the Babylonians that we owe the "sexagesimal" system that uses 60 as a base, the division of the circle into 360 degrees of 60 minutes or 3600 seconds, and also "place-value" notation.* The importance of this notation in mathematics is obvious, and as an invention it has been compared to the alphabet in value.

The superior accuracy of prediction of astronomical events was not, however, the result of any theories of the true movements of the planets, Sun, or Moon in space. The predictions were, as will have been noted, founded on the skilful use of records of past observations on an empirical basis with the help of improved mathematical methods.

The observations of eclipses by the inhabitants of the

*This consists in the use of a very limited number of symbols whose magnitudes are determined by position. By it, 21 does not mean 2 plus 1 (as in Egyptian or Roman numbers), but 2 times 10 plus 1. In the sexagesimal system 2, 1 means 2 times 60 plus 1, *i.e.*, 121.

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Mesopotamian countries over many centuries led to the discovery, made at least several centuries B.C., that eclipses recur after a period, known as the "Saros," consisting of 6585 days, or 18 of our years plus 10 or 11 days according as four or five leap-years are included. This is very nearly equal to 223 synodic months. It usually contains about 71 solar eclipses (this number varying 2 or 3 one way or the other), about 45 of which are total or annular and visible somewhere on the earth. It is much more probable that the discovery was made by study of the recorded dates of eclipses than by any process of calculation using a knowledge of the motions of the Sun and Moon. As solar eclipses, recurring at an interval of 18 years, are not generally visible at the same place, it is not clear that the Saros was taken to apply to Solar as well as Lunar eclipses.

The records of eclipses made in Mesopotamia have been put to good use in later times. For instance Ptolemy (about 140 A.D.) employed three, of the years 721 and 720 B.C., to improve his tables for prediction of the movements of the Sun and Moon; and in recent years they have been found useful in determination of a slight lengthening of the Earth's period of rotation, which has been found to be at the rate of about 1/1000th of a second per century. This has been ascertained through the comparison of ancient with modern eclipses by the late Dr. J. K. Fotheringham. The principal cause of the slowing up is the friction of the tides on the solid body of the Earth, most effective in shallow confined seas or bays.

The astronomers of ancient Mesopotamia are credited with the invention of the clepsydra or water clock. The Babylonians probably had this clock as early as the seventh century B.C. It consisted of one vessel kept full of water that was allowed to escape through a small hole in the bottom into a receptacle, the rise of level in the latter showing the passage of time by a pointer, on a float, directed to a graduated scale. It is said that in one instrument the amount which was allowed to pass in a day and night corresponded to about six drops per second.

The gnomon and the sundial appears also to have been invented by the Babylonians, both of them instruments of

the greatest value before the later improvements of graduated circles and mechanical clocks. The uncertainty of the real dates of these inventions, however, is such that some authorities consider that the water-clock in a rough form, the gnomon, and the sundial, were in use in Egypt at an even earlier period (see p. 18).

It was probably in the Mesopotamian region of the Earth that many of the constellations and the Zodiac itself were first mapped out. Fragments of the baked bricks of libraries have been found showing constellations named as they are at the present day, and the signs of the Zodiac were used for description of the journeys across the sky of the Sun, Moon, and planets.

The oldest preserved documents of Mesopotamia are clay tablets, showing on them three concentric circles, divided by twelve radii into twelve sectors. This gives 36 spaces; in each of these is found the name of a constellation, and a number the significance of which is not yet known. It has been suggested, however, that the tablets represent three regions in the sky each with twelve parts, the number being connected with some calendar scheme.

The nations of Mesopotamia were probably the first to develop astrology systematically. No doubt this provided some stimulus to a further study of Astronomy by the observation of the phenomena of the sky and of the relative movements of the Sun, Moon, and planets, leading to the acquisition of an ability to predict movements with increasing accuracy, as described earlier in this chapter.

HINDU ASTRONOMY

It is possible to give only a brief general sketch of the Astronomy of the Hindus, and this is probably not much to be regretted, as a comprehensive and accurate account is not at present practicable. There is a very large field awaiting systematic study with an extensive literature to be examined. Considerable controversy has taken place as to the origin and antiquity of what has been discovered and dealt with. One great trouble appears to be the tendency of the majority of Hindu students of the subject to claim great antiquity and priority for Hindu discoveries, and to deny foreign influence,

and the opposite tendency of the bulk of European investigators. This has been particularly the case with Hindu mathematics, but the position as regards Astronomy is not much better.

The "Vedas" are sacred works containing astronomical references. The date at which these books were written is not yet definitely settled, some authorities favouring 1200 B.C., while others consider them to be much older. The Sun and Moon are referred to, but there does not appear to be any clear reference to the planets. In other Hindu writings of later date a few constellations and bright stars are recognized, such as Ursa Major, Orion, and Sirius, eclipses being also mentioned.

Astronomy does not seem to have been cultivated to any great extent in earlier times before the invasion of India by Alexander the Great. But some knowledge was acquired of the periods of the Sun and Moon and the planet Jupiter (Vrihaspatis) for purposes of chronology the lunar motion being especially connected with the times of sacrificial ceremonies. No accurate knowledge of the movements of the planets seems to have existed before about 200 to 300 A.D. From that time Astronomy, which had previously been dealt with in poetical works, began to be treated as a science.

This was in the "Siddhântas," a series of which appeared in the following thousand years, all strongly influenced by Greek ideas. In one of them, an astonishingly accurate diameter is given for the Earth, 1600 *yojans* of 4·9 miles, or 7840 miles; and 51,570 of the same Hindu unit, or 253,000 miles for the distance of the Moon. Rules for calculating positions of the heavenly bodies are found and a knowledge of the differences between a sidereal and solar day and a sidereal and solar year is shown in others.

The evidence all indicates that from about 400 A.D., the Greek system of Astronomy was adopted by the Hindus, their astronomers having therefore absorbed Greek views although they were not accepted in European countries until much later. It may be that the time of introduction of the Greek ideas was much earlier, perhaps between the period of Hipparchus (about 150 B.C.) and Ptolemy (about 140 A.D.). It has in fact been suggested that close investigation of Hindu sources of the time might provide information about pre-

Ptolemaic Greek Astronomy, not available in the surviving Greek writings, that would give missing stages between the Babylonian Astronomy and the developed Greek systems.

Hindu Astronomy has been described as a curious mixture made up of old fantastic ideas of the influence of deities on planetary movements, and sober geometrical methods of calculation. Its worth to the progress of the science has been well summed up as follows: "We owe very little to Hindu Astronomy as far as direct influence is concerned. It has been pointed out that indirectly we are indebted to the Hindus because they imparted certain astronomical information to the Arabs, who, in turn, passed it on to Europe, and that the Hindus received their astronomical knowledge from the Greeks."[9]

References

[1] *Journal of the British Astronomical Association*, vol. 48, p. 167 (1938).

[2] *Ibid*, vol. 45, p. 252 (1933).

[3] *Publications of the Astronomical Society of the Pacific*, vol. 54, p. 95 (1942).

[4] H. Chatley, "Ancient Chinese Astronomy," *Occasional Notes, No.* 5, *Royal Astronomical Society*, 1939.

[5] Neugebauer, *Publications of the Astronomical Society of the Pacific*, vol. 45, p. 25 (1946).

[6] G. C. Lewis, "The Astronomy of the Ancients," p. 256 (1862).

[7] "Astronomische Chronologie" (1929).

[8] Neugebauer, *Publications of the Astronomical Society of the Pacific*, vol. 58, p. 24. On the other hand, the Egyptologist Breasted, considered the first fairly reliable date in history, 4241 B.C., to have been fixed by such observations of Sirius.

[9] M. Davidson, "The Stars and the Mind," p. 35 (1947).

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CHAPTER III

GREECE

FEW originals of writings from which the history of early Greek Astronomy could be obtained have been preserved. Even the old manuscripts dealing with the work of the first Greek astronomers were usually written centuries after these workers lived. One feature of the oldest Greek Astronomy is, however, undoubted; its strong resemblance to Egyptian and Mesopotamian Astronomy.

The sudden appearance of a Greek mathematical Astronomy about the sixth or fifth century B.C., out of a primitive condition, has raised some speculation. So far there is little more than surmise as to the reason for this; but it has been suggested that the explanation may be an intensification of progress such as follows contact between several types of civilization, in this case Hellenic, Egyptian, Mesopotamian, and possibly Syrian; and that competition rather than the result of borrowing was responsible.

The earliest Greek scientific work was not done in the country that we now call Greece. It was the product of colonists, known as the Ionians, in Asia Minor, who occupied the coastal areas about Ephesus and Halicarnassus, a people very favourably situated for the reception of foreign ideas. To the east they were in contact with current Mesopotamian culture, and their sea traffic with Egypt brought knowledge from the ancient civilization there. In addition they had arrived at political conditions which favoured intellectual progress, the cities of the mother country being still occupied with constitutional struggles.

It may also be said that although the Egyptians and Mesopotamians had gone further than the ancient Greeks in mathematics and in astronomical observation, their studies had an almost entirely utilitarian object. The Ionians and later Greeks took up these matters in a more purely intellectual manner and built the foundations for a science of Astronomy.

THE IONIAN SCHOOL

Thales (624-545 B.C.) of Miletus near Ephesus, commonly called the founder of Greek Astronomy, seems to have been a capable business man who visited Mesopotamia and Egypt in the course of his commercial undertakings. He studied the science of these countries and is said to have learned of the Saros cycle (see p. 21) in them. It is stated that this enabled him to forecast the occurrence of an eclipse of the Sun. The date is not now certain but 585 B.C., May 28 has been given, the phenomenon taking place during a battle between the Lydians and the Medes. He could only have predicted the date of an eclipse, but not the exact places where it would be total, by means of the Saros; but such a prediction would naturally draw a great amount of attention and may have done a lot to encourage astronomical study. It is only proper to state, however, that some authorities strongly doubt whether the prediction was actually made at all!

Thales knew the length of the year, the inequality in length of the seasons, the positions in the sky of the Solstices and Equinoxes, and the signs of the Zodiac, most if not all of this information having been got from the Mesopotamian and Egyptian astronomers. He held the opinion that the Sun, Moon and stars are solid bodies and not merely shining spots on the sky, that the Moon shines by the reflected light of the Sun, that eclipses of the Moon are caused by the shadow of the Earth, and eclipses of the Sun by the passage of the Moon between it and the Earth. His knowledge of the true shape of the Earth as spherical appears to be in some doubt nevertheless; some writers state that he believed it to be flat and floating on water.

A pupil of his, Anaximander (611-547 B.C.) also of Miletus, introduced the gnomon and sundial from Babylonia. He was familiar with the Obliquity of the Ecliptic and believed that the Earth's shape was that of a cylinder with its height a third of its breadth. He appears to have been the first to speculate on the relative distances of the heavenly bodies; and he supposed the heavens to be of a fiery nature, spherical in form, enclosing the atmosphere "like the bark of a tree," this enclosure forming a number of layers between which the

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Sun, Moon and stars are situated at different distances. In this conception, however, he believed the Sun to be the most distant and the stars nearest to us. The Sun was a wheel or ring 27 or 28 times as great in diameter as the Earth; the hollow rim of the wheel was filled with fire which was only seen through a hole in the rim equal in size to the Earth. The same sort of explanation was advanced for the Moon and stars, and the lunar ring was 19 times the diameter of the Earth. The Moon was self-luminous; regular partial stoppages of the aperture in the ring caused the Moon's phases and occasional stoppages were responsible for solar and lunar eclipses. These ideas are difficult to comprehend, but they may have suffered in transmission to us through the later writings of others by which they became known. His greatest and most useful practical work was the making of charts of the Earth's surface, probably based on the Egyptian and Mesopotamian maps he had seen.

The third astronomer of the Ionian school was another native of Miletus, Anaximenes (550-475 B.C.). His ideas were equally primitive. He thought that the stars are attached like nails to a sky which is of solid and crystalline structure, and that the firmament turns round the Earth "like a hat which can be turned round the head." The Sun and stars do not go under the Earth after setting; they pass behind a northern elevated part of it. The Sun's heat is caused by the rapidity of its motion, but no heat is got from the stars because of their distance.

It will be evident that the Ionians did not progress much towards a rational idea of the universe. The Earth was flat; fixed stars were attached to the vault of the sky; and ideas of the Sun and Moon were very fanciful. Indeed their ideas seem to have retrograded after Thales himself.

Heraclitus (540-475 B.C.) of Ephesus had similarly fantastic notions. According to him the Sun is produced by ignition of moist exhalations from the Earth, caught in a hollow basin with its cavity facing downwards. This basin rises from the sea in the east and the fire it contains is extinguished at sunset in the west. The Moon is also a bowl of fire shining more dimly; eclipses are caused by the turning of the non-luminous

sides of the basins towards us. Other ideas of his seem equally strange to a modern mind.

THE PYTHAGOREANS

Pythagoras (582-500 B.C.) was originally an Ionian, born in the island of Samos. He travelled widely in the East and settled at Croton in Southern Italy where he founded a school or sect. He did not leave anything in writing, but is supposed to have held that the Earth, Moon, planets and fixed stars all revolve round the Sun which itself revolves round an imaginary central fire. The Pythagoreans seem to have been the first to maintain that the Earth and the heavenly bodies are spheres. They appear to have arrived at their conclusions from mystical views such as that the sphere is the perfect shape, circular motions the most perfect motions, ten (which $=1+2+3+4$) is the perfect number (number being the real substance of all things), and so on. Pythagoras was undoubtedly a great mathematician for his time. One opinion of his appears likely to have been based on observation, however; this was that Phosphorus and Hesperus, the morning and evening stars, are one and the same body (Venus). In the fifth century B.C., one of his disciples, Philolaus, put forward an interesting hypothesis. He abandoned the idea that the Earth is the centre of the universe, and suggested that it has the same kind of movement as the planets, and that they all revolve round a central fire. This fire is always invisible as that part of the Earth inhabited by man is continuously turned away from it. In order to balance his system, he invented a "counter-Earth" which made the number 10—Sun, Moon, Earth, five planets, counter-Earth, and central fire. This conception of a moving Earth influenced Copernicus nearly two thousand years later.

THE ATHENIANS

By the end of the fifth century B.C. the intellectual life of the Greeks was passing from Asia Minor, and the western schools of the colonies in South Italy and Sicily, to Athens; and the life of Anaxagoras (500-428 B.C.), born in Clazomenae, who left Asia for Athens as a young man, coincides with this

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period. He attempted to produce scientific accounts of eclipses, shooting stars, and the rainbow. He considered that the Sun is a mass of white-hot metal larger than the Peloponnesian peninsula, that all the heavenly bodies were thrown off when the Earth was formed and that they shine by taking fire from movement in the celestial ether. His views of the causes of solar and lunar eclipses were correct, but he explained the rising of the Sun in the east each morning as a reappearance after going under the flat Earth during the night. The explanation that he gave of the Milky Way is peculiar. The Earth's shadow, according to him, extends through space for a very great distance, this extension being due, he thought, to the small size of the Sun which causes the shadow. More stars can be seen in that part of the sky covered by the shadow (*i.e.*, the Milky Way Zone) than outside it, since the light of the stars there is not overpowered by that of the Sun. Anaxagoras was prosecuted for the impiety thought to be shown by some of his views. He was successfully defended by his friend the statesman Pericles and withdrew to Asia Minor.

In the year 432 B.C., at Athens, Meton (born about 460 B.C.) observed the summer solstice, and the following year he announced the relationship known as the Metonic Cycle (see p. 13). This discovery was acclaimed at the Olympic Games, and the cycle commencing July 16, 433 B.C., was adopted in Greece and its colonies. About a hundred years later Callippus (370-300 B.C.) made a slight improvement in this Cycle, suggesting one four times as long (940 lunations, 76 years), but one day short of four times the original Metonic value.

PLATO

Plato (about 428-347 B.C.) although not an astronomical theorist, made references to the subject at various places in his writings. He evidently did not think close study of celestial movements to be a worthy task, but considered the matter to be aesthetically interesting because of the suggestion of ideal motions possible, and in its geometrical aspects. In one of his dialogues he gives a short account of the heavenly bodies, their arrangement and motions. Starting at the

nearest body, the order he gives is Moon, Sun, Mercury, Venus, Mars, Jupiter, Saturn, the stars. The outer planets move more slowly, but Mercury and Venus perform their revolutions in the same time as the Sun; this appears to indicate that Plato was aware that the motions of these two planets are of a different type from those of the others. It is stated that he set his pupils the problem of propounding rules by which the movements of the Sun, Moon and planets could be reduced to a combination of uniform circular or spherical motions. It is possible that in doing this he was not thinking of getting an accurate theory of celestial motions so much as of the construction of a simple harmonious geometrical plan not disagreeing greatly with the facts of observation. According to Dampier ("A History of Science," p. 31), Plato is said to have realized in his old age that a moving Earth would give a simpler account of the phenomena.

EUDOXUS AND CALLIPPUS

Eudoxus of Cnidus (409-365 B.C.), acting probably on this suggestion of Plato (with whom he is said to have travelled in Egypt), tried with some success to represent the most striking features of the celestial motions by means of a combination of uniform circular motions. He assumed the fixed stars to be on a celestial vault, and the Sun, Moon and planets to be on similar vaults or spheres, all revolving uniformly. The apparent motions were thus partly explained; the system entailed 27 spheres, one for the stars, three each for the Sun and Moon, and 20 for the five planets (four each). The motion of each planet was resolved into its components, a separate sphere for each component. Callippus further complicated this scheme by adding two spheres each for the Sun and Moon and one each for Mercury, Venus, and Mars, making a total of 34 revolving spheres. There was one for each separate motion: the daily rotation; the monthly, annual, or other periodic revolution; the motion in declination (*i.e.*, northward or southward with respect to the stars); and other irregularities as they came to be noted. The fixed stars lie on a sphere which revolves once per day about an axis through the Earth. The motion of each of the other

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bodies is produced by a combination of other spheres, the centre of each sphere lying on the surface of the preceding one. There seems to be little question that Eudoxus really did not believe in the objective existence of these spheres, but looked upon them merely as a mathematical conception useful in the making of tables for prediction of the movements in the sky of the Sun, Moon, and planets. It seems certain, however, that in later times a real existence was often attributed to these spheres, and to many others added to account for various irregularities of movement.

ARISTOTLE

Aristotle (384-322 B.C.) adopted the idea of Eudoxus and Callipus, but he added 22 spheres to allow for what he thought were disturbing effects of the spheres on one another, thus increasing the total to 56. He treated them as material entities and the result was, from a mechanical point of view, a very confused conception. He believed the shapes of the heaven and the various celestial bodies to be spherical, basing this in the case of the Moon on its phases which are only possible in a spherical body illuminated by the Sun. His writings may be said to have summed up the state of astronomical knowledge and ideas of the period. The Earth's rotundity was proved to his mind by the circularity of its shadow on the Moon during eclipses, and by the change in altitude of the stars in the sky as an observer moves north or south. But he was not a believer in the revolution of the Earth round the Sun, owing to the absence of any observed consequent displacement of the stars in the sky. The Sun and Moon were considered to be nearer to us than the planets ; he had observed an occultation of Mars by the Moon and he knew of similar observations by Egyptians and Babylonians.

It was an unfortunate circumstance for the attainment of more correct ideas of the celestial movements that the authority of this man, universally acclaimed as one of the greatest who had ever lived, should have been for hundreds of years always at the disposal of opponents of new views. But it does not seem right that this should almost be directed as a charge against him. All our conceptions or scientific

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theories may be temporary, to be given up when shown necessary. He himself advised his readers at the beginning of his description of the planetary movements, to compare the views which they arrived at themselves or found in other writings, with the ones he advanced. And this was good advice which the "Aristotelians" of mediaeval times might well have taken.

The schemes he propounded, although not original, were in substance and by their symmetry and beauty, good enough to last for more than two thousand years. That they were not effectively criticized seems to show that the men who followed him were, in comparison, dwarfs to his giant stature.

THE ALEXANDRIANS

From about 300 B.C. the centre of Greek scientific activity was in Alexandria, Athens taking a secondary place. Among the first and most famous of the astronomers there, was Aristarchus of Samos (310-230 B.C.), of whom Aristyllus and Timocharis were nearly contemporaries. Aristarchus believed that the Sun and the fixed stars are motionless, with the Sun in the centre of the occupied space, that the Earth is a globe rotating on an axis, and that it revolves round the Sun. He measured the apparent diameter of the Sun and found it to be half a degree (nearly correct), and derived the comparative distances of the Moon and the Sun. He did this by ascertaining that when the Moon is half full the angle made by the lines joining the Earth and Moon with the Sun was 3°.* This gave the ratio of the distance from the Earth to the Sun, to that between the Earth and Moon, as 19, which is much too small (it is less than a twentieth of the true value) the discrepancy being due to the great difficulty, owing to the Moon's surface irregularities, in deciding the moment when it is exactly half full. The method is nevertheless a most ingenious one and a great credit to its inventor. It is noteworthy that Aristarchus held that the fixed stars are very much more

*The angle which he actually measured was that between the Sun and the Moon when he judged the latter to be half full. He obtained 87°; its correct value is 89° 52′. The angle measured is the complement of the angle he required.

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distant than the Sun. His views and observations, even with their faulty result, constitute a marked advance.

Aristyllus and Timocharis were probably the first to measure and record the positions of the brightest stars. They also made observations of the planets, Sun and Moon which were found useful by later astronomers.

Eratosthenes of Cyrene (276-196 B.C.) was the chief librarian at Alexandria and probably the most learned man of his time. His most important astronomical work was a measurement of the size of the Earth. He found that at noon on midsummer day at Syene (the modern Aswan) an upright rod casts no shadow, a deep well being lit up to the bottom by the Sun's rays. This meant that the vertical there, or line direct to the Earth's centre, is then in line with the Sun. By means of the length of the shadow cast by an upright rod at Alexandria, 5000 stadia to the north of Syene, he ascertained that at the same instant the Sun's ray is inclined to the vertical, and therefore to the line joining Alexandria with the Earth's centre, at an angle equal to a fiftieth of a complete circle. Owing to the Sun's great distance its rays at Syene and Alexandria are practically parallel to each other. From these data he deduced that the distance between Alexandria and Syene subtends an angle of the amount just mentioned, or in other words, that 5000 stadia is a fiftieth of the Earth's circumference, which is thus 250,000 stadia. Unfortunately we do not know what was the length of the stadium he used. According to some historians the value was such that the result for the circumference is 20 per cent. too great. Pliny's value of about 10 to the mile would make it nearly correct. Eratosthenes also made a remarkably accurate measurement of the Obliquity of the Ecliptic, 22/83 of a right angle; and this is only about seven minutes of arc greater than the real value at the time.

HIPPARCHUS

Undoubtedly the greatest astronomer of antiquity was Hipparchus of Nicaea (190-120 B.C.). He does not appear to have belonged to the school of Alexandria but it is thought that he may have visited and observed the stars there. At

an observatory which he built in the island of Rhodes, he did the greater part of his work. Hipparchus may be regarded as the founder of systematic observational Astronomy. Greatly developing the study of trigonometry and its application to astronomical problems, he made a very large number of observations, and collected and collated records of the work of previous observers in order to discover, if possible, any astronomical changes which might have taken place. These earlier observations extended back beyond those of the Alexandrian School and older Greeks, and included some of the still more ancient records of Babylonia.

In comparing his own observations with those of Aristyllus and Timocharis about 150 years before his time, he found that there had been changes in the distances on the sky of certain stars from the two points of intersection of the circle of the Ecliptic and the celestial Equator, known as the points of the Equinoxes. These changes were of a kind only to be explained by a motion of the Equinoxes in the direction of the apparent daily movement of the stars east to west, and not by actual movement on the sky of the stars concerned. Thus Spica, the chief star in the constellation of the Virgin, had become further separated from the Autumnal Equinox by an angle of 2° in 150 years or 48 seconds of arc per year. The correct amount of this movement is 50·3 seconds annually; it is due to the revolution of the Earth's pole round the pole of the Ecliptic in a period of 26,000 years in a direction opposite to that of the Earth's revolution round the Sun. A consequence is that the Sun, in its annual journey round the sky in the Ecliptic, returns to each equinoctial point a little earlier each year with respect to the stars, so that the Equinoxes occur slightly sooner than they otherwise would. The movement has thus received the name, Precession of the Equinoxes. (See p. 83).

It may be noted, however, that according to J. K. Fotheringham the movement of the equinoctial points had been apparently noted before the time of Hipparchus by Kidenas the Chaldaean about 400 B.C.

The appearance of a comet (see p. 15) during the year 134 B.C. in the constellation Scorpio is believed to have led to the formation by Hipparchus of a catalogue of 1080 stars,

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giving the celestial latitude and longitude of each star and classifying them according to brightness into six magnitudes. It seems likely that the Precession of the Equinoxes was discovered during the compilation of this catalogue. It was the most important and the standard one until 1600 years later in the time of the Tartar astronomer Ulugh Beigh (1394-1449 A.D.).

Hipparchus measured the Obliquity of the Ecliptic, arriving at the same result as Eratosthenes. An important discovery made by comparison of his own observations with those of Aristarchus was that the Tropical Year or time enclosed by two successive passages of the Sun through the Spring Equinox, 365¼ days, hitherto accepted, was several minutes too long.

Appolonius of Perga (*circa* 250-200 B.C.) had suggested that the motions of the heavenly bodies can be represented more simply and accurately by combinations of circular motions than by the revolving spheres of Eudoxus. When Hipparchus examined the movements of the planets, he therefore considered two theories of the circular motion—that of "eccentrics" and that of "epicycles." Some of his predecessors had adopted the latter, that is to say, they had held that each planet moved uniformly on a smaller circle (an epicycle) the centre of which itself moved uniformly upon a greater circle (known as the "deferent") at the centre of which is the Earth. In the eccentrics theory it was assumed that in the case of the Sun, for example, the centre about which it was supposed to move in a circular orbit round the Earth was situated at a little distance from the Earth. Hipparchus finally adopted systematically a system of eccentrics, and to a less extent, of epicycles. He developed this for the Sun and Moon with considerable success; but his attempts with the planets were not followed up strongly owing largely to inadequate numbers of satisfactory observations; and he contended himself with the collection systematically of observations for use of his successors. The methods of Hipparchus and the tables constructed by their means enabled his successors to predict the times of eclipses of the Moon within an hour or two, and eclipses of the Sun somewhat less closely, both much more accurate than previously possible.

Hipparchus also noted that the seasons are unequal (Vernal Equinox to Summer Solstice is 186½ days; Summer Solstice to next Vernal Equinox is 178¾ days); and he measured the inclination of the Moon's orbit to the Ecliptic as 5°. He made an estimate of the size and distance of the Moon by an eclipse method. By observing the angular diameter of the Earth's shadow on the Moon, and comparing it with the known angular diameters of the Sun and Moon he could calculate a relation between the distances of the Sun and Moon, from which either distance may be found when the other is known. He knew that the distance of the Sun is very much greater than that of the Moon; trying more than one distance for it he obtained results apparently showing that the Moon's distance was nearly 59 times the Earth's radius. Combining this with the estimate of Aristarchus that the Sun's distance is 19 times that of the Moon (see p. 32) a distance for the Sun of about 1100 times the Earth's radius is obtained. This is more than 20 times too small; the true distance is about 23,400 times that unit. The size of the Moon found was a quarter the diameter of the Earth.

An invention of the greatest value to observational Astronomy, probably due to Hipparchus, was the Astrolabe, an instrument for finding the latitude and longitude of a celestial body relative to another body for which these co-ordinates are known.*

POSIDONIUS

Very little was achieved in Astronomy for several centuries after the time of Hipparchus. Some text-books were compiled during the period with little or nothing new in them. Posidonius (born about 135 B.C.), a Syrian, made a measurement of the circumference of the Earth, finding it to be 240,000 stadia. His method (similar in principle to that of Eratosthenes) involved finding the difference in latitude between Rhodes and Alexandria from the altitudes in the sky of the star Canopus when on the meridian at the two places. As

*Celestial latitude is the angular distance North or South of the Ecliptic; celestial longitude is the arc of the Ecliptic between the Vernal Equinox (First Point of Aries) and the foot of a great circle drawn from the pole of the Ecliptic, through the object, to the Ecliptic.

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this star was practically on the horizon at Rhodes, and as the distance between the two places he adopted was perhaps too great (he used 5000 stadia), it is probable that the dimension of 240,000 stadia which he found is considerably out—unless, of course, his errors compensated one another. We do not know the value of the stadium adopted, however; and we thus have no means of checking his accuracy in this measurement or in another much smaller result of 180,000 stadia that he also obtained.

SOSIGENES

An Alexandrian astronomer, Sosigenes (*circa* 45 B.C.), should here be mentioned as the technical adviser of Julius Caesar in the reformation of the calendar, by which $365\frac{1}{4}$ days was taken as the average length of the year. This length is a little too great, leading slowly to a discrepancy between dates and seasons which was not rectified until 1582 when Pope Gregory XIII ordered a change. Ten days were omitted from that year and it was arranged to omit for the future three of the leap years in four centuries. This, the Gregorian Calendar, or New Style, was not adopted in England until 1752, when eleven days had to be dropped. In Russia and in Rumania the years of adoption were as late as 1918 and 1919 respectively.

PTOLEMY

There is no really important name in the history of Astronomy between Hipparchus and Claudius Ptolemaeus, or Ptolemy (*circa* 140 A.D.) who lived in or near Alexandria. The time of his activity as an observer is from 127 A.D. to 151 A.D., the years of his first and last recorded observations. His chief written work is known as the Almagest (an Arabic title derived from the Greek, meaning The Greatest). This book is based on the results of previous astronomers, especially those of Hipparchus. Ptolemy adopted the hypothesis of a fixed spherical Earth and assumed that it is much the largest of the heavenly bodies, although merely a point in comparison with the distance of the fixed stars. In his system, a planet revolves uniformly in a circle (epicycle) the centre of which revolves uniformly on another circle (the deferent) round a

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point not coincident with the Earth (eccentric). A fair approximation to a correct representation of the planetary motions is got by adjusting the proportions of the radii of these two circles, the velocity of the planet, and the eccentric. He put the centres of the epicycles of Mercury and Venus on a line passing through the Sun and the Earth; between these two bodies Mercury and Venus revolved in their epicycles in their own periodic times, and in their deferents round the Earth with a period of a year. The centres of the epicycles of Mars, Jupiter and Saturn were put at a distance further from the Earth than the Sun. They revolved in their epicycles once a year, and in their deferents round the Earth in their planetary periods. The radii of the epicycles at the end of which these three major planets are situated always preserved parallelism to the line joining the Earth and the Sun. In the case of the Sun he assumed that it revolved round the Earth in a year, with the centre of its circular orbit outside the Earth. The Moon revolved in an epicycle with its centre on a deferent, the centre of which is in the Earth, and the periods a month. To account for some of the irregularities of the planets' motions it was also found necessary to suppose that their deferents and epicycles, although circular, are eccentric, with the Earth not exactly in their centres. In later times, with more accurate knowledge of the planetary motions, the Arabians had to add epicycle upon epicycle with consequent great complication.

The positions for these orbit centres on this hypothesis were, however, such as would produce variations in the apparent sizes of the planets, Sun, and Moon, not in accordance with what is to be observed. For the planets (and even for the Sun) the actual changes in apparent diameter are of course, too small to be noted without optical aid, although there is a great variation in brightnesses. But this was not so for the Moon which should have been sometimes nearly twice as far away as at others, with an apparent diameter consequently only half as great on some occasions as on others. It seems probable that Ptolemy noticed this but was unable to deal satisfactorily with the discrepancy, and it may be that he looked upon his theories of the movements of the celestial bodies as means of calculating their positions on the sky

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without pretending to represent the true system of the world. The accuracy of the predicted positions did not need after all to be very great, as he and his contemporaries possessed only crude instruments, incapable of fixing positions which had not sometimes errors of as much as 10 minutes of arc (a third of the Moon's diameter).

Ptolemy obtained the distance of the Moon by a method which is essentially the same as that still employed, using observations of its position in the sky as seen from two positions on the Earth. He formed a catalogue of 1028 stars which appear to be nearly the same as those of Hipparchus. It has indeed been maintained that it does not represent original observations, but that it was compiled by correcting the positions of the stars given by Hipparchus, for precession, with only a few modifications based on observations by Ptolemy or others. One circumstance supporting this idea is that Ptolemy does not include in his catalogue any stars which would be visible to him at Alexandria but not to Hipparchus further north at Rhodes.

The phenomenon of refraction of light was first noted by Ptolemy. By this the light of a heavenly body, on entering the Earth's atmosphere and passing to its lower and denser parts, is gradually bent so that the apparent direction of the body is raised in the sky, the maximum effect being at the horizon, where the amount exceeds half a degree. It was first applied as an astronomical correction by Walther (1430-1504). In the Almagest Ptolemy gives a description of the course of the Milky Way across the constellations which makes at least interesting reading to the modern astronomer.

The apparently clumsy system of Ptolemy was in use during many centuries for approximate prediction of astronomical events. Some difference of opinion exists as to Ptolemy's merits. During the Middle Ages and until the time of Copernicus (1473-1543 A.D.) his writings, in conjunction with those of Aristotle, were regarded as the final authority in all astronomical matters. He was evidently not great as an observer,* but he was a first-class original

*According to Dreyer, Ptolemy's star positions are less accurate than those of Hipparchus, their average errors being about $\frac{1}{4}°$ and $\frac{1}{6}°$ respectively.

mathematician. He rendered a very great service by preserving the theories of Hipparchus in the Almagest, and in fact developing and improving on them. In order to reconcile more closely the observations with the attempts at prediction he had to overcome difficulties which Hipparchus had been unable to surmount. His views were those which the great founders of modern astronomy, beginning with Copernicus, had to substitute by a better system.

During five subsequent centuries the Alexandrian school continued to exist; but very little was produced in the way of writings except a few works by commentators and compilers, and it may be said that the history of the Astronomy of Greece and Alexandria practically stopped after Ptolemy. Observations seem to have been abandoned, so much so generally that only eight are known to have been recorded until about the time of the Arabian astronomers in the ninth century. But the school of Alexandria, and Greek science, passed out of existence when the city was taken by the Arabs in the year 640; and all that remained by then of the famous library, of which Eratosthenes had been chief nearly nine centuries before, was destroyed.

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CHAPTER IV

MOHAMMEDANS. TARTARS. MEDIEVAL EUROPE

To the astronomers who followed Ptolemy, there seemed to be only the task of further observations to fix, more accurately than he had done, the values of the diameters of deferents and epicycles, the distances from the centres of the deferents and epicycles to the centres round which revolution took place, the periods of revolution in deferents and epicycles, and so on. Consequently, although the methods of observation were to some extent improved, the theories of Astronomy remained the same and became less than ever concerned with governing laws or forces. Astronomy therefore was at a very low level for centuries.

MOHAMMEDAN ASTRONOMY

But an extraordinary revival of interest in science generally, and particularly in Astronomy, took place among the Mohammedan conquerors. When they were able to turn from fighting to civil affairs, they began to improve considerably the culture of the peoples they had defeated, and their capital city of Baghdad was the centre of intellectual progress from about the eighth century onwards. The Caliph Al Mansur, who ruled from 754 to 775, was a lover of science and invited learned men to his capital from other countries. As a result an Indian treatise on Astronomy, based largely on Greek knowledge, was translated into Arabic and became a text book for students. From Al Mansur's time many translations of scientific works were made at Baghdad, and Astronomy became the commonest of the sciences studied by his subjects and by their descendants. This was no doubt at least partly because the Mohammedan religious ceremonies require knowledge of the direction of their Holy City, Mecca, and also as correct fixation as possible of religious dates by means of their lunar calendar. A revival of belief in astrology was probably also a contributing cause.

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The Almagest was translated several times into Arabic during the ninth century, and several other Greek works were similarly treated. The Caliph Al Mamun (about 810 A.D.) was responsible for much of this, also for the erection of a fine observatory at Baghdad with instruments practically of the Greek type but larger and of better construction. By his directions the astronomers there made observations to verify Ptolemy's estimate of the size of the Earth and also measured the Obliquity of the Ecliptic.

But a greater astronomer than any of these was Al Battani (better known as Albategnius), a Syrian prince, who observed for forty years until 918. After studying the methods of the Greeks, he checked Ptolemy's results, obtaining more accurate values of precession and of the Obliquity of the Ecliptic. He discovered that the direction of the point furthest from the Earth in the Sun's supposed orbit (the Apogee), was different from that given in the Almagest and left it to be inferred that it was slowly moving, and he derived a more correct amount for the distance between the Sun's deferent and the Earth (the eccentricity). Owing largely to the accuracy of his observations, his tables for predicting the places of the Sun, Moon, and planets were more reliable than those of Ptolemy. The Persian nobleman Al Sufi (903-986) belonged to the same group of astronomers. He wrote a "Description of the Fixed Stars" which was a thoroughly revised edition of Ptolemy's Catalogue and based on his own careful scrutiny of the sky. The estimates of brightness of the stars in this catalogue have been thought so much of that some modern astronomers have used them to try to find whether there have been any secular changes in the light of certain stars. Al Sufi was the first to mention the visibility of the hazy spot of light now known as Messier 31, a great external galaxy (See p. 303).

Abul Wefa (939-998) wrote a book also called the Almagest. It is not a copy of Ptolemy's work, being written on a different plan. He improved on mathematical methods of astronomers of his time, and appears (although this is doubted) to have discovered a feature of the Moon's motion known as the "Variation." This was not taken up by any subsequent astronomer; it was independently discovered six centuries later by Tycho Brahe.

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Ibn Yunis (? -1008) published a record of Arabian observations during nearly two hundred years. This included two solar eclipses and one lunar eclipse seen by himself near Cairo in 977, 978, 979. By these, what is known as the "secular acceleration of the Moon's mean motion" was established. He published a set of astronomical and mathematical tables, which are known as the Hakemite Tables, the Caliph of the time being named Hakem.

Arabian astronomers in Spain (then a Mohammedan dominion), and the opposite part of North Africa, performed some important work. Arzachel of Toledo, who suggested elliptical rather than circular planetary movement, calculated the Toletan Tables (1080); and to others of the same schools may be attributed improvements and methods of observation and calculation, also some good criticism of Ptolemy, but without alternative suggestions.

After the capture of Baghdad by Hulagu Khan (1258), Nasir Eddin (1201-1274), a native of Khorassan, came to that city where he gathered together a band of experts for the construction of new planetary tables. An observatory at Meraga near the present Persian frontier, was built and Nasir Eddin was put in charge. The instruments were the best that had ever been made, and were not improved upon until those of Tycho Brahe nearly 300 years later. A revised set of astronomical tables known as the Ilkhanic Tables, based on the Hakemite Tables, was issued there, and a very accurate value of the annual precession obtained. (51″: true value 50″·3).

TARTAR ASTRONOMY

In the fifteenth century, Ulugh Beigh (1394-1449), a Tartar ruler, grandson of Tamerlane, was an active patron of, and worker himself in, Astronomy. About 1420 he built a great observatory at Samarkand. New tables of the planets were produced there, also a star catalogue dealing with nearly the same stars as those in the catalogues of Hipparchus and Ptolemy, but newly observed. The errors of position in this catalogue have been found, by comparison with modern results, to be relatively small, which shows that the instruments, methods, and observers, must all have been of high

quality. Ulugh Beigh was murdered by his son in 1449. When his reign of forty years thus came to an end, Tartar astronomy ceased to exist.

These Arab and Tartar astronomers cannot be said to have advanced any original theories. They were very accurate and painstaking observers, and competent calculators. The introduction of the decimal notation with a consequent enormous simplification of arithmetic was due to the Arabs. But, as stated above, they failed in the speculative side; and their powers of analysis were apparently directed rather to astrology than to Astronomy. We also owe to them the names of many of the brightest stars, and of astronomical terms such as "almanack," "zenith," "nadir."

Perhaps the greatest service rendered by them is the prevention of any total break in the cultivation of Astronomy throughout the passing of the Dark Ages in Europe. It is difficult for a modern mind to realize the danger of disappearance of accumulated knowledge existing in earlier times. Science then had no organization; generations might pass before a new idea was effectively taken up by some one able to use and develop it. Centres of knowledge were widely isolated and apt to be overwhelmed by conquering peoples of very backward mentality. The astronomical achievements of the Mohammedans from about the eighth to the fifteenth century are therefore all the more valuable.

MEDIEVAL ASTRONOMY

For several centuries following the period of the break-up of the Roman Empire (about 375-500) there was no progress in Astronomy in Europe. Indeed the writings of the early Christian Fathers tended to the complete obscuration of any real scientific views previously held. These writers did not all go equally far in their condemnation of the Greek Astronomy but none of them suggested any alternative system in detail as a substitute. About 540, however, an Egyptian monk and traveller Cosmas Indicopleustes published a work, "The Christian Topography of Cosmas," criticizing the Greek systems and presenting an elaborate alternative based on the design of the Jewish Tabernacle, and full of absurdities to a modern intelligence.

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One or two writers such as Philoponus of Alexandria (end of sixth century) and Isidore, Bishop of Sevile (born 570), published views which showed more application of common sense and less religious bias, occupying a position somewhere between those of the Greek "philosophers" and the bigoted patristic writers. During a long period the views of the latter were generally accepted; but somewhat more enlightened opinions were held by such men as the two Englishmen, the Venerable Bede (672-735) and the monk Alcuin (735-804). The latter was appointed head of a school of instruction set up by the Emperor Charlemagne. And in the tenth century an eminent mathematican, Gerbert, who became Pope Sylvester II for the last four years of his life, was famous for his knowledge of Astronomy and his skill in making astrolabes and other astronomical instruments, acquirements probably gained as a young man from the Mohammedan astronomers in Spain.

At about the beginning of the twelfth century, the Mohammedan influence on European Astronomy began to be more strongly felt. Many Arabic writings, or Arabic translations of Greek works, were turned into Latin during the next hundred years, including the Almagest and the Toletan Tables. And at the University of Naples, founded in 1224 by the Emperor Frederick II, who had learned something of Mohammedan science, a number of scientific men were brought together and directed to make fresh translations.

But it was chiefly from the Spanish Moors that Christian Europe got its first tuition in ancient science. A set of tables known as the Alfonsine Tables was compiled at Toledo in 1252 by experts, appointed by Alfonso X of Leon and Castile, under the direction of a Jew named Hassan. The tables were intended to replace the Toletan Tables. They caught Western attention and drew European intelligence to the science of Astronomy. They continued in great repute for about three hundred years as the best planetary tables, although they existed only as a number of manuscript copies scattered throughout Europe until 1483 when they were first printed.

The celebrated scholars Albertus Magnus (1193-1280) and Roger Bacon (1214-1294) both held opinions of more rational type than the majority of their contemporaries. Bacon

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advocated experiment and mathematical reasoning in place of the customary blind adherence to Aristotle. He seems to have known the elementary facts of optics and has even been with some reason credited with a knowledge of the telescope. The following extract from his "Opus Majus" seems to support this idea : "We can place transparent bodies in such a form and position between our eyes and other objects that the rays shall be refracted and bent towards any place we please, so that we shall see the object near at hand, or at a distance, under any angle we please ; and thus from an incredible distance we may read the smallest letter, and may number the smallest particle of sand, by reason of the greatness of the angle under which they appear."

That astronomy was now interesting many people throughout Europe is made evident by the astonishing popularity from the first of a small treatise "Sphaera Mundi" composed about 1230 by the Yorkshireman John Halifax or Hollywood (also known as Sacrobosco). This work, an elementary treatise on astronomical facts dealing chiefly with the more obvious consequences of the diurnal motion of the celestial sphere, may be termed a medieval "best-seller." It was translated into French, German, Italian and Spanish and many commentaries on it appeared. It had at least 65 Latin editions between 1472 and 1647 and it was probably the edition of the last-named year which, about the year 1665, turned the mind of the future first Astronomer Royal John Flamsteed, to Astronomy. (See p. 97).

With the revival in classic learning, the Almagest, previously known only in poor Latin translations from Arabic versions themselves not faultless, began to be read in the original Greek, rousing fresh enthusiasm. George Purbach (1423-1461) and his extremely able pupil Johannes Müller (1436-1476) of Königsberg, Franconia (known as Regiomontanus), were in succession professors of astronomy and mathematics at Vienna where they applied themselves to the examination and improvement of the Ptolemaic system. They made observations and were stimulated in their efforts to improve and reform the current astronomical theories by the discovery of large errors in the predicted places of the heavenly bodies obtained by use of the Alfonsine Tables : an

eclipse of the Moon, for example, being an hour late, and Mars about 2° (four times the Moon's diameter) out of place on one occasion.

They were asked to go to Rome by Cardinal Bessarion to study a Greek manuscript of the Almagest, secured among many others at the fall of Constantinople in 1453, and were about to start when Purbach suddenly died. Regiomontanus himself carried out the projected visit, however; and he did valuable research and teaching work while in Italy where he remained for seven years.

Ultimately Regiomantanus settled at Nuremberg where he worked with a wealthy amateur named Bernard Walther (1430-1504). On his own printing press there he produced nautical almanacs which were of great assistance to the Portuguese and Spanish navigators a few years later. He died in 1476, while on another visit to Rome, where he had gone at the invitation of the Pope to assist in a reform of the Calendar. Walther continued the work for twenty-eight years; he is credited with having been the first to make corrections in observed celestial positions for the effect of the refraction discovered by Ptolemy (see p., 39) and was probably the first to use a mechanical clock in an astronomical observatory.

Leonardo da Vinci (1452-1519), the universal genius who is celebrated for his knowledge and proficiency in all the arts and sciences, wrote of the rotation of the Earth as being common knowledge; he was the first to explain the illumination seen on the Moon's disc inside its crescent when very narrow, as the effect of sunlight reflected on to it from the Earth. This phenomenon is known as the Earthshine on the Moon. It is a "reflection of a reflection." The less the Moon appears illuminated by the Sun from the Earth, *i.e.*, the narrower the crescent, the more illuminated is the Earth as seen from the Moon. The Earth is Full to a lunarian when the Moon is New to us, and *vice versa*.

Fracastoro (1483-1543) a Veronese (who as a professor at Padua probably knew Copernicus when the latter was a student there) wrote an elaborate and complicated theory of spheres intended to be an improvement on Ptolemy. Apian (1485-1552) was also a copious writer on astronomical subjects.

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He made observations on comets, noting (as the Chinese had also seen, see p. 15) that a comet's tail points away from the Sun. Nonius (1492-1572) studied the phenomena of twilight and solved some problems connected with it. Fernel (1497-1558), a French doctor, made a measurement of the size of the Earth about 1528; the error was only one per cent., suggesting an accidental accuracy.

On all sides the debates and discussions of scholars and their teachers must now have tended to weaken convictions of those who were familiar with the Ptolemaic system; and true progress, which is much less due to destruction of theories than to reorganisation or substitution, was about to be accomplished by the great men who may be fitly termed the Founders of Modern Astronomy.

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CHAPTER V

COPERNICUS. TYCHO BRAHE. KEPLER

COPERNICUS

A NEW spirit of enquiry marked the beginning of the sixteenth century whereby the authority of Ptolemy and even of Aristotle was being actively questioned. To this enquiring generation belonged Niklas Koppernigk (1473-1543), or as he afterwards became known, Nicholas Copernicus. Born in Thorn on the river Vistula, where his father was a merchant originally from Cracow, he was at ten years of age, on his father's death, put under the care of his maternal uncle afterwards Bishop of Ermland. He was at school in Thorn until the age of 17 when his uncle, intending him for an ecclesiastical career sent him to the University of Cracow. His first special studies were in medicine but he made them secondary to astronomy and mathematics. In order to have an income enabling him to live while following his studies, he took orders. After several years spent at Cracow he lived for a year or two at home and then from 1496 to 1506 studied in Italy, chiefly at Bologna and Padua, but graduating at Ferrara. During his absence abroad he was appointed to a canonry at Frauenburg. He returned there later, but soon joined his uncle at Heilsberg, where he seems to have first put together some of the revolutionary ideas in Astronomy which he probably had begun to hold when working as assistant to Domenico Novara, Professor of Astronomy at Ferrara. When his uncle died he finally settled at Frauenberg in 1512, where he lived until his death in 1543.

A large amount of his time was devoted to astronomical studies, and some observational work formed part. But he was also busy in duties connected with the Church, and in some medical attention given to friends and to the poor of the town. He was not great as an observer, possessing only the roughest of instruments, much inferior to those of the Arabians of several hundred years previously. But although, until the latter part of his life he published no important work,

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he was well-known throughout intellectual circles of Europe as an astronomer and mathematician after the time of his visit to Italy.

The complete account of his studies and their results was not made public until just before his death. In fact it is stated that a copy of his newly printed book "De Revolutionibus Orbium Celestium" was given to him on his death bed, and probably never opened by him. Its principal contents will now be briefly summarized.

The first of his theories was that of the rotation of the Earth. Hitherto the celestial phenomena of the rising, passing across the sky, and setting of all but circumpolar objects had been attributed, except by a few thinkers no doubt regarded as cranks, to a real movement of the bodies themselves. Copernicus saw that the phenomena could either be explained in that way or by the Earth's rotation round an axis in the opposite direction. Although Ptolemy had noted the great difficulty involved in the first explanation by the tremendous velocities of the stars which it entailed, he had accepted it rather than the other because of the absence of movements of loose objects or of the atmosphere which he thought would take place on a rotating Earth.

But Copernicus realized that the atmosphere and any loose bodies must accompany the Earth's surface in the rotation supposed, having naturally the same movement themselves; and he adopted the idea of the rotation of the Earth as correct. He believed it to be much more likely that this was so than that the enormously greater surrounding universe should be in rotation. In this he was strengthened by the opinions of some eminent philosophers of the past such as Philolaus and Aristarchus. (See pp. 28 and 32).

Once this step had been taken some difficulties connected with the movements in the sky became less, and this encouraged him to further enquiry into the motions of the Sun, Moon and planets.

Having adopted a rotation for the Earth he found it easy to believe that it had another motion, this time round the Sun, and that the planets also moved in the same direction in circumsolar orbits, as had been held by several philosophers formerly. The theory would have several advantages: it

would account for the greater brightness of Mars, Jupiter and Saturn when in opposition, as they would then be nearer to the Earth; and it would explain more simply why planets then retrograde in their motions in the sky.

But the theory of Copernicus was far from a really simple one. He still retained the old idea of uniform circular motion and had to assume different centres for planetary orbits, outside the Sun; also that epicycles were necessary as they had been with Ptolemy's system. Several epicycles had to be introduced, one upon the other, to explain observed irregularities in the Moon's orbit, the only one left from the older system as being described round the Earth. In fact the total number of epicycles for the Earth, planets and Moon in the Copernican system as he left it was actually 34 (3 for the Earth, 4 for the Moon, 7 for Mercury, and 5 each for Venus, Mars, Jupiter and Saturn). Although this number is substantially smaller than most versions of the Ptolemaic system require (Aristotle needed 56, Fracastoro, 79!) the complication is not much less, nor the purely geometrical ideas much better than those hitherto current. In fact no *dynamic* theory was possible with such an artificially constructed system. By it, the old epicyclic theory was simplified only in the placing of the centre of the Earth's orbit at a centre of the universe, the movements of the planets being also referred to this point through which the planes of their orbits passed; and their positions of greatest and least orbital velocity were related to it. The Sun was placed in a position near to the centre of the planetary system, but it "did not seem to have any physical connexion with the planets as the centre of their motions."[1]

Perhaps the celebrity of Copernicus, greatly deserved as it is, is more than his system should have brought him. It seems possible that the magnitude of his achievement had been to some extent exaggerated, owing to the reaction after so long a period of acceptance of the Ptolemaic system. Indeed it has been rather dryly remarked that "the Copernicun theory is by no means so different from or superior to the Ptolemaic theory as is customarily asserted in anniversary celebrations."[2] It may perhaps be considered that possibly Copernicus did not ask his readers to accept his system as showing the real motions or doing more than enabling us to predict

their movements in the sky. But in this it is likely that he was in no way different from Ptolemy (see p. 38), or from Eudoxus (see p. 31).

Copernicus was fully aware that the revolution of the Earth in its orbit must cause apparent displacements of the stars in the sky ("parallactic displacements"); but he considered that the stars' distances from us were so great as to make it impossible to see the very small displacements involved.

The fact that Copernicus dedicated his great work to Pope Paul III, and also the circumstance that the preface written by Osiander, a Lutheran minister interested in Astronomy, put the theories forward as mere mathematical contrivances to represent the phenomena, probably account for the relatively quiet reception it had. But it was readily seen by Luther and others that it contradicted the usual interpretation of some Biblical passages. The system was not accepted as a true one by all learned men. For example, Francis Bacon writes of it in several places in his books, showing no real appreciation and doing no justice to it, referring to Copernicus as a "man who thinks nothing of introducing fiction of any kind into nature, provided his calculations turn out well." But among astronomers it was gradually adopted as a great advance.

Erasmus Reinhold (1511-1553) prepared a new set of tables of the celestial motions to take the place of the Alfonsine set. They were published in 1551 and given the name "Tabulae Prutenicae," or Prutenic (Prussian) Tables, in honour of the author's patron, Duke Albrecht of Prussia. Their main object was the development of the work in the "De Revolutionibus," and they were not very much better than the set they replaced. But they nevertheless marked a forward step; and they were at least notable for much more accurate computational work than that of the Alfonsine Tables. The accuracy of all tables up to this time was very poor compared with later productions. Copernicus himself once remarked that he would be very pleased if he could make his theory agree with observation within about ten minutes of arc (a third of the Moon's diameter). But even greater errors than that were sometimes found in the predictions.

The Copernican system soon spread into other parts of the

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continent with varying degrees of success, and into England. But no contributions of importance were made to it for the half century after its publication.

Among astronomers of note at this period were Michael Mästlin (1550-1631) the teacher of Kepler. He believed in the Copernican system and was probably the first to interest Kepler in its details. One small item of interest regarding Mästlin is that his eyesight seems to have been particularly good; fourteen stars instead of the usual six or seven were visible to him in the Pleiades cluster.

Another German Copernican was Christopher Rothmann, chief astronomer to the Landgrave of Hesse, William IV (1532-1592). This prince took a great interest in Astronomy from an early age; he built an observatory at Cassel which was the first to have a revolving roof, now a regular feature of an observatory. His chief astronomer was assisted by Joost Bürgi (1552-1632), originally a clockmaker, but a man of remarkable mechanical and mathematical abilities. At Cassel the principal work was the formation of a star catalogue in which allowance was regularly made for refraction; and clocks constructed by Bürgi were employed.

TYCHO BRAHE

Tycho Brahe (1546-1601) the next prominent figure in the history of Astronomy, was of ancient Danish family. His first interest in the science was aroused during his boyhood, by a partial eclipse of the Sun, October 21, 1560. He then bought a copy of Ptolemy's works, still in existence with his marginal notes in the library of Prague University. His aristocratic relatives were far from pleased with this newly developed interest, as they thought it below the dignity of a nobleman; he was therefore sent off at 16 years of age to study law at Leipzig.

The attempt to divert him was not successful, and he continued his astronomical reading. From a study of the Alfonsine and Prutenic Tables he found by a mere inspection of the sky that the actual positions of the planets were much different from those predicted, and at this early age he became one of the first students of the science to be convinced of the

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need for prolonged and systematic observations of the planets before theorising further regarding their motions. In the year 1563 his first proper observation was made, although a very rough one. This was by means of a pair of ordinary compasses; by looking along the legs from the pivoted centre he was able to measure the angular distance between Jupiter and Saturn or between either planet and a star, at a conjunction of these planets in the month of August; and he noted that the times predicted for the phenomenon showed errors in both the Alfonsine and the Prutenic Tables, in the former a month, in the latter several days.

Some years were occupied by travels in Germany. At Augsburg, along with friends, he built some astronomical instruments, one a huge wooden quadrant with a radius of 19 feet. But in 1570 he returned to Denmark where, in 1572, the astronomical event which probably finally fixed his life interest in Astronomy, the great Temporary Star in Cassiopeia, was observed by him. He wrote a book on it, "De Stella Nova." In this work are recorded the systematic observations of the star's changes in brightness and colour, its position in the sky, and absence of movement or of parallactic shift which showed that it belonged to the distant fixed stars.

In 1575 the Landgrave of Hesse was visited at Cassel. The result was of great importance to Tycho's future career as this prince strongly recommended to the King of Denmark, Frederick II, that the services of the brilliant young astronomer be secured for his country. This was followed by the establishment of the young Dane in 1576 on the island of Hven on the Sound, about 16 miles north of Copenhagen. Here were built the two grand observatories Uraniborg and Stjerneborg, that spread his fame all over the civilized world. Magnificent instruments were installed which he used with consummate skill and some pomp, donning robes of state for the purpose. Many pupils were trained, and princes (among them our own James I), ambassadors, and men of learning came to visit the island.

By his refined and improved methods of observation very great accuracy was obtained as compared with the results of former astronomers—positions within a minute or even

half a minute against five, ten or even more minutes of error hitherto. This accuracy is all the more remarkable as he had no good mechanical clocks, using a sort of clepsydra, with mercury instead of water, to measure time. To allow for this deficiency in his apparatus he sometimes used procedures which were not dependent on clocks. For example, in 1582, he made use of the fact that the planet Venus was six weeks visible in daylight even before it crossed the meridian, and he therefore measured its angular distance from the Sun and from bright stars. And he repeated these observations as often as he could in the next six years.

Among his outstanding results are: the fixing of the position of a star used as a standard by astronomers (Hamal, α Arietis), within a quarter of a minute of arc of its true place: a value for Precession of 51″; discovery of some irregularities in the Moon's motion, previously unknown, and of the fact that the inclination of the Moon's orbit to the Ecliptic oscillates in value from about 5° to $5\frac{1}{3}$°.

Tycho also compiled a catalogue of 977 of the fixed stars, the positions of 200 of which were, however, not up to the great accuracy of the others; wrote a book on comets which proved that they were celestial bodies, further away than the Moon and planets, and not appearances in the Earth's atmosphere; and he made that continued and systematic series of observations of the planets which enabled Kepler to discover his famous laws (see p. 59).

There is no doubt that Tycho was the first to employ the method of triangulation which forms the basis of all trigonometrical survey and cartographical work. He made a map of Hven on this principle which is one of the earliest constructed in this way.

His theoretical work was not so strikingly good. On account of the absence of parallactic shifts in the stars and because of Scriptural prejudices he did not accept the Copernican heliocentric system. He thought, and correctly enough, that if that theory were right the stars should show a yearly movement to and fro on the sky owing to the Earth's movement in its orbit round the Sun. He knew that he could find such a displacement if it was more than a minute of arc, but he

could not see any. The diameters of the fixed stars were considered to be two or three minutes, before the use of the telescope in Astronomy, and his observations which showed that they must have parallactic shifts of less than a minute, would, he thought, entail that the stars were two or three times the size of the Earth's orbit; this seemed much too large in comparison with what he believed to be another star—the Sun.

He therefore favoured a system of his own in which the planets moved round the Sun; but the Sun (with its planets), and the Moon, revolved round the Earth, a system similar to one believed in by Julianus twelve hundred years before Tycho's time. But there were still some epicycles and deferents (the Sun's orbit was the deferent of the planets' epicycles) necessary, to allow for irregularities, and to complicate the system, much as with Ptolemy and Copernicus. In Dreyer's opinion Tycho's system "is in reality absolutely identical with the system of Copernicus and all computations of the places of planets are the same for the two systems."[3] That is to say, they are mathematically the same although Tycho would not admit this.

During a period of 20 years Tycho was at Hven, supported by gifts of lands and money from Frederick II. He appears to have been of a very autocratic and impetuous disposition and did not treat his tenants well, spending lavishly on his astronomical work. When King Frederick died, the successor Christian IV and his advisers withheld the assistance; this led to Tycho's removal from Denmark to Prague where he was appointed Imperial Astronomer. In Prague he had the assistance of one of the world's greatest astronomers, Johann Kepler. After several years he died in 1601 in that city.

The two observations Uraniborg and Stjerneborg became ruins, and almost nothing remains of them nowadays; while Tycho's instruments, many of which he had taken to Prague, were mostly destroyed during 1619 in a war in Bohemia. A later Danish King, Frederick III, bought Tycho's records of observations, and they are now in the Copenhagen Royal Library.

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GIORDANO BRUNO

At this stage it is appropriate to refer to Giordano Bruno (1547-1600), an Italian Dominican monk, who held astronomical views very far in advance of the time. He believed the Earth to be similar to the planets, circulating round the Sun; and that there are other worlds of the same kind revolving round the stars. His attitude to the Copernican theory is shown by his bold remark that the preface to the "De Revolutionibus," written by Osiander, which stated that the doctrines therein were purely hypothetical and not meant to represent the real state of affairs, could only have been written by one ass for the benefit of other equally ignorant asses.

Bruno was burnt at the stake by the Inquisition for his unorthodox scientific and theological opinions, which were strikingly closer to modern ideas than those held by any contemporary with the exception of William Gilbert (1544-1603), Queen Elizabeth's physician, who expressed very similar astronomical views in his book "De Magnete" dealing mainly with physical and electrical questions. Another exception was the Englishman Thomas Digges (?-1595), who published an almanac in 1576 giving a description of the Copernican system with a diagram showing the stars extending outwards in space to indefinite distances.

KEPLER

In the history of Astronomy there has seldom been a period when three such distinguished men as Tycho Brahe, Kepler and Galileo were active in the study of the sky. Johann Kepler (1571-1630), whose work will next be described, was born at Weil in Würtemberg. His parents were of noble family but in reduced circumstances at the time of his birth. After irregular attendance at elementary schools, he went to more advanced ones at Adelberg and Maulbronn. The latter was connected with the University of Tübingen where he obtained a B.A. degree in 1588 and M.A. in 1591, and was taught by Mästlin (see p. 53). This convinced adherent of the Copernican system introduced it in detail to Kepler, who became a convert and defended it at the college in physical

disputations with other students. In 1594 he became lecturer on mathematics and Astronomy at the high school of Gratz.

Though Kepler had as yet little knowledge of the science of the stars, his duties forced him to study it, and during some leisure time in the year 1595 he directed his mind to the consideration of the number, size, and motions of the planets. As a result of rather wild speculation he found what he thought was a significant numerical relation, connecting the distances of the planets from the Sun with the regular geometrical solids, which he wrote about in 1596 in a book, "Mysterium Cosmographicum." The relation is only a rough one, and in any case of no value to the progress of Astronomy.

He married in 1597; and in the following year, owing to religious troubles he had to go to Hungary, where he visited Tycho two years afterwards in Prague. Returning to Gratz for a few months he had to resign his position there because of his Protestant religion, and again went to Prague. There Tycho introduced him to the Emperor who conferred on him the title of Imperial Mathematician, on condition that he assisted Tycho in his calculations. This became particularly valuable to Kepler as the observations made by the Danish astronomer were what he required to carry on his own theoretical investigations. The two astronomers now undertook, from Tycho's observations, the construction of a new set of astronomical tables to be called the "Rudolphine Tables" in honour of the Emperor, who was to pay all expenses.

When Tycho died in 1601, Kepler was appointed his successor, but was shabbily treated, his salary being only half that given to Tycho, and irregularly paid. Being no observer he never acquired any of Tycho's instruments; but he secured control of the greater part of the records of the magnificent series of observations chiefly made at Hven. For the next 25 years of Kepler's life these observations were made the basis of an improved theory of the solar system and of several epoch-making discoveries.

While at work on the Rudolphine Tables, using the Copernican system and its epicycles, deferents, and eccentrics, Kepler had found poor accordance between prediction and observation; and he began to believe that these features of the system were false and impediments to real knowledge.

Examining carefully a number of hypotheses (such as that the planets revolve round centres outside but near to the Sun, that the centres of the epicycles are not situated exactly on the deferents, and so on) he still found discrepancies in position amounting to as much as eight minutes of arc. These, he declared, were impossibly great for the observations of Tycho which he was using. He also compared the Ptolemaic and Tychonic systems with observation and found them very defective.

After an incredible amount of computation, adopting many hypotheses, Kepler found the following relationships :—

I. A planet travels in an orbit which is an ellipse ;* the Sun is in one of the foci.

II. It travels in its orbit at a rate such that the "radius vector"—or line joining it to the Sun—sweeps out equal areas in equal times.

III. The cubes of the mean distances of the planets from the Sun are proportional to the squares of the periods of revolution.

The first two of these were published in his book, "Astronomia Nova," in 1609. It took him nine years longer to discover the third.

I and II explain themselves ; the third may be best explained by an example. The mean distances from the Sun of the Earth and Mars are 1 and close to 1·523 respectively ; the periods are as 1 and 1·88. The cubes of 1 and 1·523 are 1 and 3·53, and the squares of 1 and 1·88 are also 1 and 3·53.

It was afterwards demonstrated by Newton, having been guessed at previously by others, that these three relationships define precisely the conditions under which planetary revolution must proceed if governed by a force emanating from the Sun and decreasing as the square of the distance from that body increased, *i.e.*, the law of gravitation as exemplified by the Sun and its planets.

In finding the first of these three relations, it was rather fortunate that Kepler set to work first on the observations of Mars, the orbit of which differs more from a circle than those of the other planets then known (except Mercury for which

*It is worth noting that in 1080 Arzachel (see p. 43), and Reinhold (1511-1553), had suggested an oval or elliptical form.

observations were much too few). In fact, even with that advantage he calculated seven circuits of Mars in its orbit before giving up attempts to reconcile circular motions to observations by means of some artifice or other. The second and third were found by a process of trial and error which could only have been persisted in by a man of his extraordinary ingenuity and enthusiasm. The three relationships were afterwards proved to hold for planets and their satellites, as they do for the Sun and *his* satellites, the planets. Kepler did not think them applicable to comets which he considered to move in straight lines never returning to the Sun.

As well as these major discoveries, Kepler made many other contributions of value. He drew attention to the use of eclipses for determining differences of terrestrial longitudes (the differences between local times of occurrence of the eclipses gives this); and the extension of this method to occultations of stars by the Moon is still employed. He also speculated on the physical causes behind his three relationships, and suggested that these were attractions between Sun and planets varying with distance and proportional to mass. He regarded the tides as being due to a mutual attraction between the Moon and the seas of the Earth. But the state of the mechanical ideas of his time was too undeveloped and imperfect for greater success in such speculations.

On the other hand Kepler's ideas of the other celestial bodies were very far from correct. As Dreyer says: "Though he emancipated himself in so many ways from the opinions of the ancients, he shared their opinion that the fixed stars form part of a solid shell, in the centre of which the Sun is situated. The idea, held by Giordano Bruno, that the stars are suns, surrounded by planets, he regarded as improbable, as our Sun if removed to the same distance would be much brighter than the fixed stars, though it would appear as small as they do."[4] He thought that the Nova of 1604 was composed of matter run together from the starry sphere, which, when the new star faded, ran back into the starry sphere again.

Kepler was also a writer on Optics, and was the first to suggest that a telescope, with both object glass and eyepiece convex lenses (unlike Galileo's, to be presently described), could have cross wires fitted in the focus to help in fixing

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more accurately the positions of stars seen in its field. Gascoigne (1612?-1644) was the first to use the idea in practice.

On the abdication of the Emperor Rudolph in 1611 and the succession of his brother Mathias, Kepler was reappointed Imperial Mathematician and was allowed to accept a professorship at Linz.

All through his career he worried over the delays in payment of his salary, while the repeated promises of the Government prevented him from taking other employment. He hoped that his suggestions of leaving for fresh posts would rouse the Imperial Treasury to a sense of its duty and enable him to publish the Rudolphine Tables, a great work which he felt he owed to the memory of Tycho and of Rudolph. But though he was disappointed in this, an event occurred in 1619 which seemed to promise a favourable change in his circumstances. The Emperor Mathias died in that year and was succeeded by Frederick III, who not only continued Kepler in the office of principal mathematician but promised to pay up arrears, and finance the publication of the Tables.

In 1622 this Emperor, notwithstanding his own pecuniary difficulties, ordered the whole of Kepler's arrears to be paid, and supplied the means for the immediate publication of the Tables. But new difficulties appeared, to retard still longer this important work. The Wars of the Reformation, then agitating all Germany, interfered. The library of Kepler was sealed up by order of the Jesuits, and it was only his position as Imperial Mathematician that saved him from personal inconvenience. A popular insurrection followed these unfortunate happenings and the peasantry blockaded Linz, the place of Kepler's residence. It was not until the year 1627, as the title page shows, or 1628, as Kepler says elsewhere, that the celebrated Rudolphine Tables at last appeared. By means of these Tables he was able to predict the transits of Mercury and Venus across the Sun in 1631, a remarkable achievement in the eyes of his contemporaries in the case of Mercury. That of Venus occurred during the night as far as European observers were concerned.

Kepler had some further trouble regarding sums due to him. When Wallenstein, who was a great patron of astrology (of which Kepler was a practictioner sometimes, in spite of

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himself, to raise much-needed income) invited him to Silesia, he removed his family from Linz in 1629. He himself went to Prague with the object of presenting the Rudolphine Table to the Emperor and requesting permission to join Wallenstein. In this request he was successful, and he accordingly set out to Sagan in Silesia in order to settle there. But in this remote situation he found it very difficult to get the arrears of 8000 crowns still owing to him. He resolved to go to the Imperial Assembly at Ratisbon to make a final effort to obtain this money. In this he was unsuccessful and the vexation and fatigue threw him into an illness from which he died on November 5th, 1630 (O.S.) in the sixtieth year of his age.

Kepler was a Martyr of Science, perhaps as much as, if in a different way from, his contemporary Galileo, whose work will now be dealt with. But his worries and frequent anxieties and his early death were due to an over-sensitive temperament rather than to direct effects of great poverty, judging by the inventory of the property he left at death.

References

[1] Grant, "History of Physical Astronomy," p. vii. (1852).
[2] Neugebauer, *Publications of the Astronomical Society of the Pacific*, vol. 58, p. 116 (1946).
[3] Dreyer, "History of the Planetary Systems," p. 364 (1906).
[4] Dreyer, *ibid*, p. 410.

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CHAPTER VI

GALILEO UNTIL NEWTON

GALILEO

A CONTEMPORARY of Kepler's was Galileo Galilei (1564-1642), born at Pisa seven years earlier and dying twelve years later. Galileo's father intended that he should follow a medical career and sent him to the University of Pisa in 1581. It was soon evident that his tastes lay in another direction, and that his real inclinations were for study of mechanical science and mathematics. He obtained his father's sanction to give up medicine for these subjects. As early as 1582 he discovered the principle of isochronism of the pendulum and designed an instrument whereby oscillation of a suspended weight could be employed by doctors to count the rate of a patient's pulse, a device which according to Viviani, Galileo's biographer, was still in use threequarters of a century later. The principle was applied to clocks about 1650 by Huyghens a Dutch astronomer (see p. 72).

Galileo had to leave the University owing to lack of money for the normal length of course, and until 1589 when he was appointed professor of mathematics at Pisa, he spent his time chiefly at home.

At Pisa he carried out a series of investigations which revealed such a revolutionary attitude to the doctrines of Aristotle as to make his relations with the University authorities far from harmonious. This was particularly the case in regard to a demonstration of the falsity of the Aristotlelian doctrine that a heavy body must fall faster than a light one in proportion to their weights. This and other results, together with the effect of aggressive attacks on Aristotle's conclusions generally, led to his resignation in 1571. His father dying in that year, he returned to the home of his mother at Florence.

The chair of mathematics at Padua having been vacant for some time, Galileo was appointed to it in 1592 for a term of six years with a better salary than the very inadequate one

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he had been paid at Pisa. This larger payment was, however, still insufficient for proper maintenance of himself and assistance to his family, and he was obliged to continue private teaching as had been the case at Pisa. In spite of this diversion of his energies he found time for independent work in Astronomy, mechanics, the art of fortification, and the invention of scientific instruments.

BECOMES A COPERNICAN

The exact date when Galileo became a convert to the system of Copernicus, or the particular circumstances which led him to adopt it, are not easy to ascertain; but this seems to have taken place when he was comparatively young. In a book he wrote later, published in 1632, "Dialogues on the Two Chief Systems of the World, the Ptolemaic and the Copernican," one of the speakers says :—

" I was a very young man, and had scarcely finished my course of philosophy, when there arrived in this country, a foreigner, Christian Vurstisius (Wurteisen), a follower of Copernicus. He delivered, on this subject, two or three lectures, in a certain academy, and to a numerous audience, several of whom were attracted more by the novelty of the subject than by any other cause. Being firmly persuaded that this opinion was a piece of solemn folly, I was unwilling to be present. Upon interrogating, however, some of those who were there, I found that they all made it a subject of merriment, with the exception of one, who assured me that it was not a thing wholly ridiculous. As I considered this individual to be both prudent and circumspect, I regretted that I had not attended the lectures, and whenever I met any of the followers of Copernicus I began to inquire if they had always been of the same opinion. I found that there was not one of them who did not declare that he had long maintained the very opposite opinions and had not gone over to the new doctrines till he was driven by the force of argument. I next examined them one by one, to see if they were masters of the arguments on the opposite side; and such was the readiness of their answers, that I was satisfied they had not embraced this opinion from ignorance or vanity. On the other hand,

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whenever I interrogated the Peripatetics and the Ptolemeans—and, out of curiosity, I have interrogated not a few—respecting their perusal of Copernicus' work, I perceived that there were few who had seen the book, and not one who understood it. Nor have I omitted to inquire among the followers of the Peripatetic doctrines, if any of them had ever stood on the opposite side: and the result was, that there was not one. Considering then, that nobody followed the Copernican doctrine, who had not previously held the contrary opinion, and who was not well acquainted with the arguments of Aristotle and Ptolemy; while, on the other hand, nobody followed Ptolemy and Aristotle, who had before adhered to Copernicus, and had gone over from him into the camp of Aristotle; weighing, I say, these things, I began to believe that, if anyone who rejects an opinion which he has imbibed with his milk, and which has been embraced by an infinite number, shall take up an opinion held only by a few, condemned by all the schools, and really regarded as a great paradox, it cannot be doubted that he must have been induced, not to say driven, to embrace it by the most cogent arguments. On this account I have become very curious to penetrate to the very bottom of the subject."

This passage undoubtedly indicates his conversion at a relatively early age. It also shows vividly the mental attitude of most men of the time to the controversy, and the reaction of Galileo's vigorous intellect to that mental attitude.

But in compliance with the current practice he had to teach the Ptolemaic system at Pisa for some time after he had convinced himself of its unsoundness and of the substantial truth of the Copernican system. This change of mind appears to have been between 1593 and 1597.

PROFESSOR AT PISA

It would appear that Galileo may have looked upon his residence at Padua as exile from his beloved Tuscany. He seems always to have wished to go back there, and in 1610 he got the opportunity to do so. Galileo's fame had become great and the Grand Duke of Tuscany wished to have him resident in Florence in the belief that he would shed some

lustre on the Duke's dominions. The consequence was his appointment in the year mentioned as professor at Pisa and also "First Philosopher and Mathematician" to the Grand Duke, with a good salary and no particular duties in either post.

His discoveries in mechanics and the laws of motion made throughout his career were of far-reaching importance and he early became famous in his own country as a brilliant discoverer and lecturer. But it was work of a different kind which established his European reputation.

ASTRONOMICAL DISCOVERIES

Galileo's astronomical work may be briefly summarized as follows: His first contribution was on the New Star in Ophiuchus found by Mästlin in 1604. This star which attained a brightness nearly equal to that of Venus, was seen for about $1\frac{1}{4}$ years before fading to invisibility with the naked eye. There is no doubt that, like Tycho's Temporary Star of 1572, it was a Supernova (see p. 16). Galileo's observations of it and his speculations as to its cause were the subject of three lectures. From the absence of parallax he proved that the common idea that it was a sort of meteor was erroneous, and that, like the fixed stars, it was situated beyond the bounds of our own system. The popularity of the subject attracted crowds to his lecture-room; and Galileo had the boldness to reproach his audience for taking so deep an interest in a temporary phenomenon, while they overlooked the wonders of the sky which were nightly presented to their view.

In 1609 Galileo heard that a Dutch spectacle-maker, Hans Lippershey or Lippersheim (?-1619), had made an instrument consisting of two lenses which magnified distant objects. Working with this information, which was little more than a hint, he solved the problem by combining in a leaden tube two lenses, the one furthest from the eye (the object glass) being plano-convex, the eye-lens plano-concave. He made a number of these instruments, the largest magnifying 32 times. Galileo was thus not the first inventor of the telescope. This has been attributed to several: Roger Bacon (about 1260) (see p. 46); Leonard Digges (about 1550); Porta

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(about 1558); and to two other Dutchmen, Jansen and Metius, besides Lippershey, at about the same time.

The first use of a telescope in Astronomy is generally attributed to Galileo, but it seems probable that an Englishman, Thomas Harriott (1560-1621), and a German, Simon Mayer or Marius (1570-1624) were even slightly before him, although their astronomical work was trifling compared with his. Two of Galileo's telescopes and the object glass (cracked) of the one with which Jupiter's moons were found are still in existence mounted on a stand in the Tribuna di Galileo at Florence.

On directing this new optical aid to the sky he was rewarded with a remarkable series of discoveries by which his name quickly became celebrated throughout Europe. These included: the spots on the Sun which appeared to rotate round it in about 27 days; the mountains, craters and plains on the Moon; discs of appreciable size to the planets which he saw were not points of light like the fixed stars; Venus showing phases like the Moon; four satellites circulating round Jupiter; Saturn with appendages making it look like three bodies joined in a row, at one time, while at others the planet looked like one body; the "libration" of the Moon by which, according to its position in the sky (and in its orbit), we sometimes see a little way beyond its edges; 36 stars in the Pleiades where ordinary eyesight shows only 6 or 7; 40 in the cluster Praesepe, where three nebulous stars only had been previously thought to exist; and the Milky Way as composed of myriads of stars.

A few of these were noted independently by others. The sunspots had been seen by Harriott in England, by John Fabricius (1587?-1615) in Holland, and by the Jesuit Christopher Scheiner (1575-1650) in Germany; and the moons of Jupiter had been seen by Marius very shortly after Galileo, although this astronomer was for a long time accused as an impostor in regard to this observation. It has been shown,[1] however, that Marius probably saw the satellites even sooner than Galileo; but he evidently did not comprehend their real nature until after Galileo had published his account of their discovery and the explanation of their satellite connection with the planet.[2] The first to see Jupiter's belt markings was Zucchi in 1630.

Spots had previously often been seen on the Sun with the

naked eye, through mist and fog. Galileo proved that they were not bodies revolving round the Sun, as was believed by Scheiner and others, but markings on his surface showing his rotation.

Two suggestions of his were acted upon in later years: the determination of stellar distances from the displacements in the sky with regard to background fainter stars due to the Earth's movement round the Sun, which had been thought of before but not in so particular a manner; and the measurement of differences in terrestrial longitudes by means of the differences in the local times of observations of eclipses of Jupiter's satellites.

Galileo considered that some of his results proved the truth of the Copernican theory, but in fact this was not quite so. The phases of Venus, the satellites of Jupiter, and the Sun's rotation, were brilliant illustrations of the Copernican system as it might reveal itself, rather than actual demonstrations of its truth. For instance, phases of Venus would be seen according to the theory of Tycho in which that planet revolved round the Sun. The Ptolemaic theory was, however, disproved by the phases of Venus, while the slight phase effects on Mars's shape, first noted by Galileo, were a direct confirmation of the Copernican theory. It is of some interest to note that if Venus had been as big as one of the major planets, the crescent form would have been easily visible to the naked eye. From this it would have been obvious that the planet was a dark body revolving round, and illuminated by, the Sun; and belief in a system like that of Tycho or of Copernicus would have been of great antiquity.

Some of Galileo's discoveries were denied existence by opponents; and the chief consequence of his observations and deductions from them, which really should have been the overthrow of Ptolemy's and Aristotle's ideas for all time, was a stream of opposition from the Schoolmen and from Roman Catholic Church clergy. This was so strong that Galileo had to publish his expositions as hypotheses rather than actual systems. Becoming conscious that opposition to his views was increasing, he visited Rome in 1611. He was given a friendly reception by high dignitaries of the Church. But fresh objections arose, and in 1615 he was

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secretly denounced to the Inquisition. Going to Rome again in that year, he was admonished by Cardinal Bellarmine, on the Pope's instructions, to abandon his opinions; and this was made the subject of a Papal Decree in 1616.

CONTROVERSY WITH CHURCH

For the next year or two he suffered from ill health and was comparatively inactive; but in 1618 he had a controversy with a Jesuit named Grassi regarding three comets which appeared in that year. This developed into a dispute on the larger questions of Astronomy and Philosophy and their bearing on Theology. In 1623 he published his work "Il Saggiatore" (The Assayer) which dealt with the Copernican system in the indirect way which seemed necessary after his admonition of 1615 and the Decree of 1616; and later, in the year 1632, he published his "Dialogue on the Two Chief Systems of the World, the Ptolemaic and the Copernican." This book was really a strong advocacy of the Copernican system, and although Galileo tried to protect himself by an introduction (which nowadays reads like a piece of irony) and by the use of dialogue, it resulted in his examination by the Inquisition at Rome in 1633, and his famous forced retraction and sentence of confinement. This confinement was ultimately at his country house in Arcetri where he continued to do some scientific work.

In the year 1638, his eyesight, which had previously troubled him, failed completely—a failure which has been attributed to his observations of the Sun without adequate protection against the intense illumination even of his small instruments.[3]

EFFECTS OF HIS WORK

It is possible that Galileo's telescopic discoveries have been given rather too much prominence, as they were inevitable sooner or later once the telescope had been invented. But he used these discoveries in a way not very likely to have been equalled, in support of the truth of the Copernican system. His work on dynamical subjects leading to the recognition of the laws of motion and of force as the cause of motion was,

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however, entirely new and independent, and a brilliant example of the true scientific method.

Astronomical problems were thereby laid open to the reason as purely mechanical ones, and the metaphysical obscurities of Aristotle and other earlier workers removed. Planets were to be regarded as ordinary moving bodies and logical treatment about the nature of their orbits was made possible. The task of Newton was thus made clear and definite by Galileo.

For some time after the death of Galileo in 1642 few outstanding discoveries were announced, but some good progress was made. Kepler's three laws gave a great stimulus to theoretical speculation directed to ascertain their cause; and the use of the telescope by Galileo initiated a period of steady observational work with the new instrument.

NAPIER AND LOGARITHMS

Rather earlier, however, an invention in mathematics of paramount importance had been produced by a Scotsman, Napier of Merchistoun (1550-1617), enormously facilitating calculation. This was by the introduction of logarithms. Its value was such that the great French mathematician, Laplace, wrote of it in after years as "An admirable artifice which in reducing the labour of months to days, doubles the astronomer's life, and helps him to avoid the errors and annoyances always accompanying long calculations."

Napier was the discoverer of the general principle of logarithms, but confined its application to trigonometrical calculation. Extension to arithmetical operations was due to his friend Henry Briggs (1561-1631), Gresham Professor of Astronomy, and later Savilian Professor at Oxford.

BAYER

And in the year 1603 a publication had appeared which was of great help to practical observers. This was an atlas of the heavens, "Uranometria," by John Bayer (1572-1605) a Bavarian lawyer. It "was a scientific work, the result of thought, study and laborious observation. It first placed

on record the approximate positions and the magnitudes of some 500 stars in addition to the 777 which formed the renowned catalogue published by Tycho Brahe only one year previously. And although adopting for Tycho's stars, without question, the magnitude as well as the positions assigned by him, it gave for the new stars . . . . magnitudes derived directly from observations, so that these may be accepted as a truthful record, for future ages, of their aspect at that important epoch . . . . The system which he adopted for his notation was not, as has been supposed by many, that of assigning the letters in the constellation in the alphabetical order [the Greek alphabet] corresponding to the order of brightness of the several stars. On the contrary . . . .Bayer made no atempt at other discrimination of the relative brightness of the stars, than according to the six orders of magnitudes which had been transmitted from antiquity. For the stars of each order the sequence of the letters . . . depended upon the position of stars in the [constellation] figure, beginning usually at the head, and following its course until all the stars of that order of magnitude were exhausted."[4]

SCHEINER

Christopher Scheiner (1575-1650), whose independent discovery of sunspots has already been mentioned, first noted the bright cloud-like streaks on the Sun which were given the name of "faculae" (little torches). He made a long series of observations of sunspots, noting their changes of position on the Sun's disc and their varied appearance. As a consequence of this, and the work of Galileo and Fabricius, there followed the discovery of the rotation of the Sun, the determination of the time (about 27 days) in which it appeared to rotate as seen from the Earth, and the position of its axis of rotation.

GASCOIGNE AND HORROCKS

Gascoigne (1612-1644), a young astronomer who was killed at the battle of Marston Moor, has been already mentioned as the first to use cross wires in a telescope, thereby turning that instrument into one of precision in measurement of positions and angular separation, or of diameter, of objects

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in the sky; he invented a "micrometer" capable of dealing with angles of a few seconds of arc. He and his friend Jeremiah Horrocks or Horrox (1617?-1641) were the first to observe, in 1639, the passage of Venus between the Earth and the Sun across the latter's disc, a "transit of Venus." Horrocks made considerable improvements in the theory of the Moon's motion and noted periodic irregularities in the movements of Jupiter and Saturn later demonstrated to be due to their mutual attractions on each other.

HEVEL

John Hevel of Danzic (1611-1687), known better as Hevelius, made a catalogue of about 1500 stars, with positions for them rather more accurate than those in Tycho's catalogue. He continued to use the naked eye with his measuring instruments, declining to employ a telescope with them. But he used telescopes for general observation, one of which was 150 feet long, as before the discovery of the achromatic object-glass by Chester More Hall in the following century, telescopes had to be very long in relation to their aperture to obtain good images as free from outstanding colour as possible. In 1647 he published a finely illustrated book on the Moon, "Selenographia," describing and giving names to the mountains, craters and so-called seas. Some of his names are still in use but more are in accordance with the scheme of nomenclature of John Baptist Riccioli (1598-1671), or M. F. Langrenus (1600-1675), which uses the names of eminent men for craters.[5] Hevelius also wrote two books giving the first systematic account of comets observed in the past, observed all the five known planets, and studied several of the brighter nebulae and the variable stars P Cygni and Mira Ceti.

HUYGHENS

Christian Huyghens (1629-1695), a citizen of the Hague, made contributions of first rate importance to Astronomy. As mentioned earlier he was the first to construct a pendulum clock and he made improved telescopes by which he discovered Titan, the brightest satellite of Saturn, in 1655; he also found in 1656 that the appendages of Saturn which had so puzzled

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Galileo, were due to a flat ring surrounding that planet. This discovery was made with a non-achromatic refractor of $2\frac{1}{3}$ inches aperture and 23 feet focal length, the magnifying power being 100. Huyghens's invention, about 1662-6, of his two-lens form of eyepiece, was of great importance in the improvement of telescopes.

In a book published in 1698 after his death, "Cosmotheoros,' Huyghens assumed that the Sun and stars are similar bodies and that the stars are uniformly distributed in space out to infinity, each having a system of planets. By a comparison he made between the light from the Sun and that from Sirius (assumed equal to the Sun in luminosity), he estimated a distance of that star at least equal to 28,000 times the space between the Sun and the Earth, a distance which is, however, only about a twentieth of the correct value[6].

PICARD

Jean Picard (1620-1682), one of a group of astronomers attached to the Royal Observatory at Paris, inaugurated the systematic attachment of telescopes to astronomical instruments (1667) ; and he introduced the method of measuring one of the co-ordinates of position of a star or other body on the sky known as Right Ascension, by observing the time of its transit across the meridian. Adrien Auzout (1622-1691), another of the Parisian group, introduced an improved instrument similar to the Gascoigne micrometer.

RICHER

Financed by the Paris Academy of Sciences, another of the Paris Observatory staff, Jean Richer (?-1696) went at the suggestion of J. D. Cassini on a scientific expedition to Cayenne (1671-3) which produced two results of importance. It was found that a pendulum of a given length swings more slowly there than at Paris, indicating an intensity of gravity less near the equator than in higher latitudes and that the Earth is an oblate spheroid. Measures of the position of Mars in the sky were also obtained, which when compared with observations made at the same time by Flamsteed in England and Cassini in France enabled the latter to deduce

a parallax for the Sun of 9″·5, giving a distance about 8 per cent less than the truth, but much the best value up to that time.

THE CASSINIS

The long connection of the Cassini family of astronomers with the Paris Royal Observatory began at this time in the person of Jean Dominique Cassini (1625-1712). It will be convenient to describe briefly at this stage the careers of all of them although this takes us beyond the date arrived at. Jean Dominique was born in Italy, and in 1650 was professor of Astronomy at Bologna, where he made observations of comets and of eclipses and a special study of the planets, discovering the rotation of Mars and of Jupiter, although at about the same time Robert Hooke had noted the rotation of both planets and Fontana had suspected the rotation of Mars in 1638. At the invitation of Louis XIV, J. D. Cassini came to France in 1668 and became a naturalized citizen in the year 1673. He was given the general supervision of the Paris Observatory, and made further discoveries there, including four new satellites to Saturn and the principal division in its rings known by his name; also that the fifth satellite, Iapetus, varies in brightness in the same period as its revolution round Saturn, showing that it must constantly turn the same face to its primary just as the Moon does to the Earth.* The first notice of the Zodiacal Light, a nebulous cone seen extending from the Sun's place along the Ecliptic, has been attributed to him, but it was seen, twenty-two years before Cassini's observation, in 1683 by an English clergyman, Childrey, although a reference in one of Shakespeare's plays[7] suggests that it was known earlier. The name Zodiacal Light is however due to Cassini. In the last two years of his life he became totally blind.

His son, Jacques (1677-1756), did much first class work on measurement of the size of the Earth, and continued his father's investigations, particularly on the rotation of the planets. He was not given the title of director of the Observatory, as there was no official director until 1771, all

*Although Cassini thought of this he did not accept it as an explanation of his observations.

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astronomers there being Academicians and considered as of equal rank, J. D. Cassini, because of experience, having been empowered to arrange any investigations requiring the co-operation of several. But Jacques performed some of the duties of that office such as procuring new instruments. He made the noteworthy suggestion, in 1715, that Saturn's rings are composed of small particles, an idea not proved until more than a century later by Clerk Maxwell (see p. 158); and in 1738, from a comparison of his own observations with those of Richer at Cayenne in 1672 he followed Halley (see p. 101) in demonstrating that certain stars had proper motions in the sky.

Of Jacques's three sons one only, César Francois, known as Cassini de Thury (1714-1784), became an astronomer. Although his chief work was in geodesy and geography he was appointed in 1771 to be the first director of the Observatory. During his period of office the affairs of the Observatory were in a poor condition, instruments having been sent on expeditions and borrowed for small observatories, and never returned. The fourth Cassini, another Jean Dominique (1748-1845) was, at the age of 20, sent on a ship to America and Africa to test marine chronometers. He took up astronomical work on his return to the Observatory, but soon had to devote most of his time to administrative duties. He was the first of the family to accept all the Newtonian theories. Possessed of good qualities for the Directorship his career was unfortunately cut short by the French Revolution and he was arrested in 1794 as an aristocrat. But he was released from prison, being protected by the people of the Observatory quarter, who remembered his past kindnesses to them. He retired to the country where he lived another 50 years, dying at the great age of 97.

ROEMER

One famous discovery at the Paris Observatory in its early days was that of the velocity of light. The Danish astronomer Olaus Roemer (1644-1710) spent nine years from 1672 to 1681 in Paris where, in 1675, he announced his famous discovery. He made this through investigations of the eclipses of the satellites of Jupiter, particularly the first satellite. These

did not always occur at the predicted times; at opposition, when the Earth was closest to Jupiter, the eclipse took place earlier than expected; but at conjunction where the Earth was farthest from Jupiter, it occurred later than predicted. Only at places on the Earth's orbit where the distance Sun-Jupiter was equal to the distance Earth - Jupiter did the eclipses occur at the computed times. From this fact Roemer concluded that time is required for light to travel from one point to another. As his observations were not of modern accuracy he found 11 minutes for the time light takes for the mean distance Sun - Earth instead of the present accepted value of 8 minutes 18·6 seconds. This discovery of Roemer's was not accepted at first by contemporary astronomers such as Cassini, but it was adopted very soon by Flamsteed. In fact the conception of movement of light at a finite velocity in space was a new idea to most scientists although the Sicilian philosopher Empedocles (*c.* 500-400 B.C.) had suggested it. And Francis Bacon (1561-1626) had written expressing "a doubt as to whether the face of the serene and starry heavens be seen at the instant it really exists, or not till some time later .... For it seems incredible that the rays of the celestial bodies can pass through the immense interval between them and us in an instant; or that they do not even require some considerable portion of time."

During his stay at Paris Roemer took part in triangulation work for measurement of the size of the Earth and he built "orreries" (so called at a later date after the Earl of Cork and Orrery who had one made in 1715). These were machines showing the relative positions and movements of the members of the Solar system. On his return to Copenhagen in 1681 as professor of mathematics, he invented new types of instruments, the most notable of which was the Transit Instrument, and he secured observation of great accuracy. Only one series of Roemer's observations is still in existence, all others having perished in a great fire in 1728 at Copenhagen. It is no small tribute to him that this series furnishes the only observational results of these days deemed worthy of inclusion in modern research on stellar movements. It is referred to as the "Triduum" because it was the work of three successive nights in 1706.

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DESCARTES

Another great name of the earlier part of this period is that of René Descartes (1596-1650), who made great advances in pure mathematics that helped Astronomy. But his astronomical writings, which became very popular, rather retarded than advanced progress. These included his famous "Vortex Theory," an attempt without any support of an experimental nature, to explain the movements of the planetary system by a combination of vortices or eddies, an hypothesis built entirely on purely imaginary physical concepts. But it may perhaps be said of him that by making the Sun the centre of the system of planets he gave some help to the general adoption of the Copernican theory.

References

[1] J. H. Johnson, *Journal of the British Astronomical Association*, vol. 41, p. 164 (1931).
[2] P. Pagnini, *Ibid*, p. 415.
[3] Humboldt, "Cosmos," vol. iii., p. 273.
[4] B. A. Gould, "Uranometria Argentina" (1879).
[5] For a history of lunar nomenclature, see "Who's Who in the Moon," *Memoirs of the British Astronomical Association*, vol. 39, Part I, pp. 3-5.
[6] For an excellent account of Huyghens's Life and Work, see A. E. Bell, "Christian Huygens" (1947).
[7] "Romeo and Juliet," Act III, Sc. V, where Juliet says :

"Yon light is not daylight, as I know it :
It is some meteor that the Sun exhales."

CHAPTER VII

NEWTON

ISAAC NEWTON was born at Woolsthorpe, a hamlet eight miles south of Grantham, a few hundred yards from the main road to London, on Christmas Day, 1642 (O.S.), nearly a year after Galileo's death. The house in which he was born is practically unchanged and is still owned by the family to which it was sold in Newton's lifetime. He was born after the death of his father, and was a small, premature, delicate child. His widowed mother married a clergyman in 1645, leaving Isaac in charge of her mother.

EARLY EDUCATION

For about nine years he attended small village schools in the neighbourhood and then went to King's School, Grantham. When he was about fourteen years of age his step-father died and his mother returned to Woolsthorpe with three more children from her second marriage. Her first action was to send for Isaac to return and assist her by taking over control of his paternal acres. He began his new duties but he did not settle down to farming, and it was not long before he was sent back to school at Grantham. This was in 1658; for three years he worked hard at school in preparation for going to Cambridge University, where his maternal uncle had advised that he be sent. No other scholar at the King's school compared with him for ability or acquirements; and when he left in 1661 to go to Cambridge, the schoolmaster called the pupils together using the occasion not only to praise the departing scholar but to point out to the boys their good fortune in having been fellow-students with so brilliant an example.

When Isaac Newton went to Cambridge he seems to have possessed a good all-round education of the normal type for the time. He had received all the book-learning available, but it was clear that his tastes then lay in the direction of practical experimentation. He made many mechanical contrivances, waterclocks, model windmills and so on, and he

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constructed, when only nine years of age, a wall-sundial at his home. This sundial is now a treasured possession of the Royal Society in London. A notebook of his, dated 1659, which is still in evidence, contains entries on practical methods of drawing and colouring, some chemical experiments, and also notes on spelling and grammar. These all appear to have been written when Newton was from thirteen to sixteen years of age, and belong to the first part of the notebook. A later part, written just before he went to Cambridge, is chiefly concerned with mathematical and astronomical matters and shows interest in the formation of the Calendar and in the Copernican system.

AT CAMBRIDGE

Little is known of Newton's first years at Cambridge, but it is clear that he made very useful progress in mathematics and allied subjects, surprising his professors and tutors by the speed at which he absorbed all that they knew. By 1665 when he graduated as Bachelor of Arts, he had undoubtedly read most of the mathematical books of the time, and had received a solid grounding in the subject. But a great deal of his time during these first years at the University was devoted to experimental work, including experiments in Chemistry and Optics, and the polishing of lenses.

In this same year 1665, there was a serious break in his University studies which meant the loss of the use of the College library and of the experimental apparatus which had become so necessary to him ; and it also meant that for employment of his time he was largely thrown upon the knowledge of which he had mental and written notes. The cause of the break was the appearance of the Great Plague in London and its spread to Cambridge. In order to avoid the further spread of infection, all the students were sent home in August. Newton had already returned to Woolsthorpe where he stayed, except for a month or two, for two years.

MATHEMATICAL AND PHYSICAL DISCOVERIES

By the year 1666 he had made fundamental discoveries in mathematics (the Binomial Theorem, Fluxions and Inverse

Fluxions, *i.e.*, the differential and integral calculus) by which all his and other investigators' work in dynamics or mathematical Astronomy was made possible. He had also discovered, by experiments of passing light through a prism of glass, that white light is a composite of a range of colours, violet to red, and had also found the law of gravitation. His own words written years later, give an interesting account of these :

"In the beginning of the year 1665 I found the method of approximating series and the Rule for reducing any dignity of any Binomial into such a series. The same year in May I found the method of tangents of Gregory and Slusius, and in November had the direct method of Fluxions, and the next year in January had the Theory of Colours, and in May following I had entrance into the inverse method of Fluxions. And the same year I began to think of gravity extending to the orb of the Moon and having found out how to estimate the force with which a globe revolving within a sphere presses the surface of the sphere, from Kepler's Rule of the periodical times of the Planets being in a sesquialterate proportion of their distances from the centre of their orbs I deduced that the forces which keep the Planets in their orbs must be reciprocally as the squares of their distances from the centres about which they revolve : and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded Mathematics and Philosophy more than at any time since."[1]

The discovery of the action of a prism on light formed the basis of "spectrum analysis," to which so much chemical and astronomical knowledge is due. Unfortunately Newton did not realise that by use of glasses of different density and powers of refraction of light, the difficulty of chromatic aberration in telescopes would practically be got rid of. But in 1668 he invented the form of reflecting telescope known by his name, constructing one or two small ones himself. This kind of telescope, and the related Gregorian and Cassegrainian forms, have been much developed, the largest sized modern instruments being reflectors of one or the other type. The New-

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tonian has always been the favoured one for amateurs everywhere as it is less costly for a given power, is free from colour aberrations except for small defects of the kind in the eyepieces, and in some ways more convenient in use.

THE LAW OF GRAVITY

The law of gravity as discovered by Newton may be stated as follows :—Every particle in the Universe attracts every other particle with a force varying inversely as the square of the distance between them, and directly as the product of the masses of the two particles.

When considering the central force which is responsible for the movement of the planets in their orbits, it occurred to Newton that a force of the same nature as that which causes the fall of a body to the Earth, might constrain the Moon to revolve round the Earth by preventing it from flying off at a tangent, and continuously deflecting it towards the Earth. In the words of W.K. Clifford, the Moon might be said to be "a falling body, only she is going so fast and is so far off that she falls quite round to the other side of the Earth, instead of hitting it ; and so goes on for ever."[2]

Newton had already seen from Kepler's first law (see p. 59) that for planets going round the Sun in elliptical orbits with the Sun in one of the foci of the ellipse, the attracting force towards the Sun must vary inversely as the squares of the distances. Applying this inverse square law he found, from the known distance that a body falls at the Earth's surface in a minute of time, that at the Moon's distance (which he knew to be 60 times the radius of the Earth) it would fall 16 feet. Or, put otherwise, he knew that at the Earth's surface the distance in a second is 16 feet. The space is proportional to the product of the force and the square of the time. The force varies inversely as the square of the distance from the Earth, so that the space varies as the square of the time, and inversely as the square of the distance. The distance being increased 60-fold, the space fallen though should be the same in a minute at the Moon's distance as in a second at the Earth's surface. Assuming a value for the Earth's radius he calculated the distance through which the Moon is actually

drawn, to be only 14 feet. This discrepancy temporarily condemned the idea so that he laid aside the calculations. Some years afterwards, however, he used a more correct value for the Earth's radius, as found by Picard, which removed the discrepancy. This value corresponded to a value of 69 miles to a degree of the meridian in place of 60, the figure previously used. But there are reasons which lead one to believe that the delay in the discovery was not due to a wrong value of the Earth's radius, but to the difficulty Newton had at first in proving that a spherical body like the Earth attracts another body as if its matter were all concentrated at its centre.

In the meantime while Newton was engaged privately on this investigation, the question of the law behind the motions of the Moon and planets was much discussed by some scientific men. Robert Hooke (1635-1703), who had almost a habit of claiming to be first in most scientific discoveries of the day, Christopher Wren (1632 - 1723), better known as a great architect, and Edmund Halley (1656-1742), later Astronomer Royal, had all suggested with more or less definiteness, but no proof, that the cause was an attraction between the planets and the sun which varied inversely as the square of the distance.

THE "PRINCIPIA"

Ultimately Halley visited Cambridge and learnt from Newton the explanation and the mathematical proof. On a second visit he urged that the results be published; which Newton did as a communication to the Royal Society. Further persuasion from Halley led to Newton writing, in fifteen months, his great work. "Philosophiae Naturalis Principia Mathematica," and it appeared in 1687, the expenses of publication having to be met by Halley owing to the lack of funds at the Royal Society.

Once the law had been established by Newton, he proceeded to work out some of its consequences. He perceived that the shape of a massive body must, under gravitation, be globular; and that rotation of such a globe, through the centrifugal tendency of its moving parts, would produce a flattening at the poles. Making certain assumptions as to the inside of the Earth which we now know are not correct, he calculated, by

a most ingenious method, what this flattening should be, and found a value of 1/230th of its radius. This is not far from the truth, which is about 1/297th from measurement and from theory.

He also saw that the consequent protuberance of matter at the Earth's equator would be unequally attracted by the Moon, the side nearer to it more than the other, and similarly by the Sun but to a lesser extent owing to much greater distance. This would tend to incline the Earth's axis of rotation in some positions of the lunar orbit and of the Earth's solar orbit, which would cause a rotation in space of the axis, of a conical nature,* leading to a precession of the points of intersection of the plane of the Earth's equator with the plane of its orbit. He thus gave an explanation of the Precession of the Equinoxes, discovered by Hipparchus, and he calculated its period and amount.

In general he showed that a body projected in space, and acted upon by a central force in accordance with his law of gravitation, would describe a conic section, a hyperbola, parabola, or ellipse, the shape of which could be completely determined, given the initial distance of the body from the position of the central force and the direction and velocity of its initial motion. From this it followed that comets as well as planets obeyed the law, and he found that, in cases he investigated, the paths of comets were either parabolas or greatly elongated ellipses.

One important result of Newton's work was a method of obtaining the relative masses of the heavenly bodies by their action on each other. A comparison of the mass of the Sun with the masses of the planets which have satellites was thus obtained, and also the mass of the Earth as compared with that of the Moon.

Certain irregularities in the Moon's movements due to the disturbing action of the Sun were calculated by him, some of which were actually still to be found by more exact observations by others in the future.

* The explanation of the conical motion was first given by Copernicus.

THE TIDES

Newton also explained the phenomena of the Tides. The history of human thought on these is interesting. Chinese records about 1000 B.C. remark on the influence of the Moon, this being easily recognized by the occurrence of high Tides at New and Full Moon. Among the Greeks, Pythias about 350 B.C., Seleucus and Posidonius about two centuries later, and Cleomedes in the second century A.D., and various Roman writers, noted that there was a connection, and it also appears to have been known to the Arabian astronomers. But none of them had any explanation to offer of the cause. Bacon in the thirteenth century and Kepler in the seventeenth stated, however, that the attraction of the Moon caused the Tides, and in 1644 Descartes attempted to account for the occurrence, without success, by his vortex theory. But Galileo differed from this general opinion, believing that in some way the Tides were due to the Earth's motions of rotation and revolution.

Newton's demonstration on the basis of the law of gravity was, however, conclusive. He showed that the Tides are caused by the Moon (chiefly), and the Sun, producing an acceleration on the water of the Earth nearest to them, greater than that on the solid Earth, and greater on the Earth than on the water furthest from them, the general effect on the seas being the same as if the parts of the oceans nearest the Moon or Sun were attracted and those on the opposite side repelled. The recurrence of the Tides, in general twice daily, was shown to be an effect of the Earth's rotation ; at New and Full Moons the lunar and solar effects would be added together (Spring Tides), whereas at Half Moons they would tend to counteract one another (Neap Tides) ; so that the observed greater Tides every fortnight were thus explained.

Newton was elected a Fellow of his College at Cambridge in 1667, and in 1669, he was appointed Lucasian Professor of Mathematics in succession to his friend and teacher the eminent mathematician, Isaac Barrow (1630-1677). He was elected a Fellow of the Royal Society in 1672. Most of his life until about 1700 was spent at Cambridge with occasional visits to his home at Woolsthorpe.

The "Principia," as his great work is briefly named, deals

with many matters of scientific importance besides the law of gravitation. Motions of bodies in free space and in a resisting medium, the theory of Fluxions, problems of sound and wave propagation, are some of the subjects treated in it with a revolutionary novelty. The book contains the nucleus of a true system of natural philosophy, but it was not accepted as a standard for a long time.

In regard to the planetary system the doctrines of Descartes held the field for many years. His Vortex Theory has already been referred to. This suggested an explanation based on the idea that an invisible substance called ether, composed of very small particles in a continuous state of movement, filled all space. These particles were supposed to get smaller by friction among themselves, the "dust" collecting in the centre of the vortices, which were made up of whirling ether particles, and forming Suns. Even the Keplerian elliptical movements were ingeniously worked into the theory. But although Descartes' notions had no real scientific basis, the rival Newtonian Theory was accepted by very few out of England for many years.

It is interesting to note that, even though Newton was so closely connected with Cambridge all through his life, it was not there that his system of philosophy was first officially adopted. That was at the Scottish Universities of St. Andrews and Edinburgh. But before his death in 1727 all the English Universities were more or less definitely Newtonian.

As a book the "Principia sold very well, so that by 1691 a copy was very difficult to obtain. In 1713 a second edition edited by Roger Cotes of Cambridge was published, containing an Index which had not been included in the first edition. A third edition edited by Henry Pemberton, with a new preface by Newton, appeared in 1726; and an English translation, several times reprinted, came out in that year. There have also been many foreign translations.

HIS "OPTICS"

In 1704 Newton's other great work "Optics, or A Treatise of the Reflexions, Refractions, Inflexions, and Colours of Light" was published. It is in English, unlike the first editions

of the "Principia" which were in Latin, and contains, besides his work on light, the results of many researches in chemistry and further comments on gravity. It also contains two treatises on the development of the integral calculus which unfortunately led to a regrettable dispute with the Continental mathematician G. W. Leibnitz (1646-1716), as to the first invention of the calculus. This controversy reached the conclusion that both Newton and Leibnitz independently discovered it, but that Newton was first to do so.

HIS LATER CAREER

The later half of Newton's life was taken up largely with official work at the Royal Mint in London of which he was appointed Warden in 1695, and Master in 1699. He also produced some studies of a non-Scientific description, such as Biblical and General Chronology. He sat in Parliament from 1688 to 1690, spending much of this period in London. He did not resign his Cambridge professorship until 1701, when William Whiston, who had been acting as deputy, was appointed in his place. No great discovery was made by him during this time although he developed some of his earlier work and obtained a few results of interest. His achievements at the Mint in the restoration of the badly depreciated coinage were of great importance and value to his country; and he kept a very close connection with the Royal Society of which he was President from 1703 until his death in 1727. He was buried in Westminster Abbey.

Of the astronomical discoveries first announced in the "Principia," the great French mathematician Laplace observed :—

> "The imperfection of the Infinitesimal Calculus, when first discovered, did not allow Newton to resolve completely the difficult problems which the system of the world offers, and he was often compelled to give mere hints, which are always uncertain until they are confirmed by a rigorous analysis. Notwithstanding these unavoidable defects, the number and generality of his discoveries relative to this system, and many of the most interesting points of the Physico-mathematical sciences, the multitude of original

and profound views, which have been the germ of the most brilliant theories of the geometers of the last century, all of which were presented with much elegance, will assure to the 'Principia' a pre-eminence above all the other productions of the human intellect."

And Lagrange, another eminent mathematician, wrote :—

"Newton was the greatest genius that ever existed, and the most fortunate, for we cannot find more than once a system of the World to establish."

REFERENCES

[1] From a MS. in the "Portsmouth Papers."
[2] W. K. Clifford, "Aims and Instruments of Scientific Thought."

CHAPTER VIII

THE EIGHTEENTH CENTURY: GRAVITATIONAL

It has been remarked in the last chapter that, except in Britain, the Newtonian system was accepted by very few for a number of years. Such famous mathematicians and astronomers as Descartes, Leibnitz, Huyghens, and the first three Cassinis admitted the correctness of only parts of Newton's theories; none of them accepted Newton as a whole.

But just half-a-century after the publication of the "Principia," Voltaire published a short popular exposition of Newton's discoveries in Astronomy and Optics, and this gave a certain impetus to the general acceptance that soon took place, although even as late as 1730 the Paris Academy of Sciences awarded a prize to an essay on the planetary motions by John Bernouilli, written on Descartes' principles, giving second place to a Newtonian essay.

NEWTON'S FOLLOWERS

During the period intervening from the "Principia" to that time, the direct followers of Newton did not develop his discoveries to any extent. This has been explained as perhaps because of scarcity of men of outstanding ability, but it was certainly at least partly due to the difficult mathematical methods he employed. These were geometrical rather than analytical. In the former each step is part of a sustained and continuous severe geometrical reasoning, interpretable at all stages in terms of the original problem; in the latter, the work is performed by algebraic methods according to certain known rules without any question of interpretation of the intermediate steps. The new analytical methods were at first due chiefly to Leibnitz, and to the brothers James Bernouilli (1654-1705), and John Bernouilli (1667-1748), and John's son Daniel (1700-1782), and also in some degree to Newton himself. In the course of time an analytical method was produced which did not require the intellect of a Newton for its use. In other words Newton's geometrical method was of the nature

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of a Bow of Ulysses which could be used successfully by himself alone.

EULER

Leonard Euler (1707 - 1783), Alexis Claude Clairaut (1713-1765), and Jean-le-Rond D'Alembert (1717-1783), were the first astronomers to advance beyond Newton in the analysis of the planetary and lunar motions. The first named has been termed perhaps the most versatile as well as the most prolific of mathematicians of all time, his papers, in addition to several books, on mathematical, astronomical, and physical subjects, totalling about 800. His chief astronomical publications were on the Tides, the periodic irregularities in the movements of Jupiter and Saturn, two solutions of the problem of the mutual gravitational disturbances of three bodies which helped in dealing with the Moon's motions, and a number of useful contributions to the general theory of planetary motions. He introduced the idea of a planet's orbit as an elliptical path which, owing to gravitational disturbances by other planets (perturbations) is gradually changing in position, size and shape. That is to say the "elements" of the orbit are changing. These elements are : (1) the size, (2) the shape, (3) the position of the orbit in its own plane, (4) the inclination, and (5) intersection of the orbit with a fixed plane, and (6) the position of the planet in the orbit.* The theory of Euler was that the motion of a planet could be regarded at any moment as performed in an ellipse whose "constants" were continually changing under the action of other planets. With later mathematicians this conception has led to fruitful results. Euler was probably the first to publish an explanation of the principles of the achromatic refracting telescope (see page 109).

CLAIRAUT

Clairaut, who was so precocious that he presented a mathematical memoir to the Paris Academy of Sciences before he was 13, and was admitted to the Academy when 18, wrote on the figure of the Earth, the problem of three bodies, and

*In technical terms : (1) the mean distance, (2) the eccentricity, (3) longitude of perihelion, (4) inclination, (5) longitude of node, (6) epoch.

planetary theory. He calculated, with great computing labour, the date on which the comet known as Halley's would, at its next appearance in 1758, pass through its nearest point to the Sun, and found a result only a month and a day wrong. In one of his investigations on the motion of the Moon's "Perigee" (the point in its orbit nearest the Earth) he at first found only half the value given by observation, going so far as to suggest a modification of the law of gravity which would introduce another term, in addition to the inverse square, varying inversely with the fourth power of the distance. The great naturalist Buffon, however, believing in the truth of Newton's discovery, suggested that the difference found by Clairaut was due to an error in the calculations. He therefore tried to induce Clairaut to re-examine his work. After some hesitation, Clairaut did this and found that when his calculations were carried to a greater degree of accuracy, the difference disappeared and his results agreed with observation. This was the first great post-Newton triumph for the law of gravitation.

D'ALEMBERT

D'Alembert's first important work was a "Treatise on Dynamics" (1743). During following years he wrote on mathematical physics and the problem of three bodies; and in 1749 he published a work on the Precession of the Equinoxes in which that phenomenon and that of Nutation (discovered by Bradley, see page 104) were successfully dealt with. He was a constant rival of Clairaut's; both wrote important papers in which results published by the other were criticized. Although having its somewhat unpleasant features, this led to detection of weak points and subsequent improvements.

LAGRANGE AND LAPLACE

Joseph Louis Lagrange (1736-1813) was of French descent, but was born in Italy. He showed very great mathematical abilities at a youthful age. In 1764 he won a prize offered by the Paris Academy for an essay on the libration of the Moon. In 1766 he succeeded Euler as the head of the mathematical section of the Academy at Berlin where he spent the following twenty-one years of his life. During this time he

wrote about sixty important papers and his great book the "Mécanique Analytique." In consequence of the death of Frederick II in 1787, he left Berlin and went to France where he was elected to the Paris Academy.

Pierre Simon Laplace (1749-1827) was born in Normandy. He went to Paris at the age of 18 with a letter of introduction to D'Alembert, but did not succeed in meeting that mathematician until he had written a letter to him on the principles of mechanics; this impressed D'Alembert who got him a post at the Paris Military School. From that time he lived continuously in Paris, holding various official posts there. He wrote very many papers in Astronomy and mathematics, but his work was very largely contained in his great book the "Mècanique Celeste" that appeared in five volumes between 1799 and 1825. Another work of his was an excellent treatise on popular Astronomy, "Exposition du Système du Monde" (1796), and he wrote treatises on probability that formed the foundation of the subject. He held various public appointments in spite of the revolutionary changes of government of the period, such as member of a Commission for Weights and Measures, and of the Bureau de Longitude, and professor at the École Normale. When the Bourbons returned in 1814, he offered his services and was made a Marquis.

Nearly all Lagrange's and Laplace's most important work was done after Clairaut and D'Alembert had finished theirs; but Euler wrote for almost 20 years later. Lagrange and Laplace survived Euler for more than 30 years.

For a long time the progress of mathematical Astronomy consisted of little more than a series of achievements by Laplace and Lagrange, of which not more than the merest sketch is possible here.

It had been suspected for some time, from comparison of ancient with modern eclipses, that the Moon's motion was slowly accelerating, and it had been computed by R. Dunthorne (1711-1775) in 1749, and by Mayer, that the change amounts to ten seconds of arc at the end of a century. The question was considered by Lagrange in 1774, but he only succeeded in showing that the acceleration is not due to the influence of the Earth's shape on the motion of the Moon. Laplace was, however, apparently more fortunate. His researches seemed

to prove that there is an acceleration of the observed amount due to a slow diminution of the eccentricity of the Earth's orbit and a consequent shortening of the month which has been going on for a long time, although it will cease and the reverse take place after about 24,000 years. His result undoubtedly explains an acceleration in the motion of the Moon; but it was shown by J. C. Adams more than half-a-century later that the correct value by theory should have been about 5 or 6 seconds. The accuracy of the law of gravitation might have seemed to be in question, but it was later demonstrated that the difference between calculation and observation is due to a slow change in the length of the terrestrial day, caused by a retardation of the Earth's rotation on its axis produced by tidal friction. This slight lengthening of the day diminishes the number of seconds in a month, which makes the month, and the Moon's period, apparently shorter. It is now generally accepted, therefore, that the lengthening of the day by 1/1000th of a second per century (see page 21) explains the discrepancy.

A second difficulty was found in the abnormal variation of the velocities of Jupiter and Saturn in their orbits, a variation which had long been a subject of mystery. The deviation from the expected motion is rather considerable, amounting in the course of 2000 years to a difference of 3° 49′ in the celestial longitude of Jupiter, and 9° 15′ in that of Saturn, *i.e.*, about seven and eighteen times the Moon's diameter respectively. The question was carefully investigated by Laplace, who found that these deviations were due to the fact that five revolutions of Jupiter take almost the same time as two of Saturn (5 times Jupiter's period is 59·3 years, twice Saturn's is 58·9 years). This fact gives rise to a great reciprocal perturbation between the two planets with a period of about 930 years. The discovery was another verification of, and triumph for, the law of gravitation.

Laplace also examined the question of the figure of the Earth from irregularities in the lunar motions which, he showed, depended on the oblate spheroid shape of the Earth; and he computed that the amount of flattening responsible is about 1/300th (it is about 1/297th). He also did valuable work on the Tides, allowing, for the first time, for the effect of the

Earth's rotation, deducing that depth of ocean is an important factor and showing that if depth were uniform some tidal irregularities would disappear.

The extreme difficulty and triumphant results of the many investigations of Laplace and Lagrange in mathematical Astronomy and physical science make their places in the history of these sciences lower only then that of Newton. Between the two there was, up to Lagrange's death in 1813, a friendly spirit of emulation. From their elaborate calculations there emerged a result regarding the stability of the Solar system which is of great interest. Put shortly, it may be said that they found that the alterations in the inclination, eccentricity, and length of major axis, of the orbit of any of the planets, should not vary in value more than between certain limits. Laplace himself summarized this in the following short general statement: "From the sole consideration that the motions of the planets and satellites are performed in orbits nearly circular, in the same direction, and in planes which are inconsiderably inclined to each other, the system will always oscillate about a mean state from which it will deviate but by very small quantities."[1] There would be some fluctuations caused by perturbations, some long and some short in period, but they are limited in extent. If true, this meant that, apart from unforeseen circumstances, no great changes were to be expected in the conditions of the Earth due to varying distance from the Sun, or to alterations in seasonal variation of heat caused by variation of eccentricity of the Earth's orbit, or to movement in angle of the Ecliptic which, unaccompanied by a corresponding change in the equator, would increase changes in temperature throughout the year.

This general result was got by calculations that ignored mathematically the possible effects of certain very small quantities. But, even so, apart from anything of a catastrophic nature that might unexpectedly happen, it appeared reasonable to Lagrange and Laplace to believe that, up to an indefinitely distant future time, the stability of the Solar System was assured. Regarded as abstract propositions their conclusions have been confirmed by later mathematicians. There are certain factors, however, which would upset the conclusion as far as an indefinitely long period is concerned. For one

thing, the bodies of the Solar System were all assumed to be perfectly rigid, which is not correct. Ocean tides on planets, or bodily tides, were thus left out of account, and their effects are not compensatory as was found to be the case by Laplace and Lagrange for ordinary mutual gravitational disturbances between the Sun and planets. And the effects of loss of mass by radiation, particularly in the case of the Sun, or of pressure of light, were not thought of, being findings of physical science long afterwards. Consequently the propositions regarding the condition of the Solar System at an indefinitely distant date receive only a modified assent nowadays.

Laplace also showed that whatever changes time may produce in the planets' orbits or in the plane of the Ecliptic, there is an invariable plane, that remains always parallel to itself, passing through the centre of gravity of the Sun, about which the whole system oscillates within limits. This plane is inclined at 1° 38·7′ to the Ecliptic (1900) and lies between the orbits of Jupiter and Saturn, nearer the former.

But Laplace is probably more famous to the general reader for his Nebular Hypothesis of the origin of the Solar System than for his profound mathematical researches.

LAPLACE'S "NEBULAR HYPOTHESIS"

Among the first attempts at forming a theory of the origin of our system were those made by E. Swedenborg (1688-1772), and by I. Kant (1724-1804) in the eighteenth century. Their speculations were put forward in a rather more scientific form by Laplace, in his book "Exposition du Système du Monde," published in 1796. He noted that, apart from the relatively close disposition of the Sun and his planets as compared with the distances of the stars,* there are numerous regularities of arrangement which are not the necessary consequence of gravitation (as are Kepler's laws), and that these arrangements are overwhelmingly indicative of a common origin for the system. He saw that, for the seven planets and fourteen satellites known at the time, the revolutions in their orbits were in the same direction ; and also that all rotations of Sun,

*Representing the diameter of the Solar System over the orbit of the most distant planet (Pluto) by one foot, the distance of the nearest star will be about two-thirds of a mile.

planets, and satellites that had been noted were similar in direction. This meant 30 or 40 movements all going the same way. The odds against this being fortuitous were enormous. And the facts that these motions of revolution and rotation all take place in planes only slightly inclined to one another, and that the eccentricities of the orbits are all small, pointed also in the direction of a common origin.

The chief features of Laplace's hypothesis are covered by the following quotations: "From a consideration of the planetary motions, we are brought to the conclusion that, in consequence of an excessive heat, the solar atmosphere* originally extended beyond the orbit of all the planets, and that it has successively contracted itself within its present limits"; and "We may, therefore, suppose that the planets were formed at its successive limits, by the condensation of zones of vapours, which it must, while it was cooling, have abandoned in the plane of its equator."[2]

The cooling and concentration under gravity was followed by a shortening of the period of rotation, the amount of angular momentum remaining constant, a stage ensuing when the centrifugal force at the equator got too large for the gravitational control. A nebulous ring would then separate, revolving in the same period as the main body, this ring breaking up into pieces which could, however, reunite in a single revolving and rotating body, thus forming the outermost planet.

The main mass would continue to contract and rotate even more quickly, with formation of successive rings and planets, nearer to the centre and with progressively shorter periods of revolution. In these planets the same process of rotation, contraction, and ring formation, would take place on a smaller scale with birth of satellites sometimes occurring. The rings round Saturn meant that there had been no condensation into satellites there.

The hypothesis is consistent with a number of the observed facts, but there are some difficulties insuperable to it. In the first place, it may be shown that rings such as are described would condense into many bodies but not into one single body. And, secondly, the distribution of the angular momentum

*In a later version Laplace envisaged a pre-solar entirely nebulous mass.

seems impossible. Ninety-eight per cent of the present angular momentum is possessed by the planets from Jupiter outwards; the Sun's rotation accounts for nearly two per cent, only one-tenth of one per cent being contributed by the four planets, Mars, Earth, Venus and Mercury. Internal changes inside the system cannot alter the total angular momentum, and it seems impossible to conceive anything in Laplace's scheme which could arrange this observed distribution of the momentum. The large amount possessed by the outer parts in fact suggests that there could not have been any condensation at all there, even into many small bodies. There is also the circumstance that the total angular momentum of the system, unalterable except by some very improbable disturbance from outside it, is at least 200 times less than it must have been when the system extended out to beyond Pluto.

These are not the only objections, and it seems therefore that the famous Nebular Hypothesis is unsound. It has been replaced in recent times by other theories, to be referred to later (see p. 252), none of which has so far withstood criticism. But Laplace's Hypothesis was a valuable and powerful stimulant to thought on the subject all through the nineteenth century, and an excellent stepping-off place for other theories.

BRITISH ASTRONOMERS

The striking series of successful investigations dealt with earlier in this chapter was not contributed to by British mathematicians, and the reason for this has already been discussed. In order for them to have advanced they would almost have required to begin over again, and they were not willing to do this. The more adaptable and flexible method of analysis perfected on the Continent was not acquired by them until well into the nineteenth century when mathematical studies were revolutionized at the University of Cambridge.

But on the observational side great progress was made by British astronomers during the last part of the seventeenth and throughout the eighteenth centuries, a period in which several great observers were active in this country.

REFERENCES

[1] Laplace, "Exposition du Système du Monde, 1796."
[2] Laplace, *ibid.*

CHAPTER IX

THE EIGHTEENTH CENTURY: OBSERVATIONAL

THE great observing astronomers of the period extending from the latter part of the seventeenth to the end of the eighteenth century contributed results of fundamental importance to the development and testing of the theories of Newton and his successors.

FLAMSTEED

The first of these astronomers to be mentioned is John Flamsteed (1646-1720), who has been described as "Tycho Brahe with a telescope" (A. de Morgan). He was born at Denby near Derby, and was to a great extent self-taught, leaving school at the age of fifteen. While still a boy he made some astronomical observations and wrote several papers. He studied the construction of sundials and, using home-made instruments, made a catalogue giving the places in the sky of 70 stars, and also computed the circumstances of an eclipse of the Sun which was to take place on June 22nd, 1666. His knowledge steadily advanced and he continued to make observations on his own, from which he noted great inaccuracies in the positions of planets predicted with the current tables. The importance of occultations of the stars by the Moon in accurately determining its places and the laws of its motion, was early realized by him, and he calculated in advance the occultations of bright stars for the year 1670, forwarding his results to the Royal Society where they were very favourably received. Entering Cambridge University he took the degree of M.A. in 1674.

In the following year he was appointed a member of a committee set up to report on a method for finding the longitude at sea by comparing the positions of the Moon (got by observing its distance from fixed stars) with its places predicted by means of astronomical tables, as suggested by a Frenchman Le Sieur de St. Pierre. The report was unfavourable, on account of the untrustworthiness of the places of stars in the

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existing catalogues and the unreliable predictions of the Moon's place by current tables. This led to the founding in 1676 of a national Observatory at Greenwich,* on the instruction of Charles II, with Flamsteed as Astronomical Observator (later, Astronomer Royal). His chief work was to be the construction of a more extensive and accurate star catalogue than any in existence, and in addition he entered into systematic observation of the Sun, Moon, and planets with a view to a revision of the theories of their apparent motions and the preparation of tables from which more accurate positions could be predicted.

Flamsteed worked under difficulties at Greenwich such as none of his successors have had to face. He was paid the inadequate salary of £100 per annum, and even this was not regularly received. He had to make or purchase his own instruments; and to defray the expenses thus incurred he was obliged to take pupils, and to become, from 1684 onwards, Rector of Burstow in Surrey. At first he made his observations with a sextant owned by Sir Jonas Moore, a Fellow of the Royal Society, who also lent him two clocks. The Royal Society lent him some other instruments, and Flamsteed brought with him a quadrant of three feet radius from Denby. With this meagre outfit of instruments he began his labours at Greenwich on October 29, 1676.

Flamsteed remained Astronomical Observator until his death. Although his first observation was on the date mentioned, it was not until September 11, 1689, that he began regular observations of stars crossing the meridian, with a mural arc, an instrument so constructed as to swing on the vertical face of a north and south wall, and having a movement of 140 degrees on the meridian. The forty-three years of his tenure of office did not pass without some regrettable and rather undignified quarrels, due partly at least to an impatience with which others awaited publication of his results. He thus had the misfortune, in the latter part of his life, to become estranged from some of his most eminent scientific contem-

*Other important National Observatories founded at early dates were: Copenhagen, begun 1637, finished 1656; Paris, begun 1667, finished 1671. At St. Petersburg there was one in Peter the Great's time (1725); Pulkowa was opened in 1839.

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poraries including Halley (his successor in office) and Newton. At the request of the latter he had supplied places of the Moon in order that Newton's lunar theory could be derived from and compared with observation. But he appears to have held the view that when Newton requested further similar information, he was being asked to give as a right what he considered he was only called upon to grant as a favour.

The dispute which grew out of this led to many letters and documents which are hardly a credit to those concerned. It is pleasant to note, however, what Flamsteed wrote in a letter dated 1686 to an astronomical friend (Richard Towneley) regarding Newton's discoveries, which were about to be published in the "Principia." "The Cartesian philosophy will be overthrown, but we shall have demonstration and demonstrated principles in the room of it. I have lost a cause I assigned for the recess of the equinoctial points [Precession], but I shall gain infinitely by it in the assistance these discoveries will lend me in the reforming of the planetary motions, so that in the room of mourning I congratulate my own happiness."

Flamsteed made his measurements with great skill and introduced new methods to attain accuracy, using certain stars to determine errors of his instruments. He reduced his observations to a form in which they could readily be used by theorists. His catalogue of 2935 stars, which he did not live long enough to finish, was completed by the two men, J. Crosthwait and A. Sharp (1651-1742), who had been his private assistants. This was the "Historia Coelestis Britannica," published in 1725 in three volumes; it was the first important contribution from Greenwich Observatory to science, and was followed four years later by a Star Atlas. Flamsteed's work was not marked by any brilliant discoveries. It was distinguished by continuous industry and meticulous care, and by an accuracy far surpassing that of any of his predecessors or contemporaries. He realised perhaps better than anyone of his time what was then needed for the promotion of Astronomy.

HALLEY

Edmund Halley (1656-1742) was the next to hold the office of Astronomer Royal. At an early age he gave indications

of very high powers, studying Astronomy as a schoolboy, and publishing a paper on the planetary orbits in 1676, which was laid before the Royal Society. Fellowship of that Society was accorded to him in 1678 on his return from a scientific expedition to St. Helena (latitude 16°S) where he had gone, leaving Oxford University without taking any degree. His chief work had been the observations for the compilation of a catalogue of 341 southern stars, containing places (the first obtained with an instrument having telescopic aid) that were an important addition to astronomical data. The expenses of this southern expedition were paid by Halley's father, a London soap-boiler; the Government provided free passage out and home.

As has been stated in the account given earlier of the work of Newton, Halley first effectively became known to that great man in 1684. What he did then meant that (to quote de Morgan the mathematician) he "kept Newton up to his engagement, prevented him from mutilating it in disgust, undertook to see the work through the press, paid the expense of printing and made himself thoroughly master of its contents, the most difficult task of all," a performance which entitles Halley, as the man responsible for the pressure on Newton which produced the "Principia," to the thanks of astronomers for all time.

His astronomical and other scientific work covered a very wide field, and can only be given here in the form of a list. The chief items were: observation and calculation on Newton's principles of the orbits, of the comets 1680* and 1682; calculation of the orbits of 24 comets from which, through the resemblances of certain orbits and approximately equal intervals between appearances, he was led to suggest that the comet of 1682 was periodic and would return to our neighbourhood about 76 years later (Halley's comet); the acceleration of the Moon's motion (see p. 91); the periodic irregularities of the movements of Jupiter and Saturn of which he suspected the cause (see p. 92); the advocacy of use of transits of the

*This comet is remarkable as the one by which Newton showed that comets are controlled by the Sun's gravitation. Halley suspected with some reason that it had a period of 575 years, being perhaps identical with the bright comets of 44 B.C., and 531 and 1106 A.D. But Encke later showed that the period is probably much longer.

planet Venus over the face of the Sun for measurements determining the Sun's distance (see footnote, p. 108); the proper motions in the sky of the stars discovered from changes in the positions of Sirius, Procyon, and Arcturus since Greek times; the first general chart of the world showing the "Variation of the Compass"; the association of Aurorae with terrestrial magnetism. He also noted the globular star clusters in Hercules and Centaurus, and considered nebulae to be composed of "a lucid medium shining with its own proper lustre," and "filling spaces immensely great."

Halley was elected Savilian Professor of Astronomy at Oxford in 1703 and held that chair until his appointment in 1720 as Astronomer Royal. At Greenwich he found that practically all the instruments there, which had really belonged to Flamsteed, had been removed by the latter's executors. In 1721, however, he was given a grant of £500 from the Board of Ordnance, and a transit instrument was installed in the same year. Some time later he obtained a quadrant of 8 foot radius and with these instruments he began, when he was sixty-four years of age, a systematic series of lunar observations. His intention was, if he lived long enough, to continue these observations for eighteen years, the period of revolution of the nodes of the Moon's orbit, an important interval of time for study of lunar motions. The special object of this series was the improvement of knowledge of the Moon's orbit, so that more accurate predictions of position could be made with consequent determination of more correct longitudes at sea. Halley lived long enough to complete the necessary observations but the tables were only finally published in 1752, ten years after his death. These tables became at once, and for some time remained, the standard ones, and were adopted almost universally by astronomers, those in France being the only exception.

Halley was by no means so skilful as his predecessor in the practical observing work of a sky-surveying observatory. But he was in his sixty-fourth year when appointed, the habits of minute attention to details required for success in such a field are not readily acquired in advanced life, and he does not seem to have had much original aptitude for such work. In fact a later Astronomer Royal (Maskelyne) was of the opinion

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that Halley's observations were hardly better than Flamsteed's : this amounts to a rather severe criticism as there had been good progress in the accuracy of instruments.

BRADLEY

The third Astronomer Royal, James Bradley (1693-1762), was one of the ablest if not in fact the ablest, of all who have held that position. Born at Sherbourne in Gloucestershire, he was first educated at the Northleach Grammar School, proceeding to Oxford University in 1711 where he finally graduated in 1717. Much of his time while an undergraduate was spent with his uncle James Pound, for many years Rector of Wanstead in Essex, an amateur astronomer of much skill as an observer ; and after he left the University Bradley lived at Wanstead for some years, observing along with his uncle.

In 1721 he was elected as Halley's successor to the Savilian Professorship at Oxford and resigned church appointments that he had held in the interval. Until 1732, when he took a house at Oxford, he continued to reside usually at Wanstead where his uncle died in 1724. Many of his instruments were removed to Oxford, but he left the most important, the "Zenith Sector," at Wanstead. In 1742 he was appointed Astronomer Royal and during the remainer of his life he lived at Greenwich, although he retained his professorship at Oxford.

Of his two great discoveries, the "Aberration of Light," and the "Nutation of the Earth's Axis," the first was made at Wanstead and the other at Greenwich.

In 1669 Robert Hooke had observed annual displacements in the position of γ Draconis, a star which crosses near the zenith in the latitude of London. He took these displacements to be the effect of the Earth's movement in its orbit round the Sun, *i.e.*, "parallactic" shifts on the sky ; and in 1694, Flamsteed similarly interpreted a movement of the Pole Star. Both had been misled by an aberration caused by the progressive transmission of light combined with the movement of the Earth in its orbit. Bradley determined to investigate the matter, and he observed γ Draconis (which Hooke had chosen as a star free from the displacement due to atmospheric refraction, being in the zenith when observed), continuously

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from 1725 until 1728. It evidently described a small ellipse in the sky with an annual period. But its place in the ellipse was not where it should have been on the idea that the ellipse was a parallactic phenomenon. The displacement was farthest north in September, farthest south in March, while if parallactic these should have been June and December respectively. He was for some time puzzled for an explanation. But in September, 1728, during a pleasure sail on the river Thames, he noticed that the slant of the pennant at the mast head varied with changes in the boat's course, the wind remaining steady, and this gave him the clue he needed for a complete understanding.

Aberration of light may be illustrated by an analogy with falling rain. It is a familiar fact that the apparent direction of the drops as they come down is altered by the motion of a person moving through them; so that the stick of an umbrella must be held in a direction sloping slightly ahead of the holder's movement, the angle at which the drops appear to fall depending upon the ratio between the velocity of their descent and that of the moving person. Similarly the rays of light from a star appear to come from a direction ahead of the Earth's movement, and the angle is very small as the speed of the Earth in its orbit is only about a ten-thousandth the velocity of the rays of light. Bradley was before the days of umbrellas or perhaps he would have thought of the explanation sooner!

It may be noted that this great discovery could have been made from Flamsteed's observations alone. For although Flamsteed did not detect this aberration, his observations were so accurate that from a simple study of his results for stars at different times in the year, the amount of aberration can be determined almost as exactly as from the best modern observations. In fact, more than a century later, the value found by Peters from some of these observations was only about one per cent greater than the most modern result.

But there were still discrepancies in the displacements measured by Bradley. He therefore resumed work on the star and after deducting for the newly-discovered aberration, he found residuals which varied with a period of 18½ years. This suggested to him that the explanation was the varying position of the Moon's orbit which revolves bodily in a period

of 18½ years. The effect of this is a nutation or "nodding" of the Earth's axis, that may be described as a sort of quivering of the Earth, as it reels round in its precessional movement, equivalent to an ellipse of about 18 seconds of arc long and 14 seconds broad, traced out on the sky by the Earth's pole, and superposed on the large circle 47° in diameter due to the precessional movement, thus transforming a large plain circle of 47° diameter and 26,000 years period, traced out on the sky by the Earth's axis, into one with small undulations of 18½ years' period on it.

Both discoveries, Aberration and Nutation, setting aside their intrinsic importance, were necessary to obtain accuracy in fixing the places in the sky of the heavenly bodies. They are essential to the process of "reduction" of the raw material of observations to the final co-ordinates of position. Before the time of Bradley, allowances were made as closely as possible for atmospheric refraction, for the Precession of the Equinoxes and, in the case of the Moon, for parallactic displacement due to the Earth's rotation. But the final results could only be rough compared with what they could now be when Aberration and Nutation were also eliminated.

Bradley was a master in the arts of observation. He never possessed an achromatic telescope—indeed he measured the angular diameter of Venus in 1722 with a non-achromatic telescope 212 feet long, tubeless, with the object glass carried by a mast—and his graduated circles and telescope mountings were far from the equivalent of modern work. His chief instrument was an eight foot quadrant of a type long obsolete. With it, however, he accumulated a store of observations of high class. Most of them were still in manuscript until 1798, but their value was very great owing to the time which had elapsed since their date. Some were useful in perfecting the orbits of the Moon and planets, and those for 3222 stars were formed into a catalogue which was published by the German astronomer Bessel in 1818, with the appreciative title of "Fundamenta Astronomiae." The same data were used again about 70 years afterwards in work on the proper motions of the stars.

Bradley's work in other directions was also of importance. His observations on γ Draconis satisfied him that any real

parallactic shift for that star must be less than two seconds and probably not more than half a second of arc. He frequently observed the satellites of Jupiter and discovered pecularities in their motions from which improved orbits and tables for prediction of their movements could be constructed. Several comets were carefully observed, and their orbits calculated. Improvements of lunar tables were effected from his measurements of position; and revised tables for calculating atmospheric refraction, which were in standard use for nearly a century, were drawn up by him.

BLISS AND MASKELYNE

The fourth Astronomer Royal was Nathaniel Bliss (1700-1764). He was in the post for only two years, but he left observations which formed a useful sequel to those of Bradley. His successor was Nevil Maskelyne (1732-1811), whose first observation was made on May 7, 1765. He used the same instruments as Bradley but adopted a system better designed to get trustworthy results. His places for the Sun, Moon, and planets were much in demand for construction of tables and for checks on the accuracy of theories. On the fixed stars he limited his work to a carefully selected number, thirty-six, to be catalogued as reference points on the sky of much greater accuracy than ever before. His outstanding qualities have been well described as follows: "Maskelyne, as an Astronomer Royal, marks the transition between the old and new administration, between the individual investigator who kept the results more or less to himself, and the head of a public department continuously publishing its work for the benefit of all who could make use of it. . . . One of Maskelyne's first acts after his appointment was to arrange with the Royal Society for the provision of a special fund for printing the Greenwich results."[1] Perhaps his greatest claim to fame is his foundation in 1767 of the "Nautical Almanac" which he superintended himself for 44 years until his death. Maskelyne was connected with the first attempt to measure the density and the mass of the Earth by means of the deviation from the vertical of a plumb line, caused by the attraction of Schiehallien, a mountain in Scotland. For the density the deduced result, 4½ times

that of water, was not very accurate, the value found by other and more reliable methods in later years being about $5\frac{1}{2}$. This better figure was first obtained in 1798 by Henry Cavendish (1731-1810), the method being one in which a pair of heavy balls replaced the mountain of Maskelyne's experiment, their attraction on two other balls being compared with that of the Earth by means of a "torsion-balance."

MICHELL

The torsion-balance was the invention of John Michell (1724-1793), one of the most original thinkers ever known to the science. It may be claimed that he demonstrated, by probability considerations, that double and multiple systems of stars would sooner or later be found in considerable numbers. He also showed by comparison of the brightest fixed stars with the planet Saturn, that stellar parallaxes must be very small—less than a second of arc (which subsequent work has shown to be the case); and that the angular diameters of even the brightest stars must be exceedingly minute, for Sirius less than "the hundredth, probably the two-hundredth of a second" (modern methods give estimates about midway between these values of Michell's). He also forecast that some stars might have surface brightnesses of relatively small amount and therefore have larger discs (this has been found to be the case with red low-temperature giant stars like Betelgeuse, α Herculis and Mira Ceti), and that the existence of invisible companions to bright stars would be found from their effect on the proper motions (which has turned out to be so for Sirius, Procyon, and other stars). He was much in advance of his time with these ideas; and they do not seem to have attracted the attention they deserved.[2]

FRENCH ASTRONOMERS

The Paris Observatory hardly fulfilled its early promise, although the Cassini family (see p. 74) and their relatives by marriage, the Maraldis (J. P. and J. D.), made planetary observations of value, also measurements for the purpose of determining the size and shape of the Earth. This latter activity was peculiarly the work of Frenchmen during the

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century. Among those concerned may be mentioned de Maupertuis (1698-1759), Bouguer (1698-1758), La Condamine (1701-1744), Godin (1704-1760), all active in measurement of arcs of the meridian in different parts of the world, such as Lapland and Peru.

LACAILLE

During the eighteenth century, however, France did produce one great astronomical observer. This was Nicholas Louis de Lacaille (1713-1762) who was, for a few years at the outset of his career, an assistant at the Paris Observatory. He was appointed a professor at the Mazarin College where he had a small observatory, made many observations, and wrote a large number of scientific memoirs. He was sent by the Paris Academy of Sciences on a scientific expedition to the Cape of Good Hope (1750-1754) to determine the distance of the Sun by observations of Mars and Venus made simultaneously with others made in Europe and to form a catalogue of southern stars not visible in northern latitudes. He compiled a catalogue of nearly 2000 selected stars with a star map, published posthumously in 1763, from observations of more than 10,000 such stars; and his results for Mars and Venus, combined with the northern observations, gave a parallax for the Sun slightly larger than ten seconds of arc, which was about 14 per cent in error. Before his premature death in 1762, a catalogue of 400 of the brightest of his stars, and an excellent set of tables for prediction of the Sun's position, taking planetary perturbations into account for the first time, were published by him. He also improved the methods of calculating orbits of comets and computed orbits for a considerable number. While in South Africa he measured an arc of the meridian. The foregoing includes only the chief items of his work, largely performed without assistance, in addition to routine duties by which he earned his living; truly a magnificent record.

T. MAYER

In Germany, Tobias Mayer (1723-1762), an astronomer of great merit particularly as an observer, was professor of mathematics at Göttingen from 1751. Among his chief works

are a catalogue of 998 zodiacal stars, determination of proper motions of stars by comparing Roemer's star places with Lacaille's and with his own, and improvement of Lacaille's Solar Tables. But notable contributions were studies of the Moon's movements and of its surface markings. From his measurements of position of craters and other features, he derived a satisfactory position of the Moon's axis of rotation and a good explanation of its libratory movements. Thirteen years after his death, a map of the Moon by him was published along with other posthumous works. But his Lunar Tables for predicting the movements of our satellite among the stars were his most important achievement. These were a considerable improvement on any previous ones; and, having been sent to England in 1755, where Bradley reported favourably on them to the Government, they earned a reward of £3000 to Mayer's widow in the year 1765, for their value to navigation in determination of the longitude to within about half a degree of the truth.

In connection with this problem, prizes were given to John Harrison of £3,000 and later of £10,000 (1773), for improvements in the chronometer by which a different and better method for finding the longitude was rendered practicable.

TRANSITS OF VENUS

During the eighteenth century two Transits of Venus* (1761 and 1769) were observed by expeditions of astronomers sent out by Governments, Academies, and privately, to foreign stations all over the world, while observations were also made at Greenwich, Paris, Vienna, Upsala and elsewhere in Europe. The results (as Lacaille had forecast) were not very satisfactory

*Venus would always pass across the Sun's disc when between it and the Earth if the inclination of its orbit were sufficiently small. This angle is too great for that (it is about 3°.4); so the planet is only occasionally seen in transit. At the present time the occasions occur in pairs (1631 and 1639, 1761 and 1769, 1874 and 1882, 2004 and 2012). By one method the lengths of the chords of the Sun's disc traversed by the planet, and their angular distance apart, are obtained by timing the planet's entries and exits on and off the Sun as seen from different stations on the Earth. From these data the distance of Venus from the Earth is obtained and, the ratios of all distances in the Solar system being derivable from Kepler's Third Law (see p. 59), the distance of the Earth from the Sun follows.

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owing to difficulty in determining exactly the times of contact of the disc of Venus with the edges of the Sun's disc. The values of solar parallax at first obtained ranged from eight to ten seconds of arc but finally, in the following century, Johann Franz Encke (1791-1865) deduced, from all the observations, a parallax of 8·58," corresponding to a distance of 95,370,000 miles, about two-and-a-half per cent too great, but used for a long time as the standard value.

INSTRUMENTAL DEVELOPMENTS

It is desirable at this stage to refer briefly to the development of instruments during the century. The English instrument makers provided strong assistance to the observer in this country throughout the period and the quadrants and sectors of Graham, Sisson, Cary, Bird and Ramsden had no equals on the continent. Bradley's results were obtained with instruments by Graham and by Bird; the former made the "Zenith Sector" with which the aberration of light was discovered; and with the 8 foot quadrant by Bird the excellent Greenwich stellar observations were secured. The art of accurate division of circular limbs into small parts was invented and perfected in England, so that continental observatories of the time got their best instruments from this country. Ramsden was the first effectively to substitute complete circles for quadrants; and Edward Troughton efficiently continued the tradition and brought instruments up to the modern standard. However, in 1804 Reichenbach's Institute was founded at Munich and this provided instruments for Germany and other continental countries equal to the best British make.

During the century the achromatic refracting telescope was discovered by Chester More Hall about 1733, but the necessary combination of crown and flint glass and its introduction in manufacture for sale were not effected until John Dollond (1706-1761), originally a Spitalfields weaver, re-invented the achromatic object glass and made instruments for the market.[3]

The reflecting telescope is a British invention almost throughout. It was first invented by Newton who made

small instruments, and was improved by John Hadley (1682-1744). This was the Newtonian* form, but James Short (1710-1768) of Edinburgh made many Gregorian telescopes which were of high quality. The Newtonian with silvered glass mirrors instead of metallic ones (the idea of the silver-on-glass being due to a Frenchman, Foucault, in the next century) has been the favourite astronomical instrument of the amateur. The history of reflectors as real instruments of discovery may be said to have been started by William Herschel.[4]

REFERENCES.

[1] Mrs. A. W. Lane Hall, *Journal of the British Astronomical Association*, vol. 43, p. 75 (1932).

[2] For an account of his views see J. E. Gore, "Astronomical Essays," p. 112-123 (1907).

[3] H. C. King, "The Invention and Early Development of the Achromatic Telescope," *Popular Astronomy*, vol. 56, p. 75 (1948).

[4] For an account of the history of makers of reflecting telescopes, see C. D. P. Davies's Presidential Address, *Journal of the British Astronomical Association*, vol. 37, pp. 6-15 (1926).

*In a "Newtonian" reflector an image is formed by the large mirror (a paraboloid), which is obliquely reflected by a small plane mirror to the side of the tube, into an ordinary eyepiece. In the "Gregorian," the observer looks straight forward, the image being thrown back by a small concave mirror through a central hole in the large paraboloid mirror, where the eyepiece is fitted. The "Cassegrain" is like the Gregorian with the exception of a convex small mirror for the concave one.

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CHAPTER X

WILLIAM HERSCHEL

EARLY YEARS

FREDERICK WILLIAM HERSCHEL (1738-1822), known to fame as William Herschel, was born at Hanover where his father was a musician in the band of the Hanoverian Guards. At the age of fourteen, having shown great musical talent, he also joined the same band. This ended his school career, but for more than two years he was given private lessons which included the subjects of languages, ethics, logic and metaphysics. In 1755 an invasion of England by the French was feared, and the Guards with other regiments were sent overseas to England, Herschel, his father and his elder brother, also a musician, accompanying their regiment which camped successively at Maidstone, Coxheath and Rochester. A year afterwards Herschel returned to Hanover. But the breaking out of the Seven Years' War in 1757 proved decisive as to his future life. The hardships of active service told upon his health, and after the defeat at Hastenbeck he was again sent to England by his father. By this he risked treatment as a deserter, although his father contended that he never had been a fully-enlisted soldier. A paper of discharge is still in existence[1] and it is evident that this disposes of the story that he deserted from the Hanoverian Army and that he was formally pardoned by George III in 1782.

MUSICIAN AT BATH

After a year or two spent in London, Durham, Leeds and other places for various periods, he went to Bath in 1766 where, following a year as member of an orchestra he was appointed organist of the Octagon Chapel, and became very busy as a musician and teacher of music. His father died in 1767, and five years later in 1772 he arranged that his sister Caroline (1750-1848) should come to England to act as his housekeeper. She became his active musical, and (later)

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astronomical, assistant, as well as manager of his domestic affairs.

TELESCOPE MAKING

About this time Herschel began to study Astronomy seriously, and in 1773 he started trying to make telescopes for himself. At first these were non-achromatic refractors; but finding the long tubes intractable in use he took up the reflecting telescope, hiring a two foot long Gregorian which was found much more convenient. He next decided to try to get a mirror suitable for a five or six foot length telescope, but found that there was none on the market of so great a size. Herschel then heard of a Quaker in Bath who had been trying to make reflecting telescopes but without success, and had decided to sell his unfinished mirrors with the tools he had used; and the stock having been bought, telescope mirror making, chiefly for the Newtonian form of instrument, began to occupy much of Herschel's time.

While carrying out the duties of a very busy career as a musician and music teacher, Herschel worked from the winter of 1773 for a number of years very enthusiastically at his new hobby. His younger brother Alexander, for whom he had found musical work in Bath, had considerable mechanical aptitude, and effectively assisted him in his telescope making. It was not long before he had made the mirrors for a small telescope and early in 1776 he finished a larger one five-and-a-half feet long.

FIRST OBSERVATIONAL WORK

His first recorded observation had been made on March 1st, 1774, when he viewed "the lucid spot in Orion's sword" (the Orion nebula), and the ring of Saturn. His success encouraged him to make others of the Newtonian form and he eventually made a seven foot telescope of about six inches aperture with which on May 1st, 1776, he observed the rings and two belts on the ball of Saturn "with great perfection."

Herschel's earliest observational work was on the planets and the Moon. From 1774 he carefully observed Saturn and other planets making drawings of Mars and the belts of Jupiter. The Moon was examined in 1776, and in 1779 he made measure-

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ments of the lengths of the shadows of lunar mountains from which their heights were derived. In 1777 he began a number of estimates of the brightness of the variable star Mira Ceti. His idea that the changes in light of this star could be best explained by supposing rotation on an axis to bring a bright side, and a dark side obscured with spots like the Sun's but larger, alternately into view, was, considering the undeveloped state of physical science at the time, not apparently an impossible one.

By 1778 he had thought of attempting to determine the annual parallax of a star using the double star method suggested by Galileo. The principle of this is a simple one, depending upon the perspective shifting to a spectator in motion, of objects at different distances from him. A star observed from opposite sides of the Earth's orbit is sometimes found to alter its situation very slightly by comparison with another star close to it in the sky, but indefinitely remote from it in space. Half the small oscillation thus executed is called the star's "annual parallax." It represents the minute angle under which the radius of the Earth's orbit would appear at the star's actual distance. From it the distance of the star is determined. Before the time of Herschel, some attempts had been made to measure stellar parallax* ; but because of the enormous distance of even the nearest stars with the resulting small displacements, and owing to the imperfection of instruments, there had been no success ; and the required results were quite beyond Herschel's instruments and methods. His endeavours were not altogether without consequence for Astronomy, however ; for he was thus led to the study of double stars, and this was followed after many years by his discovery of physically connected pairs, or binary systems (announced in 1802).

Herschel's chief purpose in his astronomical work soon settled into an investigation of the nature of the objects in the sky rather than their motions (although some of his researches, such as measurements of relative positions of the

*The method of Galileo which had also been suggested by James Gregory (1673) and by Wallis (1693), was tried by Huyghens (about 1680) and by Long of Cambridge (about 1740), but of course with no success owing to their limited instrumental means.

components of double stars and of satellites and their primaries, were similar to the positional work of the professional astronomers) ; and his Solar, Lunar, planetary and stellar observations were firmly directed to that end. He may thus be said to have been the chief originator of descriptive, and the founder of sidereal, Astronomy. Beyond measurements of position, little had been done of a systematic nature except some observation of the planets, temporary or new stars, nebulae and clusters, and variable stars.

Halley had found about half-a-dozen of the dim objects called "nebulae," Lacaille had made a list of forty-two he had seen while at the Cape of Good Hope in 1750-1754, and Charles Messier (1730-1817) had published a catalogue of forty-five (1771) and an enlargement to one hundred and three (1781). These had been more or less "come across" or "picked up" in the course of other work. But Herschel's object was very different. It was an exploration or "review of the heavens" to be as thorough as he could make it.

FIRST SKY REVIEW

His first review was in the nature of a preliminary. It was made in 1775 with a seven foot Newtonian and extended only down to the fourth stellar magnitude. The result convinced him that more optical assistance was required and he set himself to making larger instruments of ten foot, twelve foot, or twenty foot focal length. It is said that his activity was such that he made, before 1781, not less than 200 seven foot, 150 ten foot and about 80 twenty foot mirrors. He and his brother Alexander made many complete instruments of these dimensions which were sold for substantial prices in later years. These prices seem to have been based largely on the means of the purchasers and they ranged about as follows : a complete 7-foot, 100 guineas ; a 10-foot, 275 to 350 guineas ; a 20-foot, 1,000 guineas ; one 24-inch aperture and 25-foot focus made for the King of Spain, 3,000 guineas. Recent tests of some of Herschel's mirrors still in existence have shown quite good figures even as judged by modern standards.

The review of 1775 was the first of four made with tele-

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scopes of increasing power. Each was a complete scrutiny of the whole sky visible in these latitudes; everything that appeared in any way remarkable was noted, and if thought desirable, more closely examined.

SECOND SKY REVIEW. DISCOVERS URANUS

He began the second review on 17th August, 1779. This was made with an improved 7-foot reflector of 6·2 inches aperture and included down to stars of the eighth magnitude, one of its chief objects being to compile a list of double stars apparently suitable for Galileo's parallax method. In his journal at the date 13th March, 1781, which was during the progress of this review, there is the following entry: "In the quartile near Zeta Tauri, the lowest of two is a curious either nebulous star or perhaps a comet. A small star follows the comet at two-thirds of the field's distance." He submitted a paper later to the Royal Society in which he wrote that "in examining the small stars in the neighbourhood of H. Geminorum I perceived one that appeared visibly larger than the rest; being struck with its uncommon magnitude I compared it to H Geminorum and the small star in the quartile between Auriga and Gemini, and finding it so much larger than either of them, suspected it to be a comet." On March 17th he "looked for the comet or nebulous star, and found that it is a comet, for it has changed its place." On Monday, the 19th, he ascertained that the supposed comet "moves according to the order of the [Zodiacal] signs, and its orbit declines but little from the ecliptic."

This discovery was soon communicated to Greenwich and Oxford Observatories, and Continental astronomers were informed. If Herschel's idea of a comet had been correct the discovery would have been of much less interest than it turned out to be. When further observations were obtained in this country and abroad, efforts were made to calculate an orbit for it by several, including Laplace, but these were fruitless; it was found that no ordinary cometary orbit would fit its observed motion. Within several months of the discovery, however, Anders Johann Lexell (1740-1784), a St. Petersburg mathematician, who was in England at the time of the dis-

covery, informed the St. Petersburg Academy that the object was probably not a comet, but an exterior planet, revolving round the Sun at about twice the distance of Saturn. It transpired later on that the new planet had been observed at least 17 times between 1690 and 1781 by several observers, including Flamsteed and Bradley, none of whom had ever noted any difference from an ordinary star either in appearance or movement. One of them had, in fact, observed it on four consecutive nights in 1769! It is noteworthy that Herschel's method of close scrutiny (he used magnifying powers of 227, 460 and 932) confirmed his first impression of unusualness. It is even questionable if, at the time, there was in existence any telescope but one of Herschel's with which this could have been done.

A considerable sensation was caused by Herschel's discovery, the first planetary one to be recorded, the five planets, Mercury, Venus, Mars, Jupiter and Saturn having been known from prehistoric times. The fact that the new planet had been found by an amateur added to the interest, and Herschel achieved a European reputation. The Royal Society awarded him its chief honour, the Copley Medal, in November, 1781, and he was elected a Fellow in the following month, subscriptions being waived as a mark of appreciation. King George III expressed a wish to meet him and in May, 1782, he had an audience of the King to whom he presented a drawing of the Solar System, while in the July following he showed the Royal Family the planets Jupiter and Saturn and other objects through one of his telescopes.

Herschel had communicated his discovery to the Royal Society as "An Account of a Comet." By the summer of 1781 it had been shown that the object was a planet, and by November he felt sufficiently sure of this to exercise his right as discoverer to give it a name. He proposed to do this by naming it the "Georgium Sidus" in honour of George III, but this appellation never found favour abroad and did not last long in this country. The French astronomer Lalande (1732-1807) tried to get the name "Herschel" adopted; but the title "Uranus" proposed by Bode of Berlin, gradually met with general acceptance.

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APPOINTED ROYAL ASTRONOMER

Soon after Herschel's meeting with the King he received the appointment of Royal Astronomer with the modest salary of £200 per annum. This does not appear generous (although the annual salary of the Astronomer Royal, Maskelyne, was only £300) but it was enough to determine Herschel to give up the career of musician. No doubt the knowledge that he could very probably earn substantial sums by the sale of telescopes constructed by himself and his brother Alexander made it easier for him to accept the post. The offer came at a critical period, just when Herschel had begun to consider music teaching and concert management to be a waste of time to one with his high ideals and aims in astronomical research.

THIRD SKY REVIEW

The review of the sky, in which he had found Uranus and world-wide fame, was completed in 1781; along with it he had gathered the materials for his first catalogue of 269 double stars. This had been his second review and he immediately began a third using the same instrument but employing generally a higher magnifying power (460 against 227). His third tour of the skies had for one of its special objects the ascertainment of any changes which might have taken place since Flamsteed's day about a century before; and during its progress many thousands of stars were carefully examined as to colour, singleness or duplicity, clearly defined or nebulous.

His career as a musician having come to an end with the appointment as a professional astronomer, Herschel and his sister Caroline left Bath in August, 1782. They first went to Datchet, near Windsor, for about three years, then to a place in the neighbourhood, Clay Hall, for a year; and finally to Slough in 1786 in a house known as Observatory House, which Arago, the French scientist, described as "the place in the world where more discoveries have been made than anywhere else."

FOURTH REVIEW. STAR GAUGING. FORM AND SIZE OF MILKY WAY

When he had finished the third review he began another in 1783, using a much more powerful telescope 18·7 inches in

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aperture and 20 feet focal length. In this survey he introduced his famous counting of stars or star gauges. With the eyepiece he used he saw a circular field in the sky 15 minutes of arc in diameter (about quarter the area of the Moon). Turning the telescope in different directions in the sky, he counted the stars visible in the field each time, repeating the counts in from 6 to 10 closely adjacent fields, and, taking an average, he called it a gauge. This process, repeated over the sky, would, he thought, give him the relative distance to which his telescope reached, assuming that the stars, bright or faint, were distributed throughout space in a fairly uniform way. The depth reached in any particular direction would be proportional to the cube root of the number of stars of his gauge.

In 1785 he published, in a paper to the Royal Society, the results of gauges, which showed much variation. In some, perhaps only one star at a time was counted, in others nearly 600 stars were seen, while on one occasion he made an estimate that 116,000 stars of a Milky Way region passed through his telescope field in a quarter of an hour. Generally stars were most numerous in the zone of the Milky Way and much scarcer in areas of the sky farthest from it. His final conclusion based on about 3,000 gauges was that the space occupied by the stars is shaped roughly like a disc or grindstone, approximately circular in plan, with its centre plane coinciding with the centre line of the visible Milky Way. There were many irregularities, one being a deep cleft in the grindstone corresponding to the division* between two branches of the Milky Way from Scorpio to Cygnus, and the thickness of the disc at its centre was about a fifth of its diameter. The Sun was deeply embedded in the disc at or near its centre. The actual dimensions, according to Herschel, translated into modern figures, would be about 6,000 light years diameter and 1,100 light years maximum thickness.

At the same time as he was making his gauge counts, Herschel listed all objects of interest that he saw, pairs of stars, stars that varied in brightness, groups and clusters of stars, irregular diffuse bright nebulosities, and dark or vacant spaces,

*Now known to be caused by the obscuring effect of clouds of dust and gas nearer to us than the background Milky Way stars. (See p. 196, pp. 268, 297).

also more than 2,000 of the smaller nebulae; he presented several catalogues of nebulae and clusters of stars to the Royal Society throughout his long career. The smaller nebulae referred to, many of which had been first seen by him, with his 7-foot telescopes, had a different law for their distribution. They were not found in the Milky Way zone of the sky, but were most numerous near the poles of the great circle of the sky which marks its centre line. Herschel believed that they were external independent stellar systems which generally could not be resolved into stars without more powerful telescopes than his 20-foot. In fact, at a later date he suspected that the 20-foot was perhaps not powerful enough even to penetrate to the utmost confines of our own system. Herschel always realized that the assumption of uniform distribution of stars throughout space was very unlike reality, the presence of numerous aggregations and cluster of stars in the sky being strong presumptive evidence against it. And as time passed he modified his ideas of distribution, especially to admit that the spacing in Milky Way regions was probably not the same as nearer to the Sun. The irregular diffuse bright nebulosities that, like Halley (see p. 101), he considered to be composed of "some shining fluid of a nature unknown to us" (from which he thought the stars were probably formed) and the dark vacant starless areas, were not to be explained by his work. But he never abandoned entirely his idea of the general shape of our stellar system, nor did he give up his opinion that the small nebulae, mostly outside the Milky Way areas of the sky, are generally external systems like our own Galaxy.

The theory of a limited stellar system shaped like a disc was not first proposed by Herschel. It had been suggested by Thomas Wright (1711-1786) of Durham in his "Theory of the Universe" published in 1750, by Kant, the philosopher in 1755, and by J. H. Lambert (1728-1771) in 1761; but for their hypotheses no observational foundation was provided as it was by Herschel.

"LIMITING" APERTURES

Star gauging was Herschel's device for obtaining some idea of the shape and size of the Milky Way system. But in 1817 he also employed a method for a rough estimate of the relative

distances of individual stars. This was based on two assumptions: that there is a general equality among stellar luminosities and that there is no appreciable loss of light in space. The method was carried out by ascertaining the smallest aperture of telescope with which a star could be seen. It may readily be shown that with the assumptions mentioned (neither of which is correct, however) the distances of stars would be directly proportional to this aperture. He also used a similar procedure for a rough estimation of the relative distances of "Globular" clusters (see p. 294), founded on their incipient resolvability into stars with particular telescopic apertures.

SUN'S MOTION IN SPACE

In 1783 Herschel contributed a paper to the Royal Society in which he estimated the direction and speed of the Sun's motion in space with relation to the stars in its neighbourhood. Tobias Mayer was the first to try to find, from the apparent motions of the stars, whether the Sun was moving towards any determinate part of the sky. He remarked that if there was any such movement, it would follow that the stars in that part of the sky to which the Sun's motion was directed would appear to be gradually separating from each other, whereas in the opposite direction they would seem to be closing up. He communicated a memoir to the Göttingen Academy of Sciences in 1760, based on the proper motions of 80 stars determined by comparison of the observations of Roemer in 1706 with his own and Lacaille's in 1750 and 1756. His conclusion was, however, that these proper motions did not give evidence of the movement of the Solar System towards any particular region in the sky.

Herschel's results in 1783 were on the basis of the proper motions of only 13 stars (14 if the two components of the double star Castor are taken as separate stars) and he located the point to which the Sun is moving, the "Solar Apex," at about γ Herculis which is within 10 degrees of that derived by the most recent investigations. He thought that the speed of movement could "certainly not be less than that which the Earth has in her annual orbit"; but this is about fifty per cent greater than modern estimates.

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In the same year as Herschel published his results, only a few months later, Pierre Prévost (1751-1839) deduced a similar result for the direction, from Mayer's proper motions. His "Apex" is situated considerably to the west of Herschel's (27 degrees west).

Using Maskelyne's table of 36 proper motions, Herschel made a second estimate in 1805. He used a graphical method of great circles of the sky to include the observed stellar movements and putting the Apex in the middle of their intersections. The result was not so good, lying too far west and somewhat too far north as well. His estimate of velocity was in proportion about half as much too small as his first had been too great.

It is interesting to note that in 1748 Bradley had written as follows : "If it should be found that our planetary system changes its situation in absolute space, then there may arise, in course of time, an apparent variation in the angular distances of the fixed stars. . . . The alteration in their relative positions may be dependent on so great a number of causes that, perhaps, many centuries may be required before the laws can be discovered." One of these causes is the motion of the Sun in space, the direction of which was discovered in 1783 by Herschel and Prévost less than half-a-century later. The differential rotation of the Galaxy, another of the causes, was not discovered until nearly two centuries afterwards (see p. 265), and the individual motions of stars, by determination of radial velocities and proper motions, are still being investigated.

DOUBLE AND MULTIPLE STARS

Herschel's first observations of double and multiple stars were made with a view to compiling a list of suitable stars for parallax measurements. But the number he discovered and the large proportion of all stars which were accompanied by a close comparison showed, as Michell had pointed out (see p. 106), that most pairs must really be physically connected.

In 1782 he communicated a catalogue of 269 pairs of which he had discovered 227 himself, to the Royal Society ; and another list of 434 in 1784 ; while his last paper to the same body in 1821 gave 145 more. He kept on examining a number

of these, and after 20 years' work he informed the Royal Society in 1802 that in his opinion "casual situations will not account for the multiplied phenomena of double stars," and reported that he proposed to communicate results which showed "that many of them have already changed their situation in a progressive course, denoting a periodical revolution round each other." He gave these results in two papers read in 1803 and 1804. In the first of these he discussed in detail the displacements shown by his measurements and showed that for six pairs these could only be accounted for by each of the six being "real binary combinations, intimately held together by the bonds of mutual attraction."* In the second paper he gave details for fifty pairs.

The proof of the existence of binary (and multiple) systems marked the beginning of investigations of considerable importance. It showed that a science of sidereal mechanics was certainly practicable and gave evidence of the universal prevalence of the law of gravitation.

ESTIMATES OF STELLAR BRIGHTNESS

One of Herschel's earliest difficulties in stellar observations was that there did not appear to be any method of ascertaining changes in brightness of the stars such as some differences in catalogues seemed to indicate had happened. He therefore entered into a very arduous series of observations to arrange the stars of the different constellations in sequences of brightness. He did this by setting every star between two others in his lists, one slightly brighter, the other fainter. On this principle he prepared six catalogues based on observations between 1794 and 1797. Nearly a century later, the great authority on stellar photometry, E. C. Pickering (1846-1919), examined the catalogues and carefully reduced Herschel's sequences to modern stellar magnitudes. His verdict was that Herschel had thus "furnished observations of nearly 3,000 stars, from which their magnitudes a hundred years ago can now be determined with an accuracy approaching that of the best modern catalogue."[2]

*These were : Castor, γ Leonis, ϵ Bootis, δ Serpentis, γ Virginis and ζ Herculis—all noted binary systems.

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THE FOUR-FOOT REFLECTOR

Reference has already been made to Herschel's telescope making, but it became evident fairly soon that although the sale of instruments to others was lucrative their manufacture interfered considerably with his own activities and projects. He had long had in mind the construction of a very large instrument, but this was going to be exceedingly difficult if his time was so much occupied with the building of telescopes for sale. He had in view the making of a 30 or 40-foot focal length reflector for special investigations, but he could not afford to make such an instrument at his own expense, particularly if he was to lose income by giving up time for it from the manufacture for sale to others.

Fortunately, after representations to the King by Sir Joseph Banks, President of the Royal Society, a grant of £2,000 was made in 1785 towards the cost of the proposed giant, followed two years later by another donation of the same amount; and he was paid £200 per annum, in addition to his salary, for the upkeep of the instrument, with a small annual salary of £50 to his sister Caroline as his assistant. Incidentally it should be mentioned that by this time Caroline's name was becoming well known, not only as his assistant, but as a discoverer of comets, of which she eventually found eight.

Nearly four years were taken in the building of the big telescope, which was of 48 inches aperture and 40 feet focal length; it was ready for use in August, 1789. This instrument was not of the ordinary Newtonian form, the diagonal mirror being dispensed with to save the light lost in the second reflection. Herschel had experimented with a 20-foot telescope, and had found that by omitting the secondary mirror and tilting the main one, good and brighter images could be got. The telescope was thus used as a "front-view" (later called the Herschelian type), the eyepieces being at the side of the top end of the tube while the observer looked down it, facing the mirror and with his back to the sky.

On the whole, however, the 40-foot was not as successful as hoped for. It was very heavy and cumbrous and, even with two men attendants, difficult to use; while the mirror, in spite of the utmost care, required repolishing (a formidable

job) at least every two years. Herschel estimated that, with a magnifying power of 1,000, this great instrument could be effectively used, in the English climate, not more than about a hundred hours per year; and a review of the heavens with it would have occupied eight centuries. As a matter of experience, he found the opportunities for its employment rare. It took some time to get it started up, while a telescope like the 20-foot could be in operation in about ten minutes. And the large mirror was rather liable to get frozen up in cold weather, or dewed up in moist conditions. In his later years, Herschel seldom employed it, although it remained standing on its mounting at Slough for seventeen years after his death. By the orders of his son, Sir John Herschel, it was dismantled in 1839 and laid in a horizontal position. It remained like this for many years, until a falling tree destroyed most of the tube in 1867 during a violent storm. The large mirror, the most used one of the two constructed, is still kept at Slough.

SOLAR WORK

The fame of Herschel's work on the stars and nebulae, and on the structure of the sidereal system, tends to overshadow his Solar, Lunar, and planetary researches. From an early date he took an interest in Solar phenomena and he contributed several papers to the Royal Society giving some of his observations and theories. The Sun and sunspots had been observed for more than a hundred years by Galileo, Scheiner, Fabricius, Hevelius, Cassini and others, but there was no agreement as to the physical explanations of any of the observed phenomena.

A few years previously, however, Alexander Wilson (1714-1786), Professor of Astronomy at Glasgow University, had put forward a theory of the Sun's constitution. He had become convinced that sunspots are depressions below the general solar surface, and cavities in the glowing "photosphere" through which a darker interior is seen. He thought that the "body of the Sun is made up of two kinds of matter, very different in their realities: that by far the greater part is solid and dark with a thin covering of that resplendent substance from which the Sun would seem to derive the whole of his

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revivifying heat and energy." Herschel's observations led him to agree with this view and he concluded that the solar globe is dark and solid and surrounded by two cloud-layers, the outer one being intensely hot and luminous and the inner a sort of screen which protects the interior. He wrote that the "solid body of the Sun beneath these clouds appears to be nothing else than a very eminent, large and lucid planet, evidently the first, or in strictness of speaking, the only primary one of our system; all others being truly secondary to it." It was similar to other planets and "We need not hesitate to admit that the Sun is richly stored with inhabitants." This theory, which seems fantastic to modern ideas, was the accepted one for many years; but present-day knowledge of heat and results of the work of the spectroscope, neither of which was accessible to Herschel, shows the impossibility of a cold interior body surrounded by shallow shells, one hot and luminous, and the theory is purely of historic interest now.

In connection with his studies of solar phenomena Herschel made one very curious and original investigation. This was an attempt to find any connection which might exist between the condition of the Sun's surface and terrestial weather. It was the first, but by no means the last, attempt of the kind. He could not find sufficient precise meteorological data, which then did not exist in the quantity later accumulated, and he therefore adopted the ingenious idea of using the price of wheat from year to year for a criterion of weather. As a result he thought he could find some evidence going to show that when sunspots were scarce and solar activity, according to his ideas, on a diminished scale, prices had been higher and weather presumably colder. Food and sunspots had therefore been plentiful at the same time. There is no doubt, however, that Herschel's data were insufficient to establish any such connection. Even if the weather was the only or indeed chief factor in determining the prices of British wheat, such prices could hardly be taken as a measure of the world's weather conditions. Any relationship which might exist between solar phenomena and terrestial weather could not be shown by such a simplified comparison. In fact it is more than doubtful whether full meteorological data for the entire globe could be successfully used for such a purpose.

LUNAR WORK

One of the first objects observed by Herschel was the Moon and, as has been remarked earlier, he made attempts to measure the heights of its mountains. In 1783 he thought he saw lunar volcanoes actually in eruption, but after sending a paper on the subject in 1787 to the Royal Society, he wrote no more regarding it and evidently accepted the view of leading French astronomers that what he had seen was certain light-coloured parts of the lunar surface more than usually illuminated by Earthshine (see p. 47). Herschel held the opinion that our satellite is inhabited or at least habitable. Although he admitted that there was no evidence of large seas on its surface and that the existence of an atmosphere at all like the Earth's can be ruled out, he considered that there might be inhabitants "fitted to their conditions as well as we on this globe are to ours"—a method of reasoning by which almost any celestial body except the high-temperature stars themselves may be considered habitable or inhabited.

PLANETARY OBSERVATIONS

Mercury was only viewed by him when in transit across the Sun's disc. But his observations of Venus extended from 1777 to 1793. He tried to ascertain a rotation period for it but the markings on its disc were too elusive and indefinite for the purpose. He disagreed with results published by the German astronomer Schröter that supposedly indicated the existence of mountains 20 miles or so high, and in this later observers have supported him.

His observations of Mars began the modern physical study of that planet. He noted the white polar caps and their expansion and contraction as Martian winter and summer alternated for the respective hemispheres; and he concluded that they are snowy deposits from "a considerable though moderate atmosphere," while he satisfied himself of the general permanence of the dark markings. The rotation period he obtained in 1781, 24 hours 39 minutes 21·67 seconds, exceeds by only about two minutes the most modern value. This excess is perhaps explained on the assumption that Herschel, like Cassini and Maraldi at an earlier date, gave the

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observed periods of rotation without correction. Because of the Earth's faster orbital motion inside the orbit of Mars, that planet apparently loses one rotation during a revolution from opposition to opposition, so that an average *minus* correction of rather more than two minutes has to be applied.

Jupiter was not so much observed by him as Mars but the cloud nature of the belts of this planet was noted by him and irregular drifting of some spots on these belts observed. From determinations of the variable brightness of the four Galilean satellites he concluded that as their stellar magnitudes vary with orbital position they rotate in the same periods as their revolutions round their primary. Saturn was the most observed of all the planets at Slough. Herschel wrote six papers on it for the Royal Society, the first in 1789, and the last 16 years later. He determined its period of rotation as 10 hours 16 minutes and discovered two faint inner satellites. One of these, Iapetus, he found to perform its rotation, like the moons of Jupiter and our Moon, in a period coincident with its revolution confirming Cassini's observation. He believed the rings of Saturn to be solid (an erroneous opinion); and seems to have seen the so-called "crape" ring, which is inside the other two, without specially remarking on its existence. In 1789 the ring system was disposed edgewise to the Earth and showed as a broken line of light in Herschel's 20-foot reflector. He thought that the spots of light gradually moved along this line and that this was due to a rotation of the rings round the planet with a period, measured from the movement of the brightest spot, of about $10\frac{1}{2}$ hours. This is about the period appropriate to a satellite situated in the brighter ring not far inside the Cassini division.

His own planet, Uranus, was specially observed and he measured it to be between 4 and 5 times the diameter of the Earth. In 1787 he found two moons and estimated them to be as big as the four largest moons of Jupiter. In 1797 he announced four more but these seem to have been only faint stars in the planet's neighbourhood at the time, and none of them identical with two others found fifty-four years afterwards.

REASONS FOR SUCCESS

There may be a tendency, which would be perhaps natural, to attribute Herschel's success to the lines of investigation he adopted, as he thereby entered largely unexplored territory. But the direction of his work was his own choice, and a highly creditable one, apart from the achievements he recorded in it. Or the possession of an instrumental equipment far greater than that of any predecessor might perhaps be given as the reason for it all. The fact that the equipment was of his own making was, of course, a great feat of skill and energy; and when it is borne in mind that, with the exception of some good lunar work by Schröter with a 7-foot reflector bought from Herschel, nothing of note was apparently done by any of the many purchasers of his powerful telescopes,* the reason for the great performances is seen to be otherwise.

Herschel was the founder of sidereal astronomy; he outlined principles which guided its future and accumulated data which were useful to his successors. Any one of his more important investigations would have been sufficient to make a reputation.

SCHRÖTER

A contemporary of Herschel's was Johann Hieronymus Schröter (1745-1816), who has been called the "Herschel of Germany." He was born at Erfurt on 1745, August 30, studied law at Göttingen from 1764 to 1767, and settled in a legal appointment at Hanover in 1777. Being very musical he became acquainted with the Herschel family there, and heard from Dietrich Herschel, a young brother of William, who had returned from a stay in England, the first news of William's astronomical work. A three foot long Dollond refractor was acquired in 1779, and the first observations of the Sun and Moon were made. In 1782 he was appointed first magistrate of Lilienthel, near Bremen, and in 1783 the mirrors for a four foot long reflector were ordered of W. Herschel. This instrument was mounted by himself in 1784.

*Although the suggestion that this is explained by the fact that the purchasers were mostly the "nobility and gentry" has something to commend it! (Eddington *Occasional Notes, No. 3, Royal Astronomical Society*, 1939, January).

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In the following year his observatory was erected, and he started a correspondence with Olbers and with Herschel. In 1786 he obtained a seven-foot reflector of 6 inches aperture from the latter. After 1792 he possessed a good thirteen-foot reflector of 9½ inches aperture which was generally his chief instrument, although he also had a twenty-six-foot of 19 inches aperture, both being the work of a German optician. From 1796 to 1805 he had as assistant Harding, afterwards professor at Göttingen, and from 1806 to 1810 F. W. Bessel acted in a similar capacity. The French occupation of Germany in 1810 was of most serious consequence to Schröter. He lost his appointment and income in 1811. But all this misfortune culminated in 1813 when French troops set fire to many premises in Lilienthel, resulting in the loss of most of his books and records ; and his observatory was pillaged and practically destroyed. Unable to restore what he had lost, or to accumulate observations, he did not long survive the disaster, dying three years later.

Schröter may be said to have founded the comparative study of the Moon's surface features, and his two books, "Selenotopographische Fragmente," 1791 and 1802, and other writings, contain much detailed observations, the value of which was, however, lessened by his limited powers of drawing. His planetary work was uneven in quality. He noted the differences in rotation periods of Jupiter's markings (see pp. 176, 236), but he believed that what he saw on Mars was impermanent and only a result of drifting clouds in its atmosphere.

REFERENCES.

[1] Printed in "The Collected Scientific Pupils of Sir William Herschel" (1912).

[2] For a good account of Herschel's stellar researches and theories, see J. E. Gore, "Astronomical Essays," pp. 124-200 (1907).

CHAPTER XI

NINETEENTH CENTURY: FIRST HALF

DURING this period exact observational Astronomy and theoretical Gravitational Astronomy made considerable progress; but there were few important or revolutionary discoveries. In consequence of improvement in dissemination of information, the work of those responsible for advancement was sometimes of a more concerted or connected nature than previously.

GROWTH OF AMATEUR OBSERVATION

Herschel's discoveries stimulated general interest, and the further development of descriptive Astronomy (see p. 114) as originated by him, was a marked feature of the time. This branch of the science is more readily understood and taken up by those without special technical or mathematical training, and its progress can therefore be materially helped by amateurs without elaborate mental or instrumental equipment. To the professional astronomers, although they might often be interested in the descriptive side, there usually fell the duty of carrying on the gravitational and exact measurement branches, as successors to Laplace and Bradley.

As a consequence of this there appeared numbers of amateur observers who often specialised in a certain department such as Solar, Lunar, Planetary, Cometary, Meteoric, Variable Stars, or, if possessed of adequate instrumental equipment, measurement of double stars. Much of the progress in some of these branches, notably Lunar, meteors and variable stars, was due to long-continued careful work of such amateurs, which could not have been undertaken by the limited number of professional astronomers with their routine and more technical duties.

IMPROVEMENTS IN INSTRUMENTS AND METHODS

Improvement in instruments of precision, upon which advances in exact measurement depend, was accompanied

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by increase in size of all types and by greater accuracy of the graduated circles; and new methods of mounting and of making and recording observations were introduced. The French mathematician A. M. Legendre (1752-1833) and the German C. F. Gauss (1777-1855) devised a process known as the method of "Least Squares" to extract from a number of observations the most likely value; and, from a derived "probable error," this method acted as a test of a result. F. W. Bessel (1784-1846) made great improvements in the corrections for refraction, aberration, precession and nutation to determine actual positions; and in addition he systematized the procedure of "reduction" of observation, not forgetting the important features of determination of errors for the instruments themselves and of the personal errors of the observer. ("Personal Equation").*

Most of these corrections were closely investigated in Bessel's "Fundamenta Astronomiae," based on Bradley's observations (see p. 104), and he finally described his general method of reduction, and simplification of procedure, in a book, "Tabulae Regiomontanae (1830)."

Bessel was the first to employ the "heliometer," an instrument constructed by Joseph Fraunhofer (1787-1826), who was responsible for the improvements in lenses and mountings at the firm of von Reichenbach at Munich. The heliometer was originally intended for work on the Sun, but became about the best instrument before photography for measuring dimensions of planets or angular distances between stars, and for stellar parallax work. The principle of this instrument, which is also called a divided object-glass micrometer, is the separation, by a measurable amount, of two distinct images of the same object. When a double star, for example, is under examination, the two half lenses into which the object glass is divided are shifted until the upper star (say)

*This is an error peculiar to the individual observer. Maskelyne noted that one of his assistants consistently differed from him in estimating the times of transits of stars. He failed to realize that the difference was as likely to be his constant error as the assistant's, and discharged the man as incompetent. Bessel first drew attention to its nature in 1823. A psychological phenomenon, depending on the relative rates of perception by the senses, it can be measured with an instrument for the purpose, and has been minimised by a special moving-wire impersonal micrometer.

in one image is brought into coincidence with the lower star in the other ; their angular distance apart then becomes known by the amount of motion employed in separating the halves of the object glass.

During the period under review the work of the Greenwich Observatory (John Pond (1767-1836) and G. B. Airy (1801-1892) successive Astronomers Royal), the Paris Observatory, and the Russian Pulkowa Observatory* (Imperial Astronomer, F. G. W. Struve (1793-1864)), in measurement of star position and other accurate instrumental observations, were accumulating fundamental data for use in gravitational and sidereal Astronomy. In America the U.S. Naval Observatory was organised in 1842, and Harvard College Observatory in 1839 ; the first observatory to be built in the States by public subscription was opened at Cincinnati in 1842. The Observatories at the universities of Great Britain were also active during this time, while the first one in the Southern hemisphere was founded at Paramatta, Australia, in 1821, that at the Cape of Good Hope starting in 1829. The East India Company had two in India and one at St. Helena in the first third of the century.

The progress of reflecting telescope construction was maintained by the Earl of Rosse (1800-1867), with one of 3 feet aperture in 1840 and a larger one of 6 feet in 1845, and by William Lassell (1799-1800), with a 2 feet and later a 4 feet aperture instrument, all of these having metallic mirrors. But the difficulties, chiefly that of making optical glass in large discs of good quality, had kept down the size of the refractor to 9·6 inches (Dorpat, 1824), 11½ inches ("Northumberland" telescope at Cambridge, 1834), 12 inches (London, Sir James South, 1830 ; later at Dunsink, Ireland), 13¼ inches (Markree, Ireland, 1834), 15 inches (Pulkowa, 1840), and 15 inches (Harvard, 1847), the large object glasses of the time being procurable only on the Continent. The Dorpat telescope was "equatorially" mounted and driven by clockwork mechanism so that celestial objects could be followed across the sky in their diurnal motion. Hooke, Roemer, Bradley, Ramsden (1735-1800), and others had thought of such an

*Destroyed by the Germans in the 1939-45 World War.

arrangement and had tried it out (Ramsden had a patented form on sale 50 years earlier); but the Dorpat refractor was the first telescope for which the idea was made a practical feature, and was the beginning of similar mechanical arrangements in most large instruments. One of the first reflectors to be equatorially mounted was a 9-inch aperture instrument made by Lassell in 1840.

MATHEMATICAL ASTRONOMY

In mathematical Astronomy, Gauss produced new and improved processes of calculation the chief being a method of computing the orbit of a planet or comet from three or more observations of position. This method was published in his book "Theoria Motus" (1809). Hitherto the prediction of the future movement of a newly discovered body was very uncertain unless the observations extended over a period of time and were numerous and accurate. Gauss's method, and his application with it of the principle of Least Squares, soon showed their value in regard to the place of a new minor planet (see p. 141) which had been observed for only about six weeks before moving into that part of the sky occupied by the Sun, and had become lost afterwards. Although the recorded observations covered barely 3 degrees in the sky, it was found nearly a year later close to the predicted position.

Felix Savary (1797-1841) was the first to apply gravitational principles to the computation of orbits of binary stars, when, in 1827, he derived an orbit for ξ Ursae Majoris, which he found to be an eccentric ellipse, with the period of revolution about 60 years. In the next few years, J. F. Encke (1791-1865), and William Herschel's son, John F. W. Herschel (1792-1871) developed other methods of computing such orbits.

DISCOVERY OF NEPTUNE

But the greatest achievement in gravitational Astronomy of the first half of the nineteenth century (and probably the greatest since Newton) was the discovery of the planet Neptune. By means of the many observations of Uranus made since its discovery by W. Herschel in 1781 (see p. 115), an orbit for that planet had been calculated from which its past and future

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positions in the sky could be obtained. It was then found that Uranus had been seen and recorded as a fixed star no less than twenty times between 1690 and 1781, four astronomers having been concerned, who had thus each had a chance of being its discoverer. But an unexpected difficulty appeared. If these early observations were correct then Uranus had departed from its calculated orbit ; and it did not seem possible to modify the orbit so that both old and new observations could be fitted in. Indeed, as time passed, the planet got further and further away from its predicted places. For example, the discrepancies were 20″ in 1830, 90″ in 1840, and 2′ in 1844, using places calculated from an orbit computed by Bouvard which was based only on recent observations.

By some astronomers inaccuracy of the older observations was considered to be probably the explanation. But others (among them Bessel and Sir John Herschel) thought that there might be a planet beyond the orbit of Uranus which was the disturbing cause. It is of interest to note that among the first to suspect this was the Rev. T. J. Hussey, Rector of Hayes, Kent, who wrote to G. B. Airy (then professor at Cambridge) in 1834, suggesting that he might try to find such a planet with his "large reflector." Airy's reply discouraged him from this, as the motions of Uranus were, "not yet in such a state as to give the slightest hope of making out the nature of any external action."[1]

There were two men who favoured the outer planet idea, quite unknown to each other. These were the Englishman J. C. Adams (1819-1892) and the Frenchman U. J. J. Leverrier (1811-1877). They independently began work on the problem of determining from the observed deviations of Uranus (which never exceeded 2 minutes of arc, a space imperceptible to the unaided eye), the location of the disturbing planet, a problem requiring mathematical skill of the highest order. The first of the two to obtain the probable path of the outer planet, and its place in the sky at the time, was Adams. By October, 1845, he had succeeded in assigning a place differing less than 2 degrees (four times the Moon's diameter) from its actual position, and communicated his result to the Astronomer Royal, Airy. Unfortunately Airy was not confident as to the prediction and seems to have been unable, with pressure of

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official work at the Observatory, to have a telescopic search instituted there.

In August, 1846, Leverrier had also succeeded in obtaining an orbit and a position in the sky for the disturbing body, and when his results, which confirmed those of Adams, were brought to the attention of the Astronomer Royal, Airy requested the Professor of Astronomy at Cambridge, J. C. Challis (1803-1862) to undertake a search with the Northumberland 11½ inch telescope. This was done by the laborious method of checking up on every star seen near the predicted place on a number of nights, to detect possible motion on the sky. In the words of S. Newcomb, "Instead of endeavouring to recognise it by its disc . . . his mode of proceeding was much like that of a man who, knowing that a diamond had dropped near a certain spot on the sea-beach, should remove all the sand in the neighbourhood to a convenient place for the purpose of sifting for it at leisure, and should thus have the diamond actually in his possession without being able to recognise it."[2]

But in the meantime Leverrier had written to the Observatory at Berlin intimating that if a telescope were directed to a point in the Ecliptic in the constellation Aquarius, at longitude 326 degrees, a planet looking like a star of about the ninth magnitude but having a perceptible disc would be seen. The Berlin astronomers were more fortunate than those at Cambridge in that they possessed a new star chart of the region;* and consequently within half-an-hour of beginning the search on September 23rd, 1846, J. G. Galle (1812-1910), found the new planet within a degree of arc of the predicted place, looking like a star of the eighth magnitude, and showing a disc, big enough to be measured two nights later. In this discovery he was assisted by H. L. D'Arrest (1822-1875), who checked on the chart the stars as they were seen by Galle in the telescope, and intimated the absence from it of the one which turned out to be Neptune. The optical discovery as well as the mathematical one was thus shared by two.

*They may be said to have been doubly fortunate as the position of the planet was just inside the area of the chart.

Challis, at Cambridge, soon found that he had already seen the new planet. "After four days of observing" he wrote to Airy, "the planet was in my grasp if only I had examined or mapped the observations." This meant that he would have discovered it several weeks before Galle. Within seventeen days of its discovery at Berlin an English amateur, Lassell, had discovered a satellite with his reflector of two feet aperture. It is said that Lassell might have found the planet itself even before Galle. Two weeks prior to its discovery at Berlin he received a letter from W. H. Dawes who had heard of Adams's results, urging him to look for a star with a disc in a certain part of the sky. Unfortunately, just at the time, Lassell had a sprained ankle and could not observe. It is perhaps a pity that Dawes himself, unrivalled as an observer, and using high-class refractors of more than six inches aperture, does not seem to have searched, as he might have recognised the planet by its difference from an ordinary star.

When an orbit had been obtained for Neptune it was found that, like Uranus, the planet had been seen previously as a star by several observers. In fact, J. J. Lalande (1732-1807), had on the 8th and again on the 10th of May, 1795, noted it as a fixed star, but suppressed the first observation as erroneous thus losing the opportunity to discover Neptune as Herschel had found Uranus fourteen years before. But it is only just to remark that the discovery of Neptune would then "merely have been the accidental reward of a laborious worker, instead of being one of the most glorious achievements in the loftiest department of human reason."[3]

LUNAR AND PLANETARY THEORY

During the period special problems of lunar and planetary theory were written on by S. D. Poisson (1781-1840). And theories of the Moon's motion were investigated by M. C. T. Damoiseau (1768-1846), A. A. Plana (1781-1864), P. G. D. de Pontécoulant (1795-1874), and J. W. Lubbock (1830-1865). But these were all superseded not long after by the results of P. A. Hansen (1795-1864) whose tables of the Moon's motion were found to be of such great accuracy as to be adopted by the Nautical Almanac and all similar publications.

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LUNAR WORK

The work of Schröter on the Moon, which extended from 1785 until 1813, started systematic "Selenography" or the study of the physical features of the Lunar surface. The only map of any real value in existence before his time was the 7½ inch diameter one of Tobias Mayer published at Göttingen in 1775. Herschel had paid some attention to the Moon's surface features and had tried to measure the heights of some of its mountains. His ideas as to the Moon's physical condition have already been referred to (see p. 126). Other students of selenography had been Hevelius, Cassini, and Riccioli; but the idea of examining lunar details persistently with a view to detecting signs of change, or activity of natural forces, was Schröter's. He discovered the markings he named "rills" which appear to be clefts in the surface about half-a-mile or so wide, a fraction of a mile deep, and up to a hundred or more miles long. He found about a dozen before 1801, and by the end of the first half of the nineteenth century more than a hundred were known out of the hundreds which later examination has shown to exist. He was a very poor draughtsman, however, and his drawings of craters and other formations have much less value than they might otherwise have had as records of the Moon's surface a hundred and fifty years ago. Schröter thought that he saw evidence of the existence of a lunar atmosphere in twilight extensions of the points of the lunar crescents, but this has not been confirmed although often looked for by others.

The next worker in selenography was W. G. Lohrmann (1796-1840) a land surveyor of Dresden. In 1824 he published four out of twenty-five sections of an elaborate Lunar chart on a scale of 37½ inches to the Moon's diameter. His eyesight failed, however, and at his death in 1840 he left materials whereby the work was utilized and published nearly forty years later (see p. 161). But the first scientifically designed chart made by a proper trigonometrical survey was issued by M. Beer (1797-1850) and J. H. Von Mädler (1794-1874) in four parts, in 1834-36, nearly on the same scale as Lohrmann's but with greater and more accurate detail. The telescope used was only four inches in aperture, but the positions of 919

formations and the heights of 1095 mountains (four of which they found to be more than 20,000 feet high) were given with their map.

In 1837 Beer and Mädler published a descriptive volume, "Der Mond : oder allgemeine vergleichende Selenographie," summarizing knowledge of our satellite according to their ideas. They viewed it as an airless, lifeless and changeless globe (differing in this from Schröter, Lohrmann and others), which is probably nearer the truth than the other view. But fortunately no effect of a kind likely to discourage Lunar research followed as will be seen later.

The Moon was first successfully photographed by Bond at Harvard College Observatory in 1850, on a small scale, by means of the daguerrotype process.[4]

SOLAR WORK

In the last chapter the theory of the Sun's condition held by Herschel and by Wilson has been described (see p. 124), and it was remarked that it held the field for some time as the accepted explanation. Although the ideas in it are indeed strange to modern minds, one should remember that it was the first systematic attempt to co-ordinate solar phenomena ; and while it was completely erroneous it may have had the value to which Lord Bacon referred when he wrote that truth is more apt to emerge from error than from confusion.

Observations of Solar eclipses before this time had been used chiefly for corrections to the movements of the Sun and Moon, a check on which was provided by the times of apparent contact of the limbs of the two bodies. But from about the eclipse of 8th July, 1842, observed in Central and Southern Europe by the leading astronomers of the day, the appearances surrounding the eclipsed Sun began to be more closely noted and studied. These were the "Corona," the luminious surrounding halo of light, and the coloured "Prominences" seen on the edge of the disc. Both of these had been noted in past years and had sometimes been considered to be appendages of the Sun, but about as often believed to be lunar features. From the 1842 eclipse onwards, study of all total Solar eclipses was systematically pursued with a view to a

solution. The question of their nature, whether self-luminous Solar appendages or Lunar clouds lit up by the Sun, was not yet decisively answered. In the eclipses between 1842 and 1851 the process of discovery of the nature of these appearances was continued, but the Wilson-Herschel Theory of the Sun's constitution does not appear to have been affected as yet.

The telescopic study of sunspots during the period resulted in one epoch-making discovery. Heinrich Schwabe (1789-1875) of Dessau, began in 1826 to observe the Sun on every clear day with a small telescope, having in mind, it is said, the possibility of discovery of an intra-Mercurial planet. Every day for more than forty years he counted the number of spots, but by 1843 after years of patient work, he noted a recurring periodicity of about ten years in the daily numbers. In his tabulated figures for 1826 to 1843 the hint of periodicity was strongest in the numbers of days, for a given year, showing no spots. For example in 1828 there were no such days, five years later in 1833 there were 139, five years after that in 1838 there were again none, to be followed in 1843 by 149. And the recorded numbers of no spot days each year between these dates varied progressively. By the year 1851 Schwabe had confirmed this and he then published his results and suggested the ten year cycle. Examination of former records of spots and later observations have increased this to a period of 11·1 years. But this is correct only on an average. Intervals of from seven (1830-37) to sixteen (1788-1804) years have been noted. From Chinese records between 188 A.D. and 1638, in which time 95 observations of spots visible to the naked eye have been found, an average period of eleven years is obtained. It may be noted, however, that according to the researches of Spörer and Maunder it appears that there was a period of great scarcity of spots between about 1643 and 1715.[5] Study of the varying thicknesses of the growth rings in giant trees of Western America by A. E. Douglass also shows what is evidently the effect of this long quiet period, and a contemporary scarcity of Aurorae has been noted. The first to suggest periodicity of Sun spots, but without any idea of the time interval, was probably the Danish astronomer Christian Horrebow (*d.* 1776).

A similar period for the Earth's magnetic variations was

first noted by Lamont at Munich in 1851, and confirmed independently by several scientists in the following year, the probable connection with the Sun being soon noticed.

The granulated structure of the Sun's luminous surface, described by a famous astronomer as "like a plate of rice soup," seems to have been first noticed by Short the optician in 1748. Herschel detected it and refers to it in papers printed in 1795 and 1801. Dawes studied it closely in 1830, and Schwabe referred to it in 1831. As regards study of the individual spot structure, this received special attention from Sir John Herschel at the Cape of Good Hope in 1836-7.

The first determinations of the strength of the Sun's radiation, of a scientific character, were made in 1837 by Sir John Herschel and, a few months later, by the French physicist Pouillet, the two agreeing very well. Pouillet found that the vertical rays of the Sun on each square centimetre would raise the temperature of 1·76 grams of water one degree centigrade per minute. The modern value of this, the Solar "Constant," which varies one or two per cent, is 1·94.

MARS

In the year 1830 the planet Mars came closer to the Earth than at any other time during the nineteenth century. The occasion of this favourable Opposition* was used by Beer and Mädler to make the first systematic chart of the planet's surface features. Lines of latitude and longitude were adopted for the first time and when it was published in 1840 a foundation for the modern Science of Areography was laid.

THE MINOR PLANETS

Although in planetary Astronomy the chief event of the first half of the century was the discovery of another member of the Solar System, Neptune, as described earlier, there were other discoveries of planets of a different kind. These were perhaps of more significance with respect to theories of the origin and development of the planetary system.

*When in opposition a planet crosses the meridian at about midnight. The Sun, Earth and Mars are in line, as viewed from North or South of the Ecliptic, when Mars is in opposition.

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On the first evening of the nineteenth century, January 1st, 1801, Giuseppe Piazzi (1746-1826), Professor of Mathematics and Astronomy at Palermo, Sicily, noted the position of an eighth magnitude star in the constellation Taurus, a part of the sky which he was observing in connection with an error he had found in a star catalogue. It was his practice to deal with a set of fifty stars on four successive nights, and on the two following nights he found that the star had changed its position. He observed it when possible until February 11th when he unfortunately fell ill. Its loss in the Sun's rays and its re-discovery as a result of calculations by Gauss have been referred to earlier in this chapter (see p. 133).

The orbit found by Gauss indicated that the new body belonged to the space between the orbits of Mars and Jupiter, which had been noted as abnormally large by astronomers since Kepler's day. That some body might be found in this gap had appeared probable from an empirical relation between the sizes of the planet's orbits first pointed out by J. D. Titius (*d.* 1796). This Wittemberg professor had appended a note to a translation he had made of a scientific work, published in 1772, drawing attention to the existence of a series of numbers that expressed, with rough accuracy, the distances from the Sun of the six then known planets. The numbers are obtained by adding 4 to 0, 3, 6, 12, 24, 48 and 96, the results 4, 7, 10, 16, 28, 52, 100, fairly representing the actual distances except that 52 and 100 have to be taken for Jupiter and Saturn, with no planet at 28. The priority of Titius has been questioned and the first discovery of the so-called "Law" has been attributed to Christian Wolf of Halle (*d.* 1754). The relation gives a good result for Uranus but breaks down for Neptune and Pluto. It is not a true series as the 0 should be $1\frac{1}{2}$. The name, Bode's Law, is usual. J. E. Bode (1747-1826) of Berlin noted the absence of a planet to correspond with the number 28, and he organised a combination with five other astronomers for the purpose of trying to find what they thought might be a new planet. But Piazzi's planet, which was given the name of Ceres, forestalled any result from Bode's plan, and the discovery of nine other small planets, Pallas (1802), Juno (1804), Vesta (1807), Astraea (1845), Hebe (1847), Iris (1847), Flora (1847), Metis (1848), Hygeia (1849), by various

observers (Iris and Flora from an observatory in Regent's Park) followed before 1850, in which year four more were found. All of these had orbits in the zone previously unoccupied by any known body, and were of small size being barely distinguishable from a fixed star except in the most powerful telescopes of the time. They were the forerunners of thousands of "Minor Planets" to be discovered later mostly by photography. The largest, Ceres, is rather less than 500 miles in diameter; the majority are very much smaller. Only the first four exceed 100 miles.

H. Olbers (1758-1840) a medical man and amateur astronomer of Bremen, advanced the hypothesis that the four minor planets known to him were fragments of an exploded large planet. This mode of origin has been considered likely by some astronomers but the idea has several rather serious difficulties to contend with.

THE MAJOR PLANETS

As has already been stated, a satellite to Neptune was found by Lassell shortly after the discovery of the planet itself. The same observer, with his two feet aperture reflector, discovered a satellite to Saturn in 1848, independently found by W. C. Bond (1789-1859) with the 15-inch refractor at Harvard College Observatory. This discovery made up to eight the seven previously seen by Huyghens, W. Herschel and J. D. Cassini. In 1850 the same two observers and W. H. Dawes (1799-1868) all discovered independently the so-called "crape" ring of Saturn, lying immediately inside the main ring which is divided into two, an outer and inner, by Cassini's division (see p. 74). This dusky crape ring had been found twelve years before by Galle of Berlin, the discovery having been duly reported to the Berlin Academy of Sciences but remaining unnoticed for some reason. But it seems probable that the crape ring was seen as far back as 1720 by Hadley and even earlier by J. D. Cassini, where it crosses the ball. In 1837 Encke drew attention to the division in the outer bright ring, which goes by his name, a marking which seems to vary in distinctness.

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COMETS

Before the introduction of the telescope in 1609 about 400 comets are recorded; all, being visible to the naked eye, bright objects. But many more were detected with optical aid so that towards the latter half of the eighteenth century nearly one comet per year was the average. During the first half of the nineteenth century, when the average was about two per year, many remarkable comets were seen. About thirty were recorded as visible to the naked eye, the years of the appearance of nine of the brightest being 1809, 1811, 1819, 1823, 1830, 1835 (Halley's), 1843 (visible in daylight), 1845 and 1847,* and two or three telescopic comets per year were also found, one comet-seeker J. L. Pons (1761-1831) detecting 27 from 1802 to 1827. He discovered 36 altogether.

But in some ways the most remarkable comets of the period were not the large naked eye ones referred to. One comet which was seen by Pons in 1818 was given the name, Encke's Comet, because of the great amount of mathematical work done by Encke on its orbit. It is periodic and has the shortest period so far known, 3·30 years. Encke found that, after making all necessary allowances for perturbations by the planets, there remained an outstanding continuous but variable shortening of the period, which averages about one and a sixth days per revolution, that has been attributed to the effect of resistance of some kind to the comet's movement. If a resisting medium is responsible, the fact that the other short-period comets do not show a shortening of period may perhaps be explained by the medium not extending beyond the orbit of Mercury, Encke's being the only short-period one which passes inside that orbit when close to the Sun. On the other hand the apparent absence of effect on the motion of parabolic or very long period comets which go even closer to the Sun has to be noted in this connection.

A comet called Biela's Comet, period $6\frac{3}{4}$ years, was observed in 1772, 1805, 1826, 1832, without anything remarkable. But

*The number of comets recorded as visible to the naked eye per half century since 1500 has varied from 2 (1600-1650), 4 (1700-1750) and (1700-1800), to more than 40 (1900-1947). Until about 1800, however, only comets visible in northern latitudes are included in these figures.

in 1846 it became pear-shaped and actually divided into two parts which travelled together for over 3 months separated by a distance of 160,000 miles, each part developing a nucleus and a tail. When observed in 1852 the separation was nearly ten times as much and neither part has again been seen.

Another remarkable cometary occurrence was the passage of the Earth through the tail of the comet of 1819 with no noticeable consequences.

The 1835 appearance of Halley's comet was (like that of 1910) best seen in the Southern hemisphere, where it was well observed by Sir John Herschel at the Cape of Good Hope. The 1843 comet, visible in daylight, seems to have been the most brilliant of the comets seen during this period.

Some outstanding points of interest in Cometary Astronomy at this time may be mentioned. Three of these were: the discovery of the first "short-period" comets, of which six were known by 1850, with periods ranging from 3·3 to 7·4 years, thus introducing a new type of comet, the shortest periods previously known being those of Pons's comet, 1812, 71 years, Olbers's comet, 1815, 74 years, and Halley's comet, 76 years; the suggestion of Olbers in 1812 that the tails of comets are rapid outflows of highly rarefied matter most of which becomes permanently detached from the comet's head or nucleus, their formation being due to some form of Solar electrical repulsion; and the observations that made it clear that the substance of comets is very tenuous, no case of dimming of the light of stars by their passage having been noted.

METEORIC DISCOVERY

Up to about the beginning of the nineteenth century the commonest idea of the nature of meteors or shooting stars was that they are really meteorological phenomena caused by ignition of vapours in the lower atmosphere. Halley apparently held the belief that they came from outside space, but that view was certainly not a general one. The physicist, E. F. F. Chladni (1756-1827), however, in 1794 expressed the opinion that meteors are small bodies circulating through space, becoming visible through incandescence on entering the Earth's atmosphere. Acting on his suggestion in 1798 two

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students of Göttingen University, Brandes and Benzenberg, observed a number of meteors simultaneously from separate observing points, and using the base line thus provided they calculated from this survey operation that meteors appeared in the atmosphere at great heights moving with speeds like those of the planets in their orbits.

Some astronomers, including Laplace in 1802, maintained that meteors were ejected from craters in the Moon, but no progress was made in elucidating the subject for the first thirty years or so of the century. Such ideas as existed were, however, clarified by the investigations following the extraordinary shower of meteors on November 12, 1833, when nearly a quarter of a million were estimated to have been seen by observers in America during nine hours display. All of these shooting stars appeared to come from the same part of the sky, the constellation Leo, with their paths diverging from one "radiant point." As a result it was soon deduced that these meteors moved in parallel paths round the Sun, apparently divergent in our sky because of perspective.

A. von Humboldt (1769-1859) at Cumana in Venezuela, had seen nearly as rich a display in the year 1799, and had noted that the meteors also diverged from a radiant; but nothing in the way of an explanatory theory had been attempted. The first to suggest that these "Leonid" meteors might be particles in a swarm moving round the Sun was Olmsted of Yale University. He proposed for them a period of about 6 months in an orbit nearly intersecting that of the Earth at the point occupied by it on November 12th. The next to propose an explanation was Olbers. In 1837 he suggested a period of 34 years basing this on Humboldt's 1799 observation, and he thought that the next display would probably be in 1867.

Another periodic meteoric system was shown to exist, by Quételet (1796-1874) in 1836. This was what are now called the August Perseid meteors, long recorded as an annual display in that month, and known popularly as the "tears of St. Lawrence," as they are most frequent at about St. Lawrence's day. Biot had remarked that in fifty-two appearances of shooting stars recorded in Chinese annuals, the dates that recur most frequently are from 20th to 22nd July, Old Style,

corresponding to the early part of August, modern dating. The study of shooting stars, or Meteoric Astronomy, may thus be said to have become a definite department of the science from the first half of the nineteenth century.

Actual falls of meteorites to the Earth's surface occurred at l'Aigle in France in 1803 (between 2000 and 3000 small stony meteorites), and at Stannern in Austria in 1808 (about 200 stones); while among many falls of single meteorites at the time were one in 1830 at Taunton and one in 1835 at Aldsworth. These drew attention to another aspect of the study, in the examination and analysis of the constitution of such bodies.

SIDEREAL ASTRONOMY

The sidereal investigations of Sir William Herschel sketched out in Chapter X extended into the nineteenth century. It will have been noted that they covered a wide field and that he was a pioneer in much of the range.

His son, Sir John Herschel, was his closest follower in general telescopic exploration. Leaving Cambridge University as Senior Wrangler, he first studied for the Bar, but after his father's death in 1822 he stopped his legal studies and took up double-star work for a time. Between 1825 and 1833 he systematically examined the sky visible in this latitude verifying his father's discoveries of clusters and nebulae, and adding more than 500 new ones to the total of over 2300 already listed by Sir William; and he also found 3347 double and multiple stars.

He then decided to attempt to complete a survey of the whole sky, and, for the purpose, went to Cape Colony with his family, taking an 18-inch reflector and a five-inch refractor. The task was completed in four years (1834-38) but the full results were not published until 1847. They included descriptions of over 1700 nebulae, at least 300 new; and also measurements of 2100 double stars. The gauging process of star counts of his father (see p. 118) was repeated for the southern skies, and the two Magellanic Clouds—large aggregations of clusters, nebulae and stars—were carefully explored.

Among his minor pieces of work was his measurement, by a specially designed "Photometer," of the light of 191 of the

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brightest stars visible at Cape Town, and of the relative light of the Full Moon in terms of a standard star. From these in turn he derived the ratios of the brightness of the Sun to those of the stars measured, the comparative brightness of the Sun and Full Moon having already been determined by Wollaston who had found a ratio of 801, 072 to 1, later values by Zöllner, Steinheil, Bond and others reducing this to about 450,000 to 1. John Herschel's views regarding the form and structure of the Milky Way are not definitely stated in his writings, but he seems sometimes to have suggested a ring rather than a disc formation.

He was made a knight in 1830 before his trip to the Cape of Good Hope, and a baronet on his return. The remainder of his life was spent largely in the collection and arrangement of his father's and his own work, and in the writing of valuable treatises on Astronomy and other scientific subjects.

His "Outlines of Astronomy" (1849) is perhaps the finest general astronomical treatise ever written; it went through ten editions before his death in 1871. John Herschel was one of the pioneers of photography. He discovered the action of hyposulphite of soda in "fixing" the silver grains of a photographic plate; and to him are due the introduction of the familar terms "positive" and "negative" in photography, and the use of film-covered glass negatives.

STELLAR PARALLAX

Bessel's improvements in methods (see p. 131) and his construction of a great star catalogue based on Bradley's observations, were both very notable contributions. But the most memorable of his investigations was the definite detection of the parallax of a star. For this he chose the double star 61 Cygni, not because of its brightness, one obvious criterion of nearness and therefore of large parallax, but for its large proper motion of 5·2 seconds of arc per annum, which is also an indication of relative proximity. He used Galileo's method, measuring with the heliometer (see p. 131), at frequent intervals during a year, that star's distances on the sky form two neighbouring stars, both faint and without sensible proper motions and thus in all probability much more distant from us than 61 Cygni.

At the end of 1838 he announced that the parallax was about a third of a second of arc, corresponding to a distance of 10 light years. The remarkable accuracy of this first reliable determination of parallax may be gathered from the fact that the most recent value is not ten per cent different. This first parallax was quickly succeeded by two others α Centauri, nearly a second of arc (distance $3\frac{1}{4}$ light years) found in 1839 by T. Henderson (1798-1844), and Vega, about a quarter of a second (distance 13 light years), by F. G. W. Struve in 1840. The modern values for these two stars are about three-quarters of a second and an eighth of a second respectively. The methods used were not the same as Bessel employed for 61 Cygni, being the meridian circle and micrometer respectively, neither of which has proved so accurate as the heliometer in subsequent parallax work. Earlier observations for parallax by meridian circle or similar methods made by Piazzi, Cacciatore, Brindley, Calandrelli and others during the previous fifty years gave apparent values for Sirius, Aldebaran, Procyon, Polaris and other stars of one to several seconds of arc. But these proved to be illusory and due to periodic (sometimes seasonal) errors of the instruments employed.

SUN'S MOTION IN SPACE

Sir William Herschel's determination of the movement of the Solar system in space was not favourably received by all astronomers, Bessel and others, including Herschel's own son, having been unconvinced. But in 1837 W. H. Argelander (1799-1875), at one time Bessel's assistant, published a result agreeing with Herschel's, based on the motions of 390 stars. All doubt was removed by a confirmation in 1842 by F. G. W. Struve's son O. Struve (1819-1905), and by Galloway in 1847.

F. G. W. STRUVE

F. G. W. Struve, the greatest of W. Herschel's successors in double star Astronomy, first worked at Derpat from 1813 to 1837, with the 9·6-inch refractor already mentioned (see p. 132). From 1825 to 1827 he reviewed the northern heavens and, examining 120,000 stars, he found 2,200 double stars

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among them previously unnoticed as such. For ten years he measured the relative positions of the components, and in 1837 he published his great work "Mensurae Micrometricae" containing relative positions, colours and brightnesses of 2640 double and multiple stars. This was a magnificent foundation for Double Star Astronomy, and the book also contained much valuable matter in its introduction, including a chapter on the probable parallax of the stars, giving the criteria by which the probability of a star having a parallax of measurable size can be judged, "so well conceived that we could hardly improve on it to-day."[6]

Another book by Struve, "Études d' Astronomie Stellaire," published in 1847, contains a general confirmation of W. Herschel's Milky Way disc theory. But Struve believed that although the disc is of finite thickness it is infinitely extended in the direction of its plane, and that the more distant of its star clouds are invisible through extinction of light in space. This contention received very severe criticism from Sir John Herschel and Encke which it did not long survive.

EARL OF ROSSE: SPIRAL NEBULAE

One striking discovery of the period was made by the Earl of Rosse with his 6 feet aperture reflector. This was a new class of nebula termed "Spiral" from the possession of two spiral arms proceeding from opposite sides of the central nucleus. The first was found in April, 1845, in the constellation Canes Venatici being number 51 of Messier's catalogue (see p. 114). The second example was detected a year afterwards; it is Messier 99 in the constellation Virgo. Sir John Herschel's 18-inch reflector had shown Messier 51 as a sort of split ring surrounding a nucleus; and it had been seen earlier as a double nebula. By 1850 fourteen of the spiral class had been detected, the forerunners of enormous numbers to be photographed later with large reflectors.

THE FIRST ASTRONOMICAL PHOTOGRAPHS

It is believed that Daguerre, or one of the astronomers at the Paris Observatory, took a daguerrotype of an eclipse of the Sun in 1839 or 1840. Draper took a successful photograph

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of the Moon in 1840 with 20 minutes exposure, and, in 1843, one of the solar spectrum; and in 1845 Fizeau and Foucault obtained one of the Sun showing groups of spots.

In 1850 a noteworthy achievement was recorded at Harvard College Observatory, where a daguerrotype photograph was secured of Vega and one of Castor, by means of the 15-inch refractor. One or two minutes exposure was required with the primitive photographic method and the unsteady movement of the clockwork driven instrument, second magnitude stars being beyond its power. This was the beginning of stellar photography; in seven years time the improvement in steadier movement of the telescope and through the use of the collodion process was so great that all naked-eye stars could be photographed.

REFERENCES.

[1] *Journal of the British Astronomical Association*, vol. 49, p. 185 (1939).

[2] Newcomb, "Popular Astronomy," p. 370 (1883).

[3] Ball, "Story of the Heavens," p. 288 (1891).

[4] For an historical account of Lunar photography, see S. A. Saunder's Presidential Address, *Journal of the British Astronomical Association*, vol. 15, pp. 5-18 (1904).

[5] E. W. Maunder, *Journal of the British Astronomical Association*, vol. 32, p. 140 (1932).

[6] J. Jackson, "Early Estimates of Stellar Distances," *Observatory*, Nov., 1922, p. 345. A translation of the more important parts of the chapter is published in Jackson's article.

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CHAPTER XII

NINETEENTH CENTURY: SECOND HALF

THE epoch now entered upon may fairly be called the early stage of "recent" Astronomy. New methods of research of a revolutionary type were introduced and developed; one of these, the method of spectrum analysis as applied to the heavenly bodies, produced a knowledge of their chemical composition and constitution which had been deemed impossible by some.*

THE FIRST SPECTROSCOPIC WORK

Excepting the inventions of the telescope and of photography, no discovery has given so great an impetus to Astronomy as that of the instrument used in spectrum analysis —the spectroscope. Spectrum analysis consists in the analysis of light from a luminous body by means of a prism or set of prisms or by a diffraction grating. If the light of the Sun is admitted through a narrow slit an inverted image of the slit can be formed, by a lens of appropriate focal length, on a screen opposite. When a prism of glass is interposed the image is deflected to one side; and Newton showed that the images of the different colours of which white light is composed are deflected by a varying amount, the red least, the violet most. A continuous band of light or spectrum is thus formed showing all the colours of the rainbow—red, orange, yellow, green, blue, indigo and violet.

In 1802, W. H. Wollaston (1766-1828) had observed seven dark lines across this spectrum; these he took to be the edges of the primary colours. But in 1814-15 Fraunhofer studied the spectrum much more thoroughly and found about 600 lines, mapping the position of 324. Shortly afterwards he examined the spectra of the Moon, Venus, Mars and the fixed

*Auguste Comte, the founder of the Positive Philosophy, had declared in 1835 that the chemical constitution of any body outside our Earth was unknowable. ("Cours de Philosophie Positive," vol. 2. p. 8, 1835).

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stars Sirius, Castor, Pollux, Capella, Betelgeuse and Procyon. For the Moon and planets he found spectra very similar to that of sunlight; for the stars there were differences, but a line named by him the D line could be seen in most. When artificial lights were similarly analysed it was found that incandescent solids or liquids, or (later) dense gases, gave a continuous spectrum, while incandescent gases of low density gave bright lines. It was also found that dark bands or dark lines could be produced artificially in continuous spectra by passing the light through various substances or through gases.

Earlier experiments by Melvil in 1752 had shown that the light from flames tinged with metals or salts gave characteristic bright lines, and in 1823 Sir John Herschel suggested these lines might be used as a test for the presence of the substances concerned. Various explanations for the phenomena were attempted and the first to publish a satisfactory one was G. A. Kirchhoff (1824-1887) in 1859, although a correct one had been adopted by G. C. Stokes of Cambridge who had treated it as more or less an academic question. Kirchhoff repeated Fraunhofer's experiments and found that if a spirit lamp with common salt in the flame were placed in the path of the Sun's light a dark line already in the spectrum (called D by Fraunhofer) was intensified; and he also found that if he used a limelight instead of sunlight and passed it through the flame with salt, a dark D line appeared in the otherwise perfectly continuous spectrum.* He explained this as due to the absorption by sodium vapour in the flame (sodium being a constituent of salt) absorbing the same light as it emits at high temperature, that vapour itself giving a bright line in the place in the spectrum at which the D line appears. He announced that this proved that sodium vapour exists in the Sun's atmosphere; and he found subsequently by similar experiments that iron, magnesium, calcium, and chromium are actually in the Sun, with copper, zinc, barium and nickel in smaller quantities. The number of elements now identified in the Sun is 66 (1949); there is no reason to suppose that any of the 92 known are not present.

*It appears, however, that Foucault had obtained similar results in 1849, using a voltaic arc. This was unknown to Kirchhoff. (Dampier, "A History of Science," p. 258 (1942)).

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In this way the science of Astrophysics began; and a rapid accumulation of facts regarding the constitution of the atmosphere of the Sun, stars and planets, and of the nebulae was initiated.

C. Doppler (1803-1853), an Austrian professor of mathematics at Prague, pointed out in 1842 that if a luminous body is approaching an observer, its waves of light are, as it were, crowded together with a slight shift in the direction of the violet end of the spectrum, and the opposite effect in a receding body. From this principle, by comparing the spectrum of a heavenly body alongside, say, the spectrum of iron or other element obtained in the laboratory, a displacement of spectral lines can be measured, and the velocity of approach or recession of the body ascertained. If the displacement is towards the violet the motion is towards the observer, if to the red it is one of recession. Correction is, of course, made for the motion of approach or recession of the Earth with respect to the body, which will vary with the Earth's motion in its orbit round the Sun.

Not only, therefore, was an instrument provided for determining the elements present in celestial bodies. A powerful adjunct to the study of their movements from their proper motions in the sky was also made available from which wonderful results were later to be obtained.

IMPROVEMENTS IN INSTRUMENTS AND METHODS

With respect to improvement in instruments and methods of observation much was achieved before the end of the century. Photography became the equal of eye observation in all but lunar and planetary Astronomy, and superior in study of nebulae, comets, stellar distribution, and (latterly) measurements of stellar positions. Substitution of silver-on-glass mirrors for metallic specula, an invention of the German optician C. A. Steinheil (1832-1893) and, independently, of the French physicist Foucault (1819-1868) about 1856, increased by a third or more the efficiency of reflectors as light-gathering instruments. The size of the refracting telescope increased rapidly with improvement in the manufacture of optical glass. The maximum of 15 inches aperture in

1850 became 40 inches by the end of the century, this being still the aperture of the biggest refractor (the Yerkes telescope, University of Chicago). An idea of the growth in dimensions may be found from the following examples : Newall 25-inch, 1869 ; Vienna 27-inch, 1880 ; Pulkowa 30-inch, 1885 ; Lick 36-inch, 1888 ; Yerkes 40-inch, 1897.

The increased size of the reflector was also noteworthy. Leaving out of account the six foot Rosse metallic speculum (which is now a museum exhibit), the largest reflector built by 1900 was a 60 inch aperture silver-on-glass instrument, made in 1888 by an amateur Dr. A. A. Common (1841-1902) of London, which did valuable service in the photography of nebulae.

Other reflecting telescopes of particular note were : Melbourne 48-inch, 1870 (a metallic speculum, Cassegrain) ; Paris 47-inch, 1875 ; Lick (Crossley) 36-inch, 1879 ; Cambridge 36-inch, 1890 ; Toulouse 33-inch, 1887 ; Marseilles 32-inch, 1873 ; Greenwich 30-inch, 1897. The regular growth in the maximum size of the refractor was evidently due to the progress in ability to make optical glass in large discs. In the case of the reflectors there was no such limiting factor operating ; the discs for them did not require to be optically perfect for the purpose of *transmitting* light, as with refractors.

A new development was that of the photographic refractor with its object glass corrected so as to bring the blue and violet rays of strongest actinic effect, rather than those used visually, to as near as may be the same focus. These instruments with their great focal length and large scale image are particularly useful in stellar parallax work, for which the ordinary type refractor has also been used with a colour screen to limit the range of wave-lengths acting on the photographic plate. Wide angle photographic camera objectives of three or four lenses, that give good definition over a wide field of the sky, also made their appearance, starting with ordinary portrait lenses and evolving into such an instrument as the Bruce 24-inch of relatively short focal length made in 1893 and now at the Harvard College Observatory Station at Bloemfontein, South Africa.

The instruments for exact measurement were hitherto practically confined to two types ; those set up in the north

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and south plane of the meridian so as to intercept stars at their transit, and those mounted equatorially with clock drive for following objects in their diurnal motion. Several notable additions were made to these two chief methods of using the telescope, such as the altazimuth which can measure positions out of the meridian in any part of the sky, and a prime vertical instrument set up in the plane at right angles to the meridian.

In 1891 a new instrument was introduced into the equipment of the solar investigator. This was the spectroheliograph, invented independently by G. E. Hale (1868-1938) and by H. Deslandres (1854-1948). The instrument "is one by which photographs of the Sun may be taken exclusively by light of any desired wave-length. Its essential feature is the introduction of a second slit just in front of the photographic plate, which thus permits only a nearly monochromatic beam, chosen at will, to affect the plate. If an image of the Sun is formed (by the object-glass of a large telescope) on the first, or ordinary, slit of the spectroscope, this slit (which must be longer than the diameter of the image) cuts a narrow segment out of the Solar image. An image of this segment, produced solely by light of the single wave-length which is isolated by the second slit, is formed on the plate. The Sun's image is permitted to drift slowly over the first slit while the plate is moved with the same velocity behind the second slit; thus the image of segment after segment of the disc is recorded on the plate, until a complete photograph of the Sun has been built up in the selected wave-length. In photographing the prominences the disc of the Sun is concealed by an opaque screen of proper size, and the second slit is adjusted to pass radiation corresponding to one of the strong bright lines in the prominence spectrum. Regular records of the whole circumference of the Sun are thus obtained daily at a number of observatories."[1]

One development of primary importance was a practice of selection of high-level sites for new observatories such as the Lick Observatory, Mount Hamilton, California, 4,200 feet above sea level, and the Lowell Observatory Flagstaff, Arizona, at a height of 7,310 feet. There was also an increasing desire to choose places where the amount of clear sky is great and steady air conditions prevail. This has been continued

(although perhaps not quite so much as could be desired) and careful choice has been made even more desirable by the growing need for situations not affected by the sky illumination of modern towns, and the consequent limitation in photography of faint objects because of fogging of the plates.

PROGRESS OF ACCURACY IN MEASUREMENT

In connection with accuracy of measurements, on which so much depends in Astronomy, it will be of interest to note here the progress from the earliest times. The average errors may be taken to have been as follows: Hipparchus (2nd century B.C.), 4 minutes of arc; Tycho Brahe (16th century), 1 minute; Hevelius (17th century), ½ minute or less; Flamsteed (late 17th century), 10 seconds of arc; Bradley (18th century), 2 seconds; Bessel (heliometer, early 19th century), two-tenths of a second; first photographs (mid-19th century), one-tenth of a second; modern long-focus telescope photographs, one-fortieth of a second. The improvement in accuracy shown is nearly 10,000 times.

An example of the extraordinary accuracy of modern observations is provided by the discovery and measurement of the "variation of latitude." This was found from change in the astronomically determined value of latitudes of certain observatories, the amounts being much less than half a second. Küstner of Berlin in 1888 and S. C. Chandler of Boston in 1891 were the first in this field. The phenomenon is explained by a small change in the position of the Earth's axis of rotation with respect to the Earth itself, probably due to seasonal changes in ice and snow, vegetable growth, and barometric pressures. The movements of the Pole on the Earth's surface are contained in a circle of less than 50 feet radius.

ASTRONOMICAL SOCIETIES

Throughout the nineteenth century (and into the next) the formation of astronomical societies proceeded; the principal membership of these was amateur. Among them may be mentioned: the Royal Astronomical Society (1820); the Astronomische Gesellschaft (1863); the Société Astronomique de France (1887); the Astronomical Society of the

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Pacific (1889); the British Astronomical Association (1890). In Canada, the Royal Astronomical Society of Canada (1890), and (for professionals) the American Astronomical Society in the States, are important societies; and there are now organisations of varying size and importance in Belgium, Czechoslovakia, Denmark, Greece, Holland, Italy, New Zealand, South Africa, Soviet Russia, Sweden and Tasmania. This list is given as some evidence of the widespread interest in the subject; it does not aim at completeness. A fully comprehensive one would be very long indeed. For instance in the United States there is now a Society specially for the study of variable stars and also one for meteors, and there are said to be forty or more local societies which have reached some degree of stability and permanence, including a number for mutual help in telescope making.

LUNAR THEORY

In gravitational Astronomy there were no startling results such as the discovery of Neptune, but some important researches were undertaken successfully during the period. In chapter VIII reference has been made to the work of Laplace on the acceleration of the Moon's motion and the fact that J. C. Adams on going through Laplace's calculations had found that the amount should have been 5″ or 6″ instead of 10″ (see p. 92). The explanation of this discrepancy was first proposed by C. Delaunay (1816-1872) as due to the lengthening of the day by about a thousandth of a second per century. An attempt to calculate the amount of lengthening caused by tidal friction was made in 1853 by W. Ferrel; but it was not until more than a half century had passed, after calculations had been made by G. I. Taylor and by H. Jeffreys for the regions of the oceans where tides are abnormally strong,* that Delaunay's suggestion became the generally accepted explanation for at least part of the phenomenon first investigated with success by Laplace.

*It was found that on the average the dissipation of rotational energy of the Earth is equal to about 1500 million horsepower, two-thirds of this being due to the tides of the Behring Sea, where currents are strong and the water shallow. The total horsepower necessary for the lengthening mentioned would be about 2100 million.

In 1854, Hansen, and in 1858, Le Verrier, questioned the accuracy of the then accepted Solar parallax (8·58″) and distance (95,370,000 miles) of the Sun (see p. 109), maintaining that the Earth was substantially nearer to the Sun. This was based on consideration of the apparent monthly variation in the motion of the Sun across the sky, which reflects a real monthly movement of the Earth round its common centre of gravity with the Moon, and depends on the ratio of the distances of the Sun and Moon from the Earth. The amount of this variation being known, and the distance of the Moon already accurately ascertained, a value for the Sun's distance can be calculated ; and this turned out, according to Leverrier, to be nearly 4 million miles less than the accepted amount, with a corresponding parallax of over 8·9″

SATURN'S RINGS

Another mathematical result of outstanding interest was reached by James Clerk Maxwell (1831-1879) who, in 1857, showed that the rings of Saturn cannot be circulating solid or liquid appendages but can only maintain their form and stability if composed of an enormous number of small bodies revolving in independent orbits round Saturn. Something like this had been suggested by G. P. de Roberval (17th century), Jacques Cassini (1715) (see p. 75) and Wright of Durham in 1750, but their suggestions were merely speculative. Laplace's researches had only shown that the rings could not be a uniform solid body unless their weight was unsymmetrically disposed ; but he had left it at that. In 1867 Daniel Kirkwood (1815-1895) had suggested that the Cassini division between the outer and inner ring is explained by the perturbations of some of the satellites. In 1888 it was shown by Seeliger that only a ring composed of particles would show the observed surface brightness of the rings under varying angles of illumination by the Sun. J. E. Keeler (1857-1900) demonstrated by spectroscopic measurements of the radial velocities of its different parts that the constitution was undoubtedly as indicated by the mathematical researches of Maxwell. The inner edge of the rings appeared to move at 12½ miles per second in the line of sight, while the outer edge

had apparently a speed of 10 miles per second. These velocities are in accordance with what would be the case for separate satellites at the respective distances from Saturn. For a solid or liquid ring the speed of the outer edge would be *greater*, and not less, than that of the inner edge.

In 1850, Roche of Montpellier proved that no liquid satellite could exist as a coherent body inside about 2·4 times a planet's radius, if planet and satellite are equally dense. The nearest of Saturn's satellites (Mimas) is at 3·11 times its radius while the outer edge of the rings is only 2·3 radii out. This suggested that the rings might be satellite-forming material in a position where no satellite was possible. But in 1947, Jeffreys found that a *solid* satellite of rock moving near Jupiter's surface will not break up unless it is greater than about 250 miles in diameter: for a rocky satellite near to the Earth the limit is about 130 miles.

THE LEONID METEORS

It has already been remarked that for the Leonid meteor displays a periodic orbital return had been suggested, of 6 months by Olmsted, and 34 years by Olbers (see p. 145). As the time approached for the next brilliant display, judged by the interval observed between previous great ones (1799 to 1833), Professor H. A. Newton (1830-1896) of Yale, examined all past records he could find and noted that great displays had been seen between 902 and 1932 in the following years: 934, 1002, 1101, 1202, 1366, 1533, 1602, 1698 and 1799. He predicted another great display for 13th-14th November, 1866. This prediction was fulfilled although the shower was not so great as in 1833.

It had been found that great showers every 33 or 34 years could be explained by five different periods, combined with varying lengths of extension of the swarm along the orbit, the shortest period being 354½ days, the longest 33¼ years. It required great mathematical powers to solve the problem as the gravitational disturbances of the various planets had to be computed. By a very arduous process of calculation, J. C. Adams, co-discoverer with Le Verrier of Neptune, ascertained that all the observed phenomena would be

accounted for satisfactorily only by the longest period, 33¼ years. When ancient records were examined it was found that the date of return of the shower had gradually changed, with an alteration of the orbit, the meteors at each return crossing a point in the Earth's path about half a degree further on in the direction of the Earth's movement. If the orbit was the largest of the five this shift would be due to the action of Uranus, Saturn, Jupiter and the Earth, while if it was one of the smaller four the planets responsible would be Jupiter, the Earth and Venus. Adams found that with the smaller ellipses it was impossible to obtain a displacement of half the amount observed, but that with the largest, Jupiter would account for two-thirds of the change and Saturn for most of the remainder with a small part due to Uranus. And Sir John Herschel pointed out ("Outlines of Astronomy," p. 716) that if the orbits were of the short period of nearly a year the meteors must have been so much encountered by the Earth as to have been completely scattered into orbits of all degrees of inclination and eccentricity. The demonstration was therefore clear that the largest of the five orbits was the correct one.

In 1866 G. Schiaparelli (1835-1910) computed the orbit of the August meteors (see p. 145) and found it was the same as that of a comet observed in 1862 ;* and in 1867 he found that the revised orbit of the Leonid meteors as calculated by Le Verrier was about identical with that of a comet seen in 1866. Biela's Comet (see p. 143) has also been found to be connected with a periodic shower of meteors with its radiant in Andromeda ; and there are several other cases of similar association between periodic comets and meteors, the comets concerned being Halley's, Encke's, Winnecke's and that known as Giacobini-Zinner.

ORIGIN OF MOON

The mathematician G. H. Darwin (1845-1912), son of Charles Darwin, found by elaborate calculations, published in 1879-81, that the Moon had probably originated by fission

*In 1948 the French astronomer Guigay showed that, along with the comet of 1862 and other comets seen in 1825, 1826, 1877 and 1932, this Perseid shower probably forms the remains of a disrupted large comet. (See p. 181).

from the Earth at a time when the rotation was performed in from two to four hours. The length of the day (and month) had been of this short duration before tidal friction had lengthened both the day and the month. Various objections have been advanced to this hypothesis and it does not appear likely that it gives a correct account of the origin and evolution of the Earth-Moon system. Darwin himself wrote of his hypothesis, "It may be that science will have to reject the theory in its full extent but it seems improbable that the ultimate verdict will be adverse to the preponderating influence of the tide on the evolution of our planet."[2] This appears to be rather more, however, than present opinion would concede, but the influence of the tides on the Moon, particularly on its rotation period, is undeniable.

LUNAR WORK

In continuation of the account of the study of the Moon given in Chapter XI, the work of Julius Schmidt (1825-1884), director of the Athens Observatory, has next to be described. In 1878 he published a map, based on Lorhmann's earlier results (see p. 137), 75 inches in diameter, the result of more than thirty years observations and over 3000 drawings. On such a scale (twice that of Lohrmann's or Beer and Mädler's maps) there could be very much more detail, about 33,000 craters of all sizes being shown on it. In 1866 he published a catalogue of 435 rills or (a better term) clefts; 278 were new discoveries by himself and nearly 1000 of these features were known to him ultimately.

Schmidt did not hold the view that there is no possibility of observing any change on the Moon; and he produced some sensation when in 1866 he issued a statement that a small crater, Linné, formerly seen by Lohrmann and Mädler thirty or so years before as deep and five or six miles in diameter, had disappeared, a white spot with a small pit only about two miles wide in it, having appeared instead. The reality of the change now seems somewhat doubtful, although an examination of the region in 1891-2 by W. H. Pickering (1858-1938) using a 13-inch refractor led to his belief that lunar volcanic action may not yet be completely extinct. The formation

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of a new crater in another area was reported in 1878, by H. J. Klein (1844-1914) of Cologne, but here the occurrence also seems doubtful, particularly as the region is a complex one where changes of shadow are very considerable.

Lunar photography had not sufficiently advanced to settle questions such as these ; but it began to be of great importance in selenography twelve years later in 1890 when photographs were taken with the large refractor at the Lick Observatory. And in 1894 similar activities were undertaken at the Paris Observatory using a Coudé refractor* of 23½ inches aperture. Reproductions of the photographs taken with these two instruments were published, the Lick series as studies by L. Weinek (1848-1913) of Prague, and those taken at Paris in an atlas on a scale of about eight feet to the Moon's diameter.

Reverting to the visual observational work of the period, mention should be made of volumes containing illustrations and maps issued by J. Nasmyth (1808-1890), the Nasmyth of the steam hammer, and J. Carpenter (1840-1899) in 1874, and by E. Neison (1851-1940) in 1876, both published before the large map of Schmidt mentioned earlier. And in 1864 a Lunar Committee was set up by the British Association for the Advancement of Science, which produced some valuable work by amateurs directed to the preparation of a lunar map 200 inches in diameter. Owing to the death of the leading selenographer concerned, W. R. Birt (1804-1881), this map was never completed. A Selenographical Society was founded in 1877 by Birt, Neison and others but existed for only a few years.

In 1895 T. Gwyn Elger (1838-1897), a well-known English amateur, published a book "The Moon" with a small but accurate map. Elger did not hold the somewhat extreme view of Beer and Mädler that the Moon is unchanging, and he rather inclined to the belief that there is some change. His views are given in the following extracts from his book : "The knowledge we possess even of the larger and more prominent objects, is far too slight to justify us in maintaining that

*A Coudé or "elbowed" telescope is one in which, by use of a diagonal mirror or mirrors, the rays of light from the object glass are bent into a tube containing the eyepiece, the observer looking down it in a fixed direction.

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changes which on Earth we should use a strong adjective to describe have not taken place with some of them in recent years," and "it has been attempted to account for some of these phenomena [changes in tints of the plain areas named 'Seas'] by supposing the existence of some kind of vegetation, but, as this involves the presence of an atmosphere, the idea hardly finds favour at the present time, through perhaps the possibility of plant growth in the low-lying districts where a gaseous medium may prevail is not altogether so chimerical a notion as to be unworthy of consideration."[3]

The Moon's possession of any permanent atmosphere even of slight extent seems very unlikely. Sir John Herschel, for example, concluded, from the small difference between the measured diameter of the Moon and that derived from occultations of stars, that there is no atmosphere at the Moon's edge as much as 1/1980th of the density of the Earth's atmosphere. And this conclusion was supported by a spectroscopic observation in 1865 by Sir W. Huggins (1824-1910) of the disappearance of the spectrum of a fourth magnitude star when being occulted. This occurred as if all wave-lengths of light were simultaneously equally affected, not a likely event if the light had passed tangentially through a lunar atmosphere. In 1897 Professor Comstock of Washburn Observatory, U.S.A., made a systematic study with a 16-inch refractor of occultations of stars, and deduced that there could be no lunar atmosphere more than 1/5000th the Earth's. The conclusions of G. Johnstone Stoney (1826-1911), published in 1867, that the Moon and other comparatively small bodies cannot permanently retain an atmosphere since the gas molecules being in rapid thermal motion would escape their gravitational attraction, seems to be decisive against any permanent lunar atmosphere.

On the other hand there is one circumstance, pointed out by A. C. Ranyard (1845-1894), which would appear to favour the existence of some atmosphere however tenuous. This is the unworn appearance of Lunar features suggesting protection from meteoric bombardment.* It should be borne in mind

*It is interesting to note the different view taken many years later by C. P. Olivier, the American authority on meteors, as to the result of meteor falls on the Moon. He remarked: "The general effect of

that a Lunar atmosphere of small total quantity, and very great tenuity, would have a density, at heights above the Moon's surface similar to those where terrestrial meteors are consumed, not unlike the density of the Earth's atmosphere there, this being due to the much smaller gravitational attraction of the Moon on any atmosphere it might have. Perhaps a very thin atmosphere, constantly being dissipated and renewed by quiet volcanic gaseous eruptions, may be present in fluctuating amount which might permit of some sort of vegetative growth as surmised by T. G. Elger.

Experiments on the radiation of heat from the Moon were first carried out by Melloni as far back as 1846, when sensible effects were noticed. In 1869-72, the Earl of Rosse measured Lunar radiation by means of his 3 feet aperture reflector, and found that the results indicated an actual heating of the Lunar surface, which he made to be nearly equal to 100 degrees Centigrade. In 1887 S. P. Langley (1834-1906), however, deduced a very much lower temperature from his experiments, in fact that of freezing water. F. W. Very's results, published in 1898, were that the Moon's surface under vertical solar radiation, is hotter than boiling water, while its temperature sinks during the 14-day Lunar night to about that of liquid air. These results by Rosse and Very have been shown by more refined modern methods, to be generally accurate (see p. 206).

SOLAR WORK

It has been stated in the last chapter that the periodic variation of terrestrial magnetic force was first noted in 1851 by J. Lamont (1805-1879), of Munich, a Scotsman naturalized as a German. He thought a period of about 10½ years fitted the observations of the varying daily range of magnetic declination. In 1852, E. Sabine (1788-1883), R. Wolf (1816-1893), and A. Gautier (1793-1881) drew attention, independently, to the similarity between sunspot variations and all features of terrestrial magnetic disturbances, including Aurorae, these activities being greatest when sunspots were most frequent, and *vice versa*.

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all this, be it great or small in amount, must be somewhat to smooth the lunar surface. Had it not acted at all, that surface would be even rougher than it is now."

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It is to the exhaustive researches of R. Wolf in 1852 among records in books, journals and proceedings of societies dating back as far as 1610, that the average sunspot period of 11·1 years (see p. 139) is due. His material was made up of casual observations of important spots and groups, and systematic surveys by experienced and by inexperienced observers using large and small telescopes, and he reduced all these to one footing by forming a representative number for each month in a formula which allowed arbitrarily for all these factors. The essential reliability of the numbers thus found is shown by their fairly close relation to the areas of sunspots measured on photographs and drawings from 1832 (Schwabe) onwards, a Wolf number being on the average about a twelfth of the area of sunspots in millionths of the Sun's disc.

A remarkable solar phenomenon was observed on September 1, 1859, by R. C. Carrington (1826-1875). This was an outburst of two patches of intense light, inside the space covered by a group of spots, which lasted five minutes. At the date of its occurrence there was a great disturbance of the Earth's magnetic forces accompanied by Auroral displays, and at the exact time, the magnetic storm was observed at Kew to become intensified, Carrington's observation was confirmed by Hodgson, and according to Brodie was repeated in a less intensified form on October 1, 1864. On July 15th, 1892, Hale and others observed a similar outburst.

As regards observation of the spots, in 1859 Carrington showed by timing the returns of spots situated at different latitudes, that the Sun's period of rotation varies from about 25 days at the solar equator to $27\frac{1}{2}$ days at about 50° north and South latitude. He also found that spots were seldom seen near the Sun's equator, more frequently in the adjoining zones to 35° latitudes north or south, and more sparsely again in the higher latitudes. The work of F. W. G. Spörer (1822-1895) followed close on Carrington's investigations, and its principal result is known as Spörer's Law which may be thus stated: At the beginning of a cycle the spots appear mostly in higher latitudes, but as the cycle goes on they tend to appear at lower and lower latitudes until a position near the equator is reached; the actual sunspot cycles overlap, however, the new cycle beginning before the expiring one has completed its course.

Spörer also showed that although there is a balance of numbers of spots between the north and south hemispheres of the Sun, in the long run, yet in three periods of the Sun's history the southern spots had predominated; and he drew attention to the fact that during the seventeenth century there was a suspension of the law of the spot cycle and the law of their appearance in latitude, for about 70 years, a time when the number of spots was abnormally small.

Wilson's theory (see p. 124) that spots are depressions seemed to be confirmed by examination of a series of photographs taken at Kew Observatory from 1858 to 1872. On the other hand, F. Howlett (1820-1907), a student of the Sun's surface for 35 years, who made several thousand drawings of spots, published his opinion in 1894 that the evidence of his observations had been against Wilson's theory. The controversy on this point died away as later study of solar phenomena seemed to indicate that the point is perhaps of little significance.

In 1873 a department was opened at Greenwich for the study of the Sun's surface day after day by photography; and not long afterwards arrangements were made to have photographs taken at other observatories connected with Greenwich, one at the Cape of Good Hope, and two in India, a nearly continuous record of the Sun's surface on photographs eight inches to the solar diameter being thus practicable. Analysis of the records of sunspots and other phenomena such as faculae (bright streaks or patches on the Sun's surface), by E. W. Maunder (1851-1928) and others, provided information of the greatest value to the science of solar physics.

Early observations of the granulated appearance of the Sun's surface have been referred to (see p. 140). In 1861 Nasmyth's announcement that the granulated appearance resulted from the interlacing of willow-leaf shaped forms was made a subject of keen discussion. Thirty years later P. J. C. Janssen (1824-1907), Director of the Meudon Observatory, France, using a 5-inch refractor with an enlarging lens giving an image of the Sun twenty inches in diameter, noted what he described as the "réseau photosphérique," or photospheric network, an appearance as of smudged or fogged areas among the granules. It was not clear as to whether this blurring is

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caused by currents in the Sun, in the Earth's atmosphere, or perhaps even in the tube of the telescope itself.

But the most important development of Solar research was by means of the spectroscope. This was first applied to the study of the dark lines in the general solar spectrum, and the presence in the Sun of many elements additional to those found by Kirchhoff was demonstrated. However, at the 1868 eclipse of the Sun, spectra of the "Prominences" (see p. 138), which were seen to be projecting parts of a continuous solar envelope named the "Chromosphere," were observed and found to consist of bright lines indicating a gaseous constitution. Soon after this eclipse, Janssen, and Sir J. Norman Lockyer (1836-1920), independently devised a method of seeing these bright lines in full daylight; and following on this Sir W. Huggins and J. K. F. Zöllner (1834-1882) showed that if the slit of the spectroscope is widened the prominences themselves could be seen without any eclipse. An increasing number of bright prominence lines were noted at eclipses, nine visually in 1868, twenty-nine photographically in 1878. Of these a bright one near the D line of sodium was afterwards identified as due to a gas, helium, discovered terrestrially by Ramsay in 1895.

The spectrum of the Corona (see p. 138) was seen in 1868 as a faintly continuous one, and in 1869 by C. A. Young (1834-1908) as continuous, with a bright green line from an unknown source which was given the name "coronium". (It is really due to highly ionised iron, see p. 218). In 1893 the spectrum was noted to be distinct from that of the chromosphere or the prominences, and to have at least seven other bright lines of wavelength shorter than that of the line in the green. Before the end of the century, study by A. C. Ranyard and others of the forms of the Corona as seen or photographed at different eclipses showed that it varied in shape with the sunspot cycle, extending out in all directions at sunspot maximum, but equatorial extensions predominating at minimum. It was also found that the prominences followed the same cycle of change. Among the workers specially active in these investigations were the Italians, Secchi, Respighi, Tacchini and Ricco.

At the solar eclipse of December 22, 1870, the outstanding

event was the spectroscopic discovery by Young of the "Reversing Layer." To give his own words : "As the crescent grew narrower, I noticed a fading out, so to speak, of all the dark lines in the field of view, but was not at all prepared for the beautiful phenomenon which presented itself when the Moon finally covered the whole photosphere. Then the whole field was at once filled with brilliant lines, which suddenly flashed into brightness and then gradually faded away until, in less than two seconds, nothing remained but the lines I had been watching."[4] The impression produced was a complete reversal of the dark Fraunhofer lines into the bright lines of the gaseous absorbing stratum at the Sun's surface which causes the absorption dark lines of the solar spectrum.

As regards the general theory of the Sun's constitution, the ideas of the French astronomer H. A. E. Faye (1814-1902) may be taken as representing advanced opinion of the time. Summarized they were : that the Sun is mainly a gaseous body, that the radiation is due to transport of heat upwards by convection currents with cooler matter descending, that the photosphere is a surface of condensation at the outer limits and that sunspots are breaks in photospheric clouds. Only the first of these ideas can be said to have survived later research.

THE SOLAR PARALLAX

During the period under review the distance of the Sun was measured by various methods. Transits of Venus in 1874 and 1882 were observed at widely scattered stations on the Earth, and from the results values were obtained which were, however, disappointingly diverse. The difficulty of consistently and accurately judging times of contact of the planet's edges with the limbs of the Sun had prevented any very reliable results. Hence the variety of the distances derived was considerable. For the 1874 transit they ranged from about 90 to 93½ million miles, and for that of 1882, from 91¾ to 92½ million.

But other methods were being exploited. In one of these the planet Mars and certain minor planets played a part. The observation of Mars at opposition was employed several times with success. One of these occasions was in 1877 when Sir

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David Gill (1843-1914) observed it at the island of Ascension.

The method employed was first thought of by Flamsteed. It was tried by W. C. Bond (1789-1859) in America and again suggested by Airy in 1857. In order to find the distance of a planet from the Earth it is necessary to observe it from two different terrestrial stations separated by a known space. This could normally be done by observing from two positions as much apart in latitude as possible. Airy's suggestion was that if observations are taken, at an interval of several hours, from the same station, the Earth's rotation would provide a base line of a known size. The apparent shift of the planet in the sky would be found by measuring its angular distances on the sky from fixed stars nearby. This method has been also used with minor planets, which, although usually further from us than Mars, are easier to measure as they have no perceptible disc, looking like a fixed star. Galle (the telescopic discoverer of Neptune) was in 1872 the first to suggest the use of the minor planets in this connection. By means of the distances from the Earth of Mars, and of the particular minor planets utilized, thus derived, the Sun's distance from the Earth can be computed. This is because Kepler's Laws governing the motions of the planets, enable the relative distances from the Sun of the planets to be inferred from the periodic times in which their orbits are described. The Solar System can, as it were, be accurately drawn to plan, and the scale of the plan is determined when any one distance in it has been measured.*

The planet Mars and the minor planets Flora, Juno, Iris, Victoria and Sappho were thus employed, the derived distances ranging from 92 to 93 million miles. The necessary observations were made in the year 1872 (Flora), 1874 (Juno), 1877 (Mars), 1888 (Iris), 1889 (Victoria and Sappho). An active worker in many of these investigations was Sir David Gill who was H.M. Astronomer at the Cape of Good Hope.

Other methods were used of a less direct description: for instance, by careful measurement of the displacement of stars by the Aberration of light (see p. 103) combined with the actual

*By Kepler's Third Law the cubes of the distances from the Sun are proportional to the squares of the periodic times of revolution (see p. 59; also footnote p. 108).

velocity of light found by physical laboratory experiment. The displacement of a star by Aberration depends upon the ratio of the velocity of light to that of the Earth in its orbit round the Sun. The latter velocity can therefore be derived from the two measurements referred to, and the length round the elliptical path and hence mean radius of the Earth's orbit, the quantity required, can be calculated. The distance of the Sun was also obtained from gravitational effects such as that mentioned on p. 158. Another method which, in the long run, may produce as great accuracy as any other, is spectroscopic. At one season of the year, the Earth in its orbital movement is approaching a star, while six months later it is receding from it. Determinations of the line-of-sight velocities for a number of stars can be made and an accurate knowledge of the Earth's orbital speed derived from which the circumference and radius of the orbit may be calculated.

The following table gives in a concise form the values of the Sun's distance current at various times :—

	Distance.	
140 A.D.	4,500,000 miles	Ptolemy
1620 More than	13,500,000 ,,	Kepler
1660	20,000,000 ,,	Hevelius
1700	86,000,000 ,,	Cassini
1700	81,700,000 ,,	Flamsteed
1760	81,700,000 ,,	Lacaille
1820	95,200,000 ,,	Delambre
1835	95,400,000 ,,	Encke
1860	91,600,000 ,,	Hansen, Le Verrier
1900	92,900,000 ,,	International Agreement

REFERENCES.

[1] From "Astronomy," vol. ii, p. 511, Russell, Dugan & Stewart (1927).

[2] From "The Evolution of Satellites," *Annual Report Smithsonian Institution*, 1897.

[3] For a valuable account of reported changes on the Lunar surface, see Presidential Address by S. A. Saunder, *Journal of the British Astronomical Association*, vol. 14, pp. 7-16 (1903).

[4] From *The American Journal of Science and Arts*, 1871.

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CHAPTER XIII

NINETEENTH CENTURY: SECOND HALF
(Continued)

MERCURY

In the field of planetary Astronomy much was achieved in this period. During the half century or more after Schröter little was done in observation, but from his results in 1800, another astronomer, Bessel, inferred a rotation period for the planet Mercury not much different from the Earth's. Little reliance was placed on this value although many years later, in 1882, W. F. Denning (1848-1931) found that it accorded with several days' observations. In that year, systematic study of Mercury was undertaken by Schiaparelli at Milan, the scrutiny of the planet being in daylight, as evening or morning observations had generally to be at low altitudes in the sky, owing to the small angular distances from the Sun, with consequent great atmospheric disturbances. Schiaparelli kept continuously observing, hour by hour; and he found faint markings that did not change their positions on the disc, indicating a much longer period than Schröter's. Finally he announced in 1889 that Mercury rotates in a period of 88 days, which is equal to its period of orbital revolution; and that it therefore keeps practically the same hemisphere of its surface turned to the Sun. These conclusions were supported by P. Lowell (1855-1916) at Flagstaff, Arizona, in 1896. To both observers Mercury appeared to have little or no atmosphere.

VENUS

In 1788 and 1811 Schröter had found a short rotation period of 23½ hours for Venus; and in 1841 F. de Vico (1805-1848) thought he had confirmed this. It will be recollected that W. Herschel had been unable to see any of the markings on which Schröter had based his result (see p. 126). From 1877 to 1890, however, Schiaparelli studied this planet, in the daytime as with Mercury, and concluded that he had found

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a rotation period of 225 days, equal to its period of revolution round the Sun. Support to this result was given by several observers, including Lowell. An attempt to settle the matter with the spectroscope made in 1900 by Bélopolsky at Pulkowa, apparently gave displacements of the spectral lines which agreed with the short period; but the results were not more than experimental, with apparatus not as yet competent to deal with such a question, as even with the rapid period the line displacements would be only those corresponding to a velocity of about half a mile per second.

Many other observers of the planet at this time thought that they found evidence of the more rapid rotation period. But the markings on the disc of Venus, which is very bright naturally, are of a vague and changeable kind, suggesting a cloud-laden atmosphere. The presence of an atmosphere was also indicated by the observed prolongation of each of the points of its image when of a thin crescent shape. This had been noted by several, including Mädler with the 9·6-inch refractor at Dorpat in 1849, who had seen them extended for thirty degrees beyond the semicircle; and C. S. Lyman in 1866 and H. N. Russell in 1898, had observed these carried right round the complete disc, Venus being then only a degree or two from the centre of the Sun. The first to note similar evidence of an atmosphere seems to have been D. Rittenhouse of Philadelphia at the entry of the planet on to the Sun during the transit of 1769, that part of the edge of the planet off the Solar disc appearing illuminated so that the whole outline of the planet was visible.

The reported visibility to several observers of the dark part of Venus (similar in appearance to the "Earthshine" on the Moon), during daytime or in bright twilight, and seldom if ever at night, appears to have been the result of some optical illusion, as such an appearance should be much more easily seen after dark when it would be less liable to effacement by the light of the sky. And to the same cause, or to instrumental defects, we may attribute the supposed observations of a satellite showing phases similar to those of the planet, by Cassini in 1672 and 1686, Short in 1740, and several others between 1760 and 1764.

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MARS

The essential permanence of the dark markings seen on the disc of Mars was established by Beer and Mädler by their observations from 1830 to 1839. Before that time Schröter and others (but not Herschel) had considered them to be changeable and cloud-like. In 1862, J. Norman Lockyer confirmed Beer and Mädler's conclusions; and from observations of that year, several drawings of W. Herschel, and one by Huyghens in 1672, F. Kaiser (1808-1872) derived a rotation period of 24h. 37m. 22·62s., about two minutes shorter than that found by Herschel (see p. 126), and only a twenty-fifth of a second greater than the most recent determination. The seasonal fluctuation of the White Polar Caps, discovered by Herschel, was confirmed; and the opinion gained ground that Mars is nearest of all planets to the Earth in physical constitution.

At the favourable opposition in 1877, Schiaparelli with an $8\frac{3}{4}$-inch refractor at Milan, began an intensive study of the planet, which extended over about thirteen years. In the course of the first year of his work he found that the brighter so-called continental areas, were crossed by dark linear markings to which he gave the Italian name of "canali," or channels, rather unfortunately translated into English as canals. These had evidently been seen by Beer and Mädler, Dawes and others as regards the most prominent of them; but they had then only been looked upon as straits. In 1879 Schiaparelli found that one was seen as a double marking and at the opposition of 1882 he noted the doubling of several more. These discoveries met with considerable scepticism; but other observers confirmed them, at Nice using a 30-inch refractor, and at Lick with the 36-inch; and users of smaller telescopes saw many of the canals. In 1892, W. H. Pickering, and in 1894 A. E. Douglass, found that they were not confined to the areas termed "continents," some crossing the darker parts which had been thought to be probably seas, a phenomenon also noted in the case of several of the continental canals by Perrotin of Nice in 1888.

In the year 1894 Percival Lowell founded the observatory at Flagstaff, Arizona, with a principal equipment at first of

an 18-inch refractor, and later a 24-inch refractor and a 40-inch reflector, the particular intention being planetary observation with Mars as the chief subject. Lowell and his colleagues considered that their observations showed narrow canals in great number. All of the "seas" were crossed by canals and they therefore could not really be bodies of water. In fact, these darker areas, according to the Lowell observers, underwent seasonal changes that suggested vegetation.

There did not appear to be any reasonable doubt that the "canal" markings had an objective origin, although some users of very large telescopes considered that they are what are seen, at or near the limit of vision, of something which is really much more intricate in detail. But the fascinating writings of Lowell appear to a reader of the present day to be rather in the nature of special pleading for the existence of features combined to make for the planet the details of a huge irrigation arrangement which is the product of intelligence; and there is a suspicion of some bias influencing the observations. It should be noted, however, that the small scale photographs, taken at Flagstaff, Lick and elsewhere, certainly show one or two of the larger canals, exactly where they are observed visually, as markings of a linear nature.

Evidence of cloud formations, probably mist or dust, was noted in 1873, 1888, 1890 and 1896 by different observers. But the general view was that the Martian atmosphere must be thinner than that of the Earth. Spectroscopic observations by W. Huggins in 1867, and H. C. Vogel (1842-1907) in 1873, seemed to show the presence of water vapour; but W. W. Campbell (1862-1938) in 1894, and Keeler in 1896 at Lick Observatory, did not confirm this.

On August 11, 1877, A. Hall (1829-1907), observing with the 26-inch refractor of the Washington Observatory, found a small satellite to Mars, to which he added a companion on the following night. They were given the names of Deimos (Fear) and Phobos (Panic). Both are very small bodies, probably less than 10 miles in diameter, and they revolve round their primary in $30\frac{1}{3}$ and $7\frac{2}{3}$ hours respectively, the latter therefore going round the planet faster than Mars rotates.

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MINOR PLANETS

The discovery of minor planets, ten of which were known before 1850, proceeded at an accelerating rate. By 1870, 110 had been detected and all but one had been given a satisfactory orbit, a number, and (by the discoverer) a name. By the beginning of 1900, 559 had been found, and 452 given orbits, numbers and names. Photography as a means of detection was introduced in 1891 by Max Wolf (1863-1932) of Heidelberg. At the end of the century he had found more than a hundred, Charlois of Nice about a hundred, Palisa of Vienna more than eighty, and C. H. F. Peters over fifty. Among all the discoveries the object of chief interest and astronomical value was that detected by Witt of Berlin in 1898. This was named Eros. Its mean distance from the Sun is the smallest known for a minor planet, 136 million miles; but it has a very eccentric orbit and it can come within 14 million miles of the Earth, when it has a very large parallax (60″). Unfortunately, one of these favourable oppositions occurred in 1894, several years before discovery; but the value of this small planet for future determination of the scale of the Solar system, and hence the Solar parallax, was at once evident.

In 1866, when the number known was only about 90, and ten years later when nearly twice as many were available for study, D. Kirkwood remarked that their distribution in distance from the Sun contained marked gaps. He suggested that these comparatively unoccupied spaces are just where the periods of revolution round the Sun are connected by a simple ratio with that of Jupiter. Such gaps are found where the period is a third, two-fifths, one-half, and three-fifths of Jupiter's; there the perturbations by that giant planet would be cumulative so that the spaces have been kept clearer than other regions.* The diameters of the four brightest were measured in 1894-5 by E. E. Barnard (1857-1923) with the Lick 36-inch and the Yerkes 40-inch refractors. His results were: Ceres, 477 miles; Pallas, 304 miles; Vesta, 239 miles; and Juno, 120 miles. About the same value for Vesta was

*Kirkwood also suggested a similar explanation for the Cassini division in Saturn's ring system, one of the satellites, Mimas, being chiefly responsible (see p. 159).

found in 1899 by measurements made at Paris with an interferometer. In 1853 Le Verrier showed, from considerations of possible disturbing action, that the total mass of all the minor planets cannot be as much as a fourth of that of the Earth, but the true value is certainly very much less.

JUPITER

Study of Jupiter and his cloud belts led during this period to the hypothesis that it and the other giant planets are in a state between the solar and terrestrial stages of development with hot interiors and very warm surfaces. This was based on the movements and changes in the belts and the fact that these surface markings have a quicker period of rotation near the equator as is found on the Sun. J. D. Cassini and Schröter had both observed this peculiarity. And measurements of the amount of light received from Jupiter seemed to indicate that there was probably a certain amount of emitted light, the results of Zöllner in 1865 and Müller in 1983 giving "albedoes"* nearly equal to that of white paper or of fresh snow. Observers such as Barnard, Keeler, Lowell and Denning, reported changes and relative movements, also colours, in the belts, which all appeared to be consistent with the "semi-sun" hypothesis, strongly advocated by R. A. Proctor (1837-1888) in his writings.

Another striking feature on the planet's disc, which apparently conformed, was the Great Red Spot, perhaps first noted by Schwabe in 1831, although it has been suggested that Robert Hooke and J. D. Cassini may have seen it in the 17th century, and a marking seen in 1676, which Roemer thought might be used in place of the occultations of Jupiter's moons for measurement of the velocity of light (see p. 75) may have been the Red Spot. It has been kept under review by many students of the planet ever since, although sometimes nearly disappearing; and currents of different velocities and rotation periods have been continuously observed. The changeable phenomena of the transits of satellites and their shadows across the disc of Jupiter were also thought by some

*The albedo of a spherical body is the ratio of the total amount of sunlight reflected from the body in all directions, to that which falls on the body.

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to lend support to the hypothesis of great heat. The general view of the structure and origin of the Red Spot was that it is, in the words of Lowell, "a vast uprush of heated vapour from the interior," a vortex form being attributed to it by some. In a later chapter it will be seen that recent research has shown the hypothesis of great heat to be wrong.

The spectroscopic observations of Huggins in 1862-64, and Vogel in 1871-73, showed a solar spectrum with bands in the orange and red, no explanation of these bands being possible to the science of the time. Photography of the planet was undertaken in 1890, 1891, 1892, at Lick Observatory and several years later at Paris; but the results, although promising, had little more than pioneer value. The Red Spot was conspicuously shown, however, and the main belt arrangement, without much detail.

In 1892 Barnard added a fifth satellite to the Galilean four, inside their orbits, with a period of nearly 12 hours, or only about two hours greater than the rotation period of the planet itself. It revolves at only 1·5 times the radius of the planet away from his equatorial surface.

SATURN

A bright spot was seen in the equatorial regions of Saturn by A. Hall of Washington in 1876. From it he found a rotation period of slightly under $10\frac{1}{4}$ hours or about two minutes shorter than had been noted by Herschel from a similar marking in 1794. In 1891 to 1893 several such spots were observed, giving the same or slightly shorter periods, a difference no doubt due to individual motion of the spots similar to what had been seen on Jupiter. And during about the same time, Stanley Williams found a continuously decreasing rotation period of the equatorial regions throughout the years 1891 to 1894 from his own observations of a number of spots.

Contemporary opinion of Saturn's constitution was that it is of a similar high temperature nature to that of Jupiter. The albedo found by Zöllner was not so high as for Jupiter but that measured by Müller was even greater. A spectrum similar to that of its fellow giant planet was found for the ball, with an unaltered solar spectrum for the rings.

In 1898, W. H. Pickering added to the eight found by Huyghens, Cassini, Herschel, and Bond, by the discovery, photographically, of a faint satellite revolving well outside their orbits. This satellite is peculiar in that its motion round Saturn is in a direction opposite to that of the others.

URANUS

Uranus is a difficult object for observation, large telescopes of high quality being necessary to show anything of interest on its small greenish disc of about $3\frac{3}{4}$ seconds of arc diameter—the apparent size of a halfpenny nearly a mile away. In 1883 Young at Princeton, U.S.A., with the 23-inch refractor there, noted dusky bands; and in 1884 observers at Paris with a $23\frac{1}{2}$-inch coudé refractor also saw two bands. Similar results followed examination, in the same year, with the 30-inch refractor at Nice, a bright spot near the equator being seen, indicating a rotation period of about 10 hours, which is not much different from the period found spectrographically nearly thirty years later. But nothing definite was visible with the 36-inch Lick refractor in 1889-90 possibly because of the variable nature of such markings. An elliptic shape to the disc was evident to many users of large instruments.

The spectrum as seen by Huggins, Vogel and Keeler, showed broad bands more conspicuous than with Jupiter and Saturn. Two new satellites were found by Lassell in 1851 with his two feet aperture reflector, making the total number for Uranus four, as only two of the six which W. Herschel thought he had discovered have been verified. The planes of the orbit of the four are inclined at 82 degrees to the plane of the Ecliptic, the movement being in a direction opposite to that usual for planets and satellites.

NEPTUNE

Neptune is of course an even more difficult telescopic object, its disc being not much more than half the size of that of Uranus, and less brightly illuminated by the Sun's light.* It has, like Uranus, a distinctly greenish colour. In 1899,

*The Sun's light or Uranus is 1/360th as intense as on the Earth; on Neptune it is only 1/900th.

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T. J. J. See believed he saw equatorial belts, using the 26-inch Washington refractor; but no definite markings were visible to Barnard with either the Lick 36-inch or the Yerkes 40-inch refractors.

Maxwell Hall, an amateur resident in Jamaica, using a comparatively small instrument, reported that he observed regular fluctuations in Neptune's brightness in 1883-84, suggesting a period of revolution of about 7·9 hours. It is perhaps significant that the period derived spectrographically half-a-century later is almost twice this (see p. 244).

As far as could be judged from the faint spectrum visible to Huggins and others its character was of the same type as that of Uranus. No satellites additional to the single one found by Lassell in 1846 were added even with the increased optical power available.

COMETS

During the second half of the century more than 200 comets were found, eight of them classed as "bright." Some of the most spectacular of these were Donati's of 1858 (specially interesting from the beauty of its curved tails and the brightness of its head), Tebbutt's of 1861 (the Earth passed through its tail on June 30th, 1861, when some observers believed they noticed a diffused glare in the sky), Coggia's Comet of 1874 (with a tail extending more than 45 degrees in the sky) the Great Comet of 1882 (visible in daylight, and seen telescopically to throw off a satellite comet), the comet of 1887 (first seen by a farmer near Cape Town; it had no head or condensation). All of these had striking tails millions of miles long, and orbits of greatly elongated ellipse form, with periods of hundreds of thousands of years, or of parabolic shape. From comparison of recorded descriptions, the three most brilliant comets of the nineteenth century would seem to have been those of 1811, 1843 and 1858. That of 1843 was probably the brightest, that of 1858 the most gracefully shaped.

In 1871 Zöllner revised the theory of Olbers (see p. 144) of the electrical formation of comets' tails, and in 1874 T. Bredechin (1831-1904) of Pulkowa, classified tails into three fairly distinct types according to the strength of the repulsive

force from the Sun, the degree of curvature being inversely proportional to this force. These types were clearly recognised for a large number of comets, and further experience confirmed the substantial accuracy of the classification with the suggestion of even more powerful repulsive forces in some cases.

The first spectroscopic observation of a comet was made in 1864 by G. Donati (1826-1873), the discoverer of the great 1858 Comet. He found three bright bands in the yellow, green and blue; and Huggins and Secchi in 1866 noted that Tempel's Comet gave the three bands and a continuous spectrum also. In later comets, dark lines of the solar spectrum were seen, reflected Solar light as well as bright radiation from a glowing gas or gases, being therefore indicated. In 1868 Huggins identified these three bands in Winnecke's Comet as due to a hydrocarbon. In a few comets the hydrocarbon bands were not present but the majority had them. For instance a comet discovered in 1892 by E. Holmes, a London amateur, had a faint continuous spectrum only, also Brorsen's Comet in 1868, and another comet in 1877. In 1882 the D line of sodium was seen in the spectrum of Wells's Comet, and later it was also visible in the spectrum of the Great Comet of that year; in both cases when the comet was near the Sun. This led to the discovery that, for comets which go close to the Sun, the order of spectrum is, first a hydrocarbon one, then at closer approach sodium lines appear; finally a number of other bright metallic lines including iron are shown, a continuous reflected solar spectrum being always present as well.

The practice of "Comet-hunting" or "Comet-seeking" of which Pons, Messier and others were notable exponents, died away after the first quarter of the nineteenth century, but was revived again about 1880, when Brooks, Barnard, Perrine and Swift, became successful searchers.

The first comet to be photographed deliberately was one in 1881, discovered by Tebbutt, the photographs being taken by Janssen and by Draper, and one was photographed, accidentally as it were, in the vicinity of the totally eclipsed Sun in 1882. In the same year the Great Comet was photographed at the Cape Observatory on a background crowded with stars, thus showing the possibilities of stellar photography.

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But the first to be discovered by photography was one by Barnard in 1892.

In 1891, F. F. Tisserand (1845-1896), Callendreau, and H. Newton, independently published the suggestion that certain comets had been captured into the Solar system, the gravitational pull of the larger planets Jupiter, Neptune, Uranus, and Saturn having changed parabolic orbits into elliptical ones. Some time before that date R. A. Proctor advocated the hypothesis that the particular comets had been ejected from the large planets. But later consideration has thrown some doubt on the reality of the existence of these comet families except in the case of Jupiter's family.*

A different kind of comet grouping has been drawn attention to, notably by Hoek of Utrecht in 1865. The most noteworthy group of the sort is made up of the long-period comets of 1668, 1843, 1880, 1882 and 1887, all of which move in greatly elongated elliptical orbits, of similar shape and disposition in space, that bring them very near to the Sun at perihelion. Member of such a group may perhaps be regarded as fragments of a large earlier comet which, like Biela's Comet (see p. 143), has broken into parts that have separated out along the original orbital path. A marked tendency to disintegration into parts observed in the comet of 1882 seems to lend some support to the hypothesis for the group of which it is believed to be a member.

METEORS

The connection of certain meteor showers with periodic comets has been mentioned earlier (see p. 160). In the case of the Leonids the next great shower was expected for 1899. But Johnstone Stoney and A. M. Downing (1850-1919) showed that perturbations of the main swarm of meteors by Jupiter and Saturn would so interfere with their orbit as probably to cause the richest part to miss the Earth, passing about two millions of miles from it. As a consequence, there was no great display in 1899 or 1900, much to the public disappoint-

*Recently, however, C. H. Schuette has grouped 52 for Jupiter's "family," 6 for Saturn's, 3 for Uranus's, 8 for Neptune's, 5 for Pluto's, and has found a group suggesting a transplutonian planet about half again as far from the Sun as Pluto. (*Popular Astronomy*, vol. 57, p. 176, 1949).

ment although the warning had been given; only stragglers were seen. A somewhat better display occurred in 1901, visible in America, probably from a part of the swarm well away from the rich main body which had provided the great showers of 1799, 1833 and 1866.

The existence of many swarms of meteors, although not identified with particular comets, was being made evident by the discovery of the great number of radiant points, or points of the sky from which meteors appear to radiate (see p. 145) on a particular night (or series of a few nights) of the year. E. Heis (1806-1877) of Munich, published in 1867 a catalogue giving the positions in the sky and dates of activity of 84 such points, and Schiaparelli issued another in 1871 with 189. The greatest observer of meteors of the time was, however, the English amateur, W. F. Denning, whose work extended into the first third of the next century. He claimed to have discovered "stationary" radiants, or radiants that did not appear to alter their position in the sky from night to night, as should be the case owing to the change of direction of the Earth's motion in its orbit except with meteors of apparently prohibitively high velocity.*

The reality of stationary radiants has been the subject of much controversy and various attempts at explanation. Recent expert opinion appears to be that they have no real existence as a general feature of meteor radiation, such as Denning claimed for them, although one may appear to exist in a restricted region of the sky for special reasons.

A well-marked shift of the radiant point in accordance with the Earth's changing direction of movement was observed for the August Perseid Shower (see p. 145). Le Verrier had pointed out that this should be the case; and it was noted that the radiant was not the same point source from night to night but appeared to have an elongated shape extending from Perseus into Cassiopeia. In 1877 Denning made a series of observations between July 19 and August 10 and found that the radiant moved eastward at about the rate acquired by the Earth's motions; and although the observed night to night shift seemed at first to be too great, mathematical

*It was estimated that a speed of 880 miles per second would be necessary to produce the stationary effect.

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analysis, making allowances for the Earth's attraction on the meteors, showed that all the observed radiants for the different nights belong to a compact group. This shift of the Perseid radiant was confirmed in later years with greater precision, and, in addition, he later detected a similar movement in the radiants of two other showers (the Lyrids and the Geminids). Two results of promise for the future were : the photographing in 1897 of the spectrum of a meteor at the Harvard Observatory Station in Arequipa, and the determination in 1898 of the path and the orbit of a Leonid meteor, from photographs at Yale College, by Elkin.

STAR CATALOGUES

In addition to the "Fundamenta Astronomiae" based on Bradley's work (see p. 104), Bessel brought up the number of catalogued stars to more than 50,000 by a series of over 75,000 observations. His assistant and successor, W. H. Argelander (1799-1875), published in 1857-63 the "Bonn Durchmusterung," a catalogue, with an accompanying atlas, of 324,198 stars in the northern sky, which was extended (1875-85) to cover part of the southern hemisphere of the sky by a catalogue of 133,659 additional stars by E. Schönfeld (1828-1891). Photographic observation of the rest of the southern sky made under the direction of D. Gill at the Cape of Good Hope, completed the entire heavens nearly to the tenth magnitude, with positions reliable to about a second of arc, the work of preparing the catalogues based on the observations being carried out, and finished in 1900, by J. C. Kepteyn (1851-1922).

Arising from this southern photographic work a great scheme for a photographic survey of the entire sky by a chart and a catalogue was decided upon at an international congress held in Paris in 1887. The work was to be shared between 18 observatories, from Helsingfors in the northern hemisphere to Melbourne, Australia. The chart was to include stars down to the 14th magnitude and the catalogue down to 11th magnitude ; all instruments to be the same, 13 inch aperture photographic refractors of 11-feet focal length. Each plate covered four squares degree of the sky (about 18 times the Moon's area) and as there had to be duplication to check any

possible error, 22,000 of them were required. By the end of the century a part of the work, which was, however, only effectively started in 1892, had been completed.

STELLAR PHOTOMETRY

The systematic measurements of stellar brightness, initiated by the Herschels (see p. 122 and p. 146) was continued in 1879 by B. A. Gould (1824-1896) with his "Uranometria Angentina" giving estimated magnitudes of 8000 stars visible from Cordoba, Argentina. In 1884, E. C. Pickering (1846-1919) of Harvard College observatory, published a catalogue of 4260 stars constructed from nearly 95,000 observations made with a "meridian photometer"* in 1879-82, and in 1885 C. Pritchard (1808-1893) of Oxford University, issued his "Uranometria Nova Oxoniensis" with magnitudes, made by a "wedge photometer,"† of all naked eye stars from the north pole to ten degrees south of the celestial equator, to the number of 2784. The agreement between the two catalogues was fairly good. In 1889-91 the Harvard photometry was extended to the south pole of the sky, and in 1891-98 it was enlarged to cover all stars brighter than 7·5 magnitude from the north pole to 40 degrees below the celestial equator. Another catalogue, the "Potsdamer Photometrische Durchmusterung," issued in parts from 1894 during the subsequent thirteen years, gives the magnitudes of more than 14,000 stars from the north pole to the celestial equator, down to 7·5 magnitude.

In these catalogues the ratio between the brightness of successive magnitudes was taken as 2·512 to 1, the so-called

*This instrument consists of a pair of fixed horizontal telescopes each with a movable mirror before the object-glass. One directs the light of the Pole Star to the eye of the observer, the other the light of the star to be measured. By means of a polarizing device the light of the brighter star is dimmed until it is apparently equal to the other. A magnitude of 2·1 is assumed for the Pole Star, and the light absorbed by the polarizing system being known, the magnitude of the star to be measured can be obtained.

†In this instrument an "artificial star" is formed by a suitable combination of lenses and diaphragms, and brought, by reflection, into the field of view of the telescope close to the image of the star to be measured. By means of a graduated wedge of neutral tinted glass moved across the artificial star's image until equality of the two is observed, the desired value of magnitude is found.

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"Pogson's Ratio," proposed in 1856 by N. Pogson (1829-1891), and based on J. Herschel's observation in 1830 that an average first-magnitude star is 100 times brighter than one of the sixth magnitude. Before the general adoption of this ratio the relative brightness of the stars was very vaguely defined; Herschel, Struve, and Argelander used scales which differed widely from each other.

STELLAR PARALLAX

The measurements of parallax by Bessel, Henderson, Struve and others during the first part of the century, were not soon followed by many results of high quality. In 1840 the three values referred to earlier were known (for 61 Cygni, α Centauri and Vega). By 1880 this had increased only to about 20, and in 1900 to about 60, the most reliable being those found with the heliometer, although photography was introduced in 1886 with promising results by C. Pritchard at Oxford.

STELLAR SPECTROSCOPIC WORK

Near the middle of the century, spectroscopy began to be prominent in stellar Astronomy. The first to apply the modern spectroscope to the stars, apart from Fraunhofer's early work (see p. 151) was the same astronomer as had done so far for comets, G. Donati (1826-1873), Florence. With his instrumented equipment he was unable to do more than note the positions of a few of the prominent lines. But the real founders of stellar spectroscopy were A. Secchi (1818-1878) and W. Huggins. Secchi's work was of the nature of a general survey for classification purposes; Huggins's was more a detailed study of particular spectra. Secchi's classification into four types, in 1863-7, was long of very great utility, until superseded by one originating at Harvard. It was applied by him to more than 4000 stars of a catalogue. The first type included the bluish-white or white stars like Sirius and Vega and had prominent lines of hydrogen; the second, yellowish stars like the Sun, Capella, Arcturus, showed many lines of metals; the third or red stars like Antares, Betelgeuse, had spectra with strong dark bands; and the fourth was a less numerous class of red stars with a different kind of absorption

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bands. A fifth class showing bright lines or bands, as well as dark lines on a continuous spectrum, was added in 1867 by Wolf and Rayet of Paris—the "Wolf-Rayet" stars.

SPECTRA OF NEBULAE

In 1864 Huggins first observed the spectrum of a diffuse nebula which he noted consisted of several bright lines; and in the next four years he had examined the spectra of 70 nebulae finding bright lines for more than twenty, the most notable being the Orion nebula. This showed the gaseous nature of a considerable proportion of nebulae. Initially only hydrogen could be identified as a constituent, one of the three lines first seen being due to that gas. A large proportion of the nebulae gave a continuous spectrum indicating that they were probably unresolved star clusters.

THEORIES OF STELLAR DEVELOPMENT

Lines of known elements such as hydrogen, iron, sodium, calcium, were first identified in stellar spectra by Huggins in 1864, to be followed by numerous other identifications by spectroscopists. In the same year Huggins advocated the principle that the colour of a star depends on the absorption of its light by its surrounding atmospheric vapours. This idea obtained favour against the more correct one that colour depends chiefly on temperature which was advanced by Zöllner in 1865, who suggested that yellow and red stars are progressive stages of cooling. In 1874, H. C. Vogel (1842-1907) adopted this suggestion, whereby Vega or Sirius was thought to be the youngest type of stars; yellow and red stars like Capella or Betelgeuse were older and cooler. His system of classification resembled Secchi's in a general way, and in 1885 he revised it to include a newly-recognized type of bluish white stars called the "helium" stars from the presence of lines of this element in their spectra.

In 1887 J. N. Lockyer (1823-1920) advanced his "Meteoritic Hypothesis" the basic proposition in which is that "all self-luminous bodies in the celestial spaces are composed either of swarms of meteorites or of masses of meteoric vapour produced by heat." The common origin thus suggested was supposed

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to result in a progressive evolutionary course whereby nebulae and stars were disposed along a temperature curve, rising from nebulae and bright-line stars through red stars of Secchi's third type, then second type yellow stars, to the Sirian first type. From this stage the descent in temperature was through second type (solar) stars to red stars of the fourth type, and then to extinction. But later spectroscopic work showed that certain supposed coincidences of spectral lines, on which the meteoritic part of the hypothesis rested, were not in fact exact enough.

THE DRAPER CATALOGUE AND CLASSIFICATION

In 1890 the "Draper Catalogue" of stellar spectra was published by Harvard College Observatory. For this catalogue the spectra were obtained "in bulk" by use of a prismatic camera" which consists of a photographic telescope with a large prism of a rather small angle, an "objective prism," in front of its object glass. The spectra appear as narrow streaks on a plate placed in the focal plane of the telescope. It included particulars and classification for more than 19,000 stars down to about the eighth magnitude. The classification scheme, which was destined with modifications to take the place of all others, was into types designated by letters of the alphabet. The chief divisions used are O, B, A, F, G, K, M, N. Class O are the Wolf-Rayet stars, B and A about correspond to Secchi's first type, G and K are broadly his second type, M his third type, and N the fourth.

STELLAR RADIAL VELOCITIES

In the year 1868 Huggins applied spectrum analysis to the problem of the motions of the stars. This was by measuring the position of the lines to find if there was any displacement which could be attributed to motion by Doppler's principle (see p. 153). He used the brightest star, Sirius, for the purpose and found distinct evidence of a motion of recession which was verified by Vogel. In the following year Huggins determined approximately the line-of-sight movements of 30 stars, and it may be said that the study of stellar radial velocities dates from these investigations. On radial or (which is perhaps a

better name) line-of-sight velocities, plus proper motions and a knowledge of distances to convert these proper motions into actual velocities, depends the solution of the speed and direction of movement in space of any heavenly body extraneous to our Solar system. Photography was later applied to line-of-sight velocity determination with greatly increased accuracy as visual observation and measurement could not generally be so accurate with the difficulties to be overcome. The first to use photography for the purpose was Vogel in 1887, and he was followed by others, notably Keeler, Frost and Campbell. The data he thus accumulated enabled Campbell to find from the line-of-sight velocities of 280 stars, values for the speed and direction of the Sun's movement in space (see p. 120); and Keeler in 1890, with the powerful telescope at Lick Observatory, was able to measure visually the velocities in the line of sight of some bright line nebulae, which Huggins had not been able to do in 1874.

PHOTOGRAPHY

The first to photograph a star's spectrum was H. Draper in 1872; and four years later Huggins used the dry plate, for the first time in such work, for the same purpose, on the same star, Vega. The first photographs of the spectrum of a nova seem to have been made in the case of Nova Aurigae 1891. As regards nebulae, Huggins was successful in photographing the bright-line spectrum of the Orion nebula in 1882, and J. Scheiner (1858-1913) in January, 1899, photographed the spectrum of the great Andromeda nebula finding it to be like what would be shown by a cluster of solar stars. Notable events in general astronomical photography were: Orion nebula photographed by Common in 1883; star fields by the Brothers Henry, in 1885; discovery of a nebula in the Pleiades, by the same, also in 1885; the Andromeda nebula, showing some traces of spiral structure, by Roberts 1886; Pleiades nebulosities, by the same, also in 1886; Orion nebula connections covering large part of the constellation, by W. H. Pickering, in 1889; star clouds of the Milky Way, by Barnard, in 1889. The instruments used were: Common, 3 feet aperture reflector; Henry Brothers, 13 inch aperture refractor;

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Roberts, 20 inch aperture reflector; Pickering, a small aperture star camera; Bernard, 6 inch aperture star camera (portrait lens).

DOUBLE STARS

Although certain stars had been seen to be double or multiple, notably ζ Ursae Majoris (one of the Plough), by Riccioli in the seventeenth century, θ Orionis (multiple) by Huyghens in 1656, and γ Arietis in 1664 by Hooke,* before Herschel's first survey of the sky in 1779, that year may be said to mark the real beginning of double star Astronomy. In this same year C. Mayer (1709-1783) published a small book in which he speculated as to the probable existence of binary systems and gave a list of 89 pairs. After Herschel's time and before the end of the nineteenth century the following outstanding workers in the subject may be specially mentioned: F. G. W. Struve (see p. 148), W. R. Dawes, O. Struve, E. Dembowski (1815-1881), S. W. Burnham (1838-1921). W. R. Dawes, an amateur using telescopes ranging from about 4 to 8 inches, made measurements of hundreds of double stars which he summarized and published in 1867. O. Struve published a catalogue of 514 of which a very large proportion were close pairs. Baron E. Dembowski, an amateur observing in Italy, measured pairs for more than 30 years, using 5 and 7-inch refractors. S. W. Burnham, beginning as an amateur in Chicago with a 6-inch refractor, probably used, as a professional, a greater range of large telescopes than any one else (nine instruments at various Observatories up to the 40-inch Yerkes refractor) discovering 1290 new doubles from 1871 to 1899. After his transfer from the Lick Observatory to Yerkes, his successors at the former, W. T. Hussey (1862-1926) and R. G. Aitken, discovered hundreds of new pairs before the end of the century. For the southern skies a catalogue was published in 1899 by R. T. A. Innes (1861-1932) containing 2140 pairs, more than 300 of which he had himself discovered;

*The discoveries and dates of some other well-known pairs are as follows :—

α Crucis, Fontenay, 1685.
α Centauri, Richaud, 1689.
γ Virginis, Bradley, 1718.
Castor, Cassini, 1678.
61 Cygni, Bradley, 1753.
ζ Cancri, Mayer, 1756.
ϵ Lyrae, Maskelyne, 1765.
α Herculis, Maskelyne, 1777.

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and in 1897, T. J. J. See found nearly 500 new southern pairs with the Lowell 24-inch refractor. Many more names might be mentioned of workers whose discoveries and measurements contributed largely to the progress of double star Astronomy during the period.

Two discoveries of much interest should be referred to. Bessel had predicted from the disturbed proper motions of Sirius and of Procyon, that each of these stars would be found to have a faint stellar companion, revolving round them in periods, in both cases, of about half a century. These were telescopically discovered; that to Sirius by A. G. Clark, the famous optician, in 1862, when testing a new 18-inch refractor, and the companion to Procyon in 1896 by Schaeberle using the 36-inch refractor at Lick.

In photography of double stars, as far back as 1857, G. P. Bond of Harvard obtained a collodion plate giving measurable images of ζ Ursae Majoris, a rather wide double. E. C. Pickering afterwards made a few attempts, and F. A. Gould obtained some results between 1870 and 1882 at Cordoba. In 1886 the Brothers Henry at Paris secured successful photographs of several pairs ranging from about $2\frac{1}{3}$ to over 5 seconds separation, and in 1894 and 1897 good results were got at Greenwich with 13-inch and 27-inch refractors giving measurable images down to $1\frac{1}{4}$ seconds apart. But photography, although providing measurements of great accuracy for such pairs as can be recorded and measured in this way, does not seem to be yet applicable for very close systems a second of arc or less apart.

SPECTROSCOPIC BINARIES

A new type of binary star was found in 1889 by E. C. Pickering at Harvard. On plates taken for the Draper Catalogue of Stellar Spectra, it was noted that a prominent line in the spectrum of the brighter component of ζ Ursae Majoris (the first star to be seen visually as a pair) occasionally appeared doubled. This was interpreted to mean that the star has two components revolving round one another; their motions to and from the Earth (line-of-sight velocities) are changing regularly, one star receding while the other is

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approaching and their spectra therefore have lines which shift with reference to one another. The period of revolution was found to be $20\frac{1}{2}$ days, much shorter than the several years of the shortest period visual binary. In 1880 Pickering had shown from the shape of the light curve of the variable Algol that the suggestion of J. Goodricke (1764-1786) as to the cause of its variation was correct, *i.e.*, that an eclipse of a brighter star by a fainter component revolving round it in the period of light change was responsible. He was also able to calculate their relative dimensions and distance apart. In the same year (1889) as the spectroscopic discovery of the binary nature of ζ Ursae Majoris, Vogel of Potsdam found that Algol also showed displacement in the lines of its spectrum thus confirming Pickering's theoretical investigation and Goodricke's previous surmise. Only the spectrum of the brighter star is visible in this case, but the shifts of the lines have a period the same as that of the light variation, about $2\frac{7}{8}$ days.

A number of stars in which, like ζ Ursae Majoris, there is no eclipse, the plane of revolution of the component stars not being in the line of sight, were soon discovered. By the end of the century thirteen of these "spectroscopic binaries" were known. Few if any of such systems are ever likely to be seen as a visual double; the components are too close for separation by the largest telescope. The power of separation of a telescope is proportional to aperture. A four-inch can separate a pair about a second of arc apart, a forty-inch separates two stars a tenth of a second from each other. The separation of these spectroscopic binaries will be usually much less than this lower figure; for Algol, probably between a four hundredth and five hundredth of a second of arc, requiring an aperture of about 170 feet! One exception is perhaps the bright star Capella, discovered to be a spectroscopic binary in 1899 by Campbell and Newall independently. In this case it has been claimed that elongation of the image in the correct direction has been seen with the 28-inch Greenwich refractor.

VARIABLE STARS

Apart from what appears to be a strong probability that the eclipsing variable β Persei was known to the Arabians to change in light (they named it Al Gol or The Demon), the first

known variable, Mira Ceti, was discovered by D. Fabricius in 1597. The early history of Mira is briefly as follows. D. Fabricius saw it as third magnitude in 1596 April, and noted that it had disappeared in 1597. Bayer gave it the Greek letter Omicron of the constellation Cetus, but did not seem to have connected it with Fabricius's observations. No further notices of it appear to have been recorded until 1638 when Holwarda found it third magnitude, invisible in the summer of 1639, and again visible in 1639, October. From 1648 to 1662 Hevelius carefully observed its changes, and since that time they seem to have been followed by some observer or other until the present day. In 1669, G. Montanari (1632-1687) discovered (or rediscovered?) the variability of Algol, R Hydrae* was found in 1670 by the same observer, and χ Cygni by Kirch in 1686. No more were found until 1782, when Goodricke detected the variability of β Lyrae and δ Cephei, and another seven were added before 1800 making thirteen in all.

An astronomer who contributed very greatly to the study of variable stars was Argelander, by introducing a procedure of "step" comparison between a variable and non-variable neighbouring stars of about the same brightness (a method of intercomparison of magnitude employed earlier by W. Herschel) and also in an advocacy of co-operation in observation especially addressed to amateurs. This was in the 'forties of last century, at a time when only about 18 variables were known. In 1854 Pogson gave a list of 53, and Schönfeld, Argelander's successor, published a list of 113 in 1865 and 165 in 1875, while in 1884 and 1888 J. E. Gore's catalogues contained 190 and 243 respectively. Soon afterwards, E. C. Pickering's introduction of comparison by photographs taken at different dates for detection of variables, greatly increased the rate of discovery; and his adoption of instrumental photometry (see p. 184) for variables in 1896 did much to encourage precise knowledge in the subject.

In 1880 the same astronomer had put forward a proposed

*The first variable found in a constellation is named R, the next S and so on to Z for the ninth. The tenth is RR, eleventh RS, on to ZZ; then SS begins a new series. RR being reached, AA is the next. This system is due to Argelander.

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system of classification of variables which forms the basis of all subsequent schemes. It may be briefly described as follows: the first class, Novae or temporary stars; the second class, long period variables, like Mira Ceti; the third, irregular variables, like Betelgeuse; the fourth, short period such as β Lyrae or δ Cephei; fifth, eclipsing variables as Algol. Later on, β Lyrae was found to be an eclipsing variable, and in 1895 S. Bailey of Harvard detected a type of very short period (less than a day) common in certain of the globular clusters of stars. It is interesting to note in passing, that the first six variables, known by 1782, include examples of the three main classes, long-period (Mira Celi, R Hydrae, χ Cygni), short period (δ Cephei), and eclipsing (Algol, β Lyrae).

Observations of three short period variables, δ Cephei, ζ Geminorum, and η Aquilae by Bélopolsky in 1894-5, suggested that these stars, which show rapid and regular fluctuation in displacements of lines in their spectra, are accompanied by companions, revolving in the same period as their light variation, but without eclipse of their primaries. Later research showed the impossibility of this, and a different explanation was given attributing pulsation to their bodies (see p. 290)

NOVAE

The novae or Temporary stars of 1572 and 1604 have been incidentally referred to earlier (see pp. 54, 66). In the seventeenth century three* were recorded, 1604, 1609 (a Chinese observation) and 1670; in the eighteenth there were none. In fact for 178 years until the discovery of a nova in Ophiuchus in 1848, of only the fifth magnitude, by J. R. Hind (1823-1895), there was no Temporary Star seen, although in the long interval the skies were certainly more carefully examined than at any previous time, with the advantage of telescopes and improved star catalogues.

The new star of 1866 in Corona Borealis found by J. Birmingham, an Irish amateur, was therefore of particular interest to astronomers. It reached the second magnitude,

*Another star of perhaps nova type, known as P Cygni, was seen in 1600 as a third magnitude star by W. Janson. It has been only about a quarter as bright for many years since.

and some time after its discovery Huggins observed bright lines in its spectrum, particularly of hydrogen. The next nova was detected by Schmidt in 1876 in the constellation Cygnus. It was of the third magnitude and showed a spectrum of bright lines of hydrogen and helium on a continuous background with strong absorption lines. In 1885 a nova, which reached seventh magnitude, was seen in the Great Andromeda Nebula in a position where no star as bright as the fifteenth magnitude had previously existed, and in 1860 another seventh magnitude nova appeared near the centre of the globular cluster Messier 80 in the constellation Scorpio. In 1892, one which attained fourth magnitude was found in Auriga, and in February, 1901, another in Perseus which at its brightest outshone Capella, both by an amateur, Anderson of Edinburgh. These two were closely studied by means of the spectroscope and showed the usual type of nova spectrum.

A number of fainter novae were found from 1893 onwards by a new method. This was from their spectra as photographed with a prismatic camera (see p. 187), but such discoveries were fortuitous rather than systematic and did not, except by accident, show the spectrum when the star was at its brightest.

Nova Persei was the brightest since the Nova of 1604. A large nebula extending south-east from it was photographed by Wolf with a 16-inch star camera. Further photographs taken at Yerkes and Lick Observatories showed a movement of condensations in the nebula away from the star. This was explained by Kepteyn as the gradual illumination, by the light of the nova, of a nebulous cloud already existing in the surrounding space.

Several attempts to account for the phenomena of a nova were put forward about this time; of these, two may be mentioned. W. H. S. Monck (1839-1915), and H. Seeliger (1849-1924) independently, in 1892 suggested that a temporary star appears when a dark body or bodies are made luminous by friction while passing through a cloud of nebulous matter; this, they thought, would give the spectrum observed, *i.e.*, continuous from the incandescent body, and bright lines from the gaseous nebula. The other suggestion was of a collision between two stars, of a grazing nature, forming a gaseous

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"third body" composed of the coalesced detached parts of the colliding stars. This was proposed by A. W. Bickerton (1842-1927) in 1879. Neither hypothesis seems to have survived later criticism.

STELLAR GROUP MOTION

In 1869, R. A. Proctor found that five of the seven stars of the Plough have a movement across the sky in the same direction and with the same angular velocity (about a moon breadth in 18,000 years), this motion being clearly not a reflex of a movement of the Solar System in space. He concluded that they form a connected moving group, and he also noted community of motion among certain stars in Taurus, years afterwards studied as the Taurus Moving Cluster. Huggins examined the five stars of the Plough with the spectroscope and found them to be moving in the line of sight away from us at the same rate of speed, thus putting the question of their connection beyond doubt. This was the first intimation of such group motion. Nevertheless the general opinion at the end of the nineteenth century was that the stars move generally at random, although in 1895 Kobold believed he had found evidence of some general motion in opposite directions in the plane of the Milky Way.

THE NEBULAE

The employment of photography for nebular research has already been mentioned. H. Draper (1837-1882), Common, Wolf, I. Roberts (1829-1903), Barnard and Keeler were all pioneers in this work which was extended to discovery as well as to detailed study. The catalogues of nebulae and clusters of the Herschels were used as the basis for the "New General Catalogue" of J. L. Dreyer (1852-1926) published in 1888. It contains the particulars for 7840 nebulae and clusters and was supplemented in 1894 by an "Index Catalogue" with 1529 additional objects. Using the 36-inch Crossley reflector at Lick Observatory, Keeler photographed thousands of nebulae in addition; and he estimated that 120,000 new nebulae were within its reach and that probably half are of the spiral type (see p. 149). These large numbers have, however, proved to be underestimates.

In 1894 Wolf discovered photographically in a nebulous region three dark markings in the Milky Way about $1\frac{1}{2}°$ east of γ Aquilae. Further east, Barnard found other curiously shaped dark markings, and in 1895 he photographed an extensive nebulosity in Ophiuchus in which there are long dark lanes devoid of nebulosity and with practically no stars. These and many other markings of the kind were taken to be of a similar nature to the "Coal Sack" in the southern Milky Way and were believed by most at the time to be openings through the star clouds.

Prior to the discovery by Huggins in 1864 of the gaseous nature of diffuse nebulae, there had been a tendency to assume that all the nebulae are clusters of stars, not resolvable into individual stars owing to distance. But many, departing from the "external universe" idea of Herschel (see p. 119), held that in any case all types of nebulae would be found to be members of the Milky Way system. As pointed out by Herbert Spencer[1] and by R. A. Proctor, this was apparently supported by the facts of their peculiar distribution—much more numerous at the Milky Way polar regions than towards the Galaxy. But the view was not held rigidly by all, even Proctor writing in 1869 that it is "not improbable that the spiral nebulae are galaxies resembling our own." The observation of continuous spectra in the Andromeda nebulae and in others situated away from the Milky Way area, was seen to be consistent with this idea; but the majority view was against the "external universe" hypothesis, especially with what appeared to be an impossibly high luminosity for the 1885 nova in the Andromeda nebula at the distance entailed.

OUR GALAXY: IDEAS OF FORM AND SIZE

Various views of the shape and extension of our Galactic system were expressed during the period. J. Herschel seems to have inclined to the idea of a ring formation, size roughly 4000 light years in diameter, although he never expressly disassociates himself from his father's disc theory in any of his writings. Proctor advocated that it consists of a stream of a twisted shape, composed of comparatively small stars,

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but gave no estimate of dimensions. J. E. Gore (1845-1910) considered it to be a ring-shaped cluster reduced to a nebulous appearance in the Milky Way through great distance, and gave various estimates of size from 6000 to 20,000 light years. H. Seeliger thought that it was a disc formation 23,000 light years in diameter, while S. Newcomb (1835-1909) believed the size to be at least 6000 light years; and C. Easton of Rotterdam, thought that our system is a double-armed spiral. The Sun was considered to be near the centre of the system in all cases referred to except the last. In Easton's view the centre of mass of the system and the nucleus of the spiral were in the star clouds of the constellation Cygnus, with the Sun situated in a comparatively vacant space near the centre of the volume occupied by the spiral and its arms.

REFERENCE.

[1] "The Nebular Hypothesis" (1858). Collected Essays, vol. 1, p. 244. This essay is an example of elaborate reasoning based on necessarily defective and insufficient data.

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CHAPTER XIV

TWENTIETH CENTURY

An account of the rapid progress in all departments of Astronomy, including several revolutionary discoveries, during the past half-century, may be appropriately introduced by a short survey of the improved and novel tools with which the observational work was done.

MODERN INSTRUMENTAL DEVELOPMENTS

At the end of the nineteenth century the largest refractor was the 40-inch at Yerkes. There were then in existence about twenty telescopes of the kind, 24 inches or more in aperture, four of them (the largest a 31½-inch at Potsdam) having object glasses corrected for photography. The Yerkes instrument is still the biggest, although another ten refractors, half of them photographic, above the limit of size mentioned, have been constructed since 1900. Until 1935 the biggest reflectors were a 72-inch at Victoria, British Columbia (1919) and the famous 100-inch at Mount Wilson, California (1917). Since the year 1900 there have been fifteen reflectors 40 inches or more in aperture, projected or completed. Besides the 100-inch and the 72-inch, these include one of 200 inches for Mount Palomar, California, a 120-inch for Lick Observatory, an 82-inch at Mount Locke, Texas, a 74-inch for Pretoria, South Africa, a 74-inch for Dunlap Observatory, Ontario, Canada, a 69-inch for Perkins Observatory, Delaware, and a 100-inch for Great Britain, to be situated at the new Royal Observatory, Herstmonceux, is to be built.

It will be seen that the favoured type for telescopes of the largest size has been the reflector. The maximum practicable for the refractor appears to have been reached with the Yerkes giant. This was due to several factors, such as the problem of support against flexure (an object glass cannot be supported at its back as reflectors can), the difficulty of obtaining very large optically perfect transparent glass discs, and the greater

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cost as against a reflector of equal or even greater light-grasping power. And the achromatism of a reflector, which brings rays of light of all colours to the same focus, is a great advantage. One improvement introduced about fifteen years ago consisted in the substitution of aluminium for silver as the reflecting coating on the surface of mirrors. The advantages are: a longer time before re-coating (years against months); superior reflectivity for the violet and ultra-violet regions of the spectrum, with shorter exposures necessary; and greater powers of photographing the spectrum in the ultra-violet beyond the range of a silver-on-glass mirror.

Telescopes of new types have been developed during the century. One of them is the Tower Telescope for Solar work, the largest of the seven of these in existence being the 150-foot at Mount Wilson. The rays of the Sun arriving at the top of the 150-foot tower are directed to a 12-inch object glass by a reflecting system of two flat mirrors 24 inches in diameter, one of which is driven by a mechanism to follow the Sun and keep its rays centred on the second mirror. The light then passes through the object glass and goes down the shaft of the tower to form a 17-inch diameter image of the Sun at the ground level, thus providing a large-scale picture of the visible activities on the Solar surface, including the main features, the sunspots. A drawing is made every clear day showing the size and form of each spot and its position on the Sun's disc, and other observations are made, useful in the study of the Sun's activity.

Another new instrument, known as the "spectrohelioscope," was invented by Hale in 1924. It provides a view of the solar surface, through an eye-piece, in monochromatic light, as the spectroheliograph (see p. 155) provides a photograph. The two slits, which move across the Sun's image, in the spectroheliograph are made to oscillate to and fro. This gives a series of visual impressions of neighbouring narrow strips of a part of the Sun's image which (as in television) become a single continuous picture owing to "persistence of vision." The monochromatic image is usually that of the red hydrogen line, as other lines, useful in the spectroheliograph, are in the short-wave blue or violet region of the spectrum to which the eye is less sensitive. The instrument is ideally suited for

observation of the disturbed regions of the Sun's surface over and around spots, and it can also be adapted to show the line-of-sight velocity of any bright hydrogen cloud through the observed displacement (by Doppler's principle) of the image to the red or blue side of the spectrum which accompanies such a movement.

Several new forms of reflecting telescopes for photography have been invented, the first and best known of these having been described in 1930 by a German amateur, B. Schmidt. The Schmidt reflector consists of a spherical mirror (a more easily made form than the paraboloid of the ordinary reflector) with a thin (and rather difficult to make) correcting glass plate of varying thickness set in front of the mirror. This plate corrects the paths of the light rays so that good images are given over a very much larger field of view than is possible with the orthodox form of the reflector, some of the correction being provided by the photographic film's attachment to a holder with a specially curved surface. This and several related forms of instrument are exceedingly powerful in discovery of faint objects or faint extensive nebulae, and the definition is so good that the original plates may be magnified considerably to study detail. There are now more than thirty Schmidt cameras in existence or projected, ranging from ten to sixty inches in aperture ; and it seems doubtful as to whether large instruments of the kind or reflectors of the ordinary type of 100 inches aperture upwards, will be for some purposes the most profitable to make in the near future.

Improvements in instruments for stellar photometry, such as the selenium cell and the photo-electric cell,[1] detailed descriptions of which are not possible in the available space, have been effected, with remarkable increase in accuracy. The selenium cell was first used by Stebbins in 1910, and the photo-electric cell by Guthnick in 1913, on variable stars. They produce results correct to a hundredth of a magnitude or less against the tenth or so possible to eye estimation and the twentieth to ordinary photometers. By means of these special photometers minute fluctuations in the light of spectroscopic binaries have been detected. One of the first was by Stebbins when he showed that Algol has a secondary minimum between its principal ones proving that the fainter component

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has an appreciable brightness of its own which is lost when it goes behind its primary.

In 1931, B. Lyot of the Meudon Observatory, France, succeeded at the summit of the Pic du Midi 8500 feet above sea level, in photographing the Sun's Corona without an eclipse. Huggins had tried to do this in London as far back as 1882 but without success. Lyot's instrument has been given the name Coronagraph. Its chief features are a disc blotting out the Sun itself, an object glass free from scratches and dust and a monochromatic light filter.

The dust-free atmosphere at the great height where Lyot took the photographs is also a necessary condition. There are now two other stations for Coronagraph work, Harvard's at Climax, Colorado (11,500 feet) and one in the Alps established by Zürich University. With improvement in the new instrument more information, for the inner Corona at least, and for the Prominences, should be available in a few years than could be collected at scores of Eclipses.

Another recent development of much interest and potential value is the application of the idea of motion pictures by McMath and Hulbert in America and Lyot in France. The Solar Prominences in motion, the progress of the shadows among the Moon's mountains and craters, and the rotation of Jupiter have been filmed, and a new tool thus introduced, especially important for solar research.

PROGRESS IN PHOTOGRAPHY

The first tentative experiments in astronomical photography were made by Draper of New York in 1840, when he took several one inch diameter pictures of the Moon. The Bonds of Harvard College Observatory followed with deguerrotypes of the Sun and Moon, and with some pioneer star photography (see p. 149 and p. 188). The earliest results of much value were obtained, however, by W. de la Rue (1815-1889) an English amateur, who took successful collodion process lunar photographs in 1853; and his pictures of the Sun taken in 1857 led to systematic solar photography (see p. 166). The first use of the dry plate was by Huggins in 1876 and he was quickly followed by others. About this time

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Abney made chemical discoveries resulting in plates which were sensitive to red and infra-red light, whereby a detailed map of the infra-red part of the Solar spectrum was secured. From that period constant experiment to increase sensitivity of photographic plates, particularly for the longer wave-lengths, has continued with much benefit to astronomical research.

Recent improvements have been so marked as to produce the statement that the Mount Wilson 100-inch reflector has already been enabled to surpass the gain in speed expected from the increased size of the 200-inch Mount Palomar instrument when it was originally planned in 1928. That is to say, as these improvements can be applied to the 200-inch, when it is in operation there will be a corresponding advantage over the "200-inch of 1928."

Two methods of treatment of plates are now in use which give significant gains for exposures of more than an hour. One of these is by treatment with mercury vapour, the plates before exposure being put into an air-tight chamber containing a large drop of mercury. The other is by putting the plates in an oven and "baking" them for several days at a temperature of 50°C.

Another great advantage has been secured by a process of deposition of a thin transparent non-reflecting film[1] on the surfaces of lenses or prisms. There is generally a loss of light of about five per cent at the air-glass surface of a lens or prism, that amount being reflected back instead of transmitted. As a beam of light from a star may have to pass through a number of air to glass and glass to air surfaces before reaching the eye or the photographic plate, the saving effected by these films can be very considerable, sometimes amounting to twenty-five per cent or more.

NEW METHODS OF RESEARCH

Entirely new methods of research in Astronomy are now at their initial stages. By means of large sheet-metal paraboloids or suitably spaced antennae, radiations of several metres wave-length have been detected which appear to come most strongly from the direction of the richest Milky Way star clouds, while very strong radiation of the same order of

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wave-length has also been noted coming from sunspot areas at times of great Solar activity. And the war-time development of radar has led to a preliminary experiment in 1946 in the United States, where radiation of 2·7 metres wave-lengths has been directed to the Moon and a reflection received after the $2\frac{1}{2}$ seconds necessary for the double journey. Further experiments with a considerably shorter wave-length are expected to be able to provide, by the measurement of the time between the transmission and reception (which can be to a millionth of a second by means of a cathode-ray tube), a determination of the Moon's distance to within a mile. This technique should provide a measurement of the Sun's distance much more accurately than any hitherto made, since, with the developments which appear possible, the method can be extended to the planets and perhaps to the Sun, thus giving dimensions of the solar system of an accuracy not dreamt of only a few years ago. By radar we should also be able to form some conception of the nature of the surfaces of the Moon and the nearer planets.

For meteoric work radar has already been successfully employed. Meteors produce paths of ionisation* of the air through which they pass; these reflect radio waves, and by means of radar the passage and positions of the meteors responsible can be studied. Carefully planned and skilfully carried out observations of the kind were made during a shower in 1946 and investigations of well-known showers are now proceeding. A new meteor shower of extraordinary richness was found by means of radar in 1947 and actually studied in daylight which, unlike in ordinary meteor observation, does not interfere with radar work.

Radar meteor research is active at the Jodrell Bank Experimental Station of Manchester University, in Canada and stations in the United States (connected with Harvard, Stanford University, Massachusetts Institute of Technology, and the U.S. Government Proving Grounds at White Sands).

The possibilities of the application of electronic methods are now attracting attention. Among these are: following devices which make a telescope or star camera point itself

*Ionisation: removal of one or more satellite electrons from an atom or molecule.

closely throughout an exposure to a star or other celestial object; automatic timing of lunar occultations of stars or of satellites by their primaries, doing away completely with "personal equation" (see p. 131) and accurate enough to enable close determination of diameters of the occulting or occulted object to be made; counting of stars to a given brightness automatically on photographs, or even directly through a telescope for statistical researches in stellar distribution; studies of auroral displays; and a "Coronaviser" for direct observation of the Solar Corona and Prominences visually without an eclipse.[2]

THE INTERNATIONAL ASTRONOMICAL UNION

In Chapter XII. (see p. 156) a brief statement of the formation of astronomical societies throughout the world was given. But reference to such bodies would be very incomplete without mention of the great co-operative one, the International Astronomical Union. This is the co-ordinating successor of a number of international bodies such as the International Geodetic Union, the Carte du Ciel organisation and the International Solar Union. It was founded in 1919 and held approximately triennial meetings in different countries from then until 1938, when its activities were suspended by the War. It was revived in 1946 when delegates from eleven countries, and one from the Vatican, met in Denmark, and there was another well-attended meeting in Zürich in August, 1948. Its work is done by means of a number of Commissions on different sections of the science, and consists mainly in organisation, consideration of what are the most important problems to be tackled and the best methods for them, and in sharing out the work, and publishing reports.

RELATIVITY: ASTRONOMICAL CONSEQUENCES

During the past forty years much modification of physical ideas has resulted from the theory of relativity put forward by Albert Einstein in 1905. Two of its consequences are of astronomical significance. In the first place, the velocity of light can have only one value as measured by an observer whether he is in motion or not, and irrespective of the direction

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the light is moving in.* Secondly, the mass of a body depends on the energy contained by it, increasing slightly where this gets greater (as by heating or by added velocity), *i.e.*, energy possesses mass. In 1915 Einstein developed the theory to include the effects of accelerated motion (the General Theory of Relativity).

There are astronomical tests of the theory. It requires that rays of light travelling past a gravitating mass should bend towards it as would material particles. For a light ray grazing the Sun the observed deflection should be 1·75 seconds of arc, and for a ray farther from it the deflection is inversely proportional to its distance from the Sun's centre. This deflection can be measured at the time of a solar eclipse by photographing the stars in the Sun's neighbourhood, and comparing the photographs with others taken a few months earlier or later when the Sun is away from that part of the sky. The results obtained at eclipses of 1919, 1922 and 1929 were in accordance with the predicted value. And, according to Einstein's theory, the orbit of a planet, if not disturbed by the attraction of other planets, will be elliptical in form with the Sun at a focus; but this ellipse slowly rotates, the rate depending on the planet's mean distance from the Sun, its period, and the velocity of light. The predicted movement of the perihelion of the orbit (the nearest point of the orbit to the Sun, *i.e.*, the end of the major axis nearer to the Sun) in accordance with relativity, beyond what is to be expected by the ordinary theory, should be detectable only in the case of Mercury. For Venus the orbit is so nearly circular that its perihelion cannot be accurately determined, while the movements of the perihelia of the Earth and Mars are too slow to be observable. For Mercury the perihelion should advance at about 43 seconds of arc per century more than the orthodox

*In 1887 A. A. Michelson and E. W. Morley attempted to measure the velocity of light with respect to the ether, the universal medium then believed to exist. With ingeniously devised apparatus, they found only small fractions of the expected effects and these were attributed to accidental errors in the observations. The experiments clearly indicate that the time for light to go from one body to another depends solely on their distance apart and their movement relative to each other, and that it is completely independent of their motions with respect to the medium through which light is transmitted (although there is reason to believe that there is no such medium).

gravitational theory indicates, and this is very close to the value (40″) found by observation.

Another prediction of relativity theory is that when light is emitted or absorbed by an atom on the surface of a massive star the wave-lengths will be greater than the normal values, with a redwards shift of the star's spectral lines. In the case of the Sun the shift is small and complicated by other displacements. But C. E. St. John, in 1924, found that when these are allowed for, the shift accords with the theory. In the case of the binary star Sirius, the companion, which is very massive for its diameter, has a predicted Enstein shift of large amount. This shift was measured by W. S. Adams in 1925 at Mount Wilson, and the amount, confirmed later by Moore at Lick Observatory, agreed with theory.

LUNAR THEORY

In the year 1923, E. W. Brown published tables of the Moon's motion, after 30 years of work on the subject, which replaced those of Hansen used since 1857 by the various national almanacs (see p. 136). Several years later he proposed an explanation of certain irregularities that appear to be superposed on the rate of slowing up of the Earth's rotation, which is considered to be on the average about 1/1000th second per century (see p. 157). These irregularities are revealed by the Moon's motion, and are also shown by the planets Mercury and Venus. Brown's suggestion was that they are caused by variations of several feet in the Earth's diameter due to internal causes which alter the period of rotation, the angular momentum being constant.

LUNAR WORK

Observations of the physical features of the Moon's surface have been largely the work of amateurs, but a number of special investigations were undertaken by professionals. Among these latter was the measurement, with a thermocouple,* of the Moon's surface temperature by the American

*In this instrument the radiation is made to fall on the junction of two wires of dissimilar metals, such as bismuth and antimony. An electric current is set up by the heating of the junction, the strength of which, proportional to the radiation received, is measured by a sensitive galvanometer.

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astronomers, S. B. Nicholson and E. Pettit, using the 100-inch reflector at Mount Wilson. From observations extending over a period since 1923, it was found that the mean temperature falls off from 101 degrees centigrade at the point where the Sun is overhead to 50 degrees below zero at the points where the Sun is near the lunar horizon. Measurements of the dark side of the Moon gave 153 degrees centigrade below zero for its temperature. These measurements and computations indicate that an observer on the Moon would find that during the long night of 14 days his thermometer would give temperatures nearly as low as that of liquid air. In the week in which the Sun rises from the horizon to the zenith the temperature would rise to that of boiling water, and fall again to liquid air temperatures in the following week as the Sun sets. During lunar eclipses in 1927 and 1939 it was found that the temperatures of the surface fell from about 100 degrees centigrade just before the eclipse, to 70 degrees below zero at the beginning, and 110 below at the close, of totality. From this extraordinarily rapid cooling it has been deduced that the Moon's surface materials are of a highly porous nature like pumice-stone. In support of this, the researches, published in 1929, of B. Lyot, who measured the amount of polarization of the light from the mountains, plains, and the brighter or darker areas, may be quoted. With varying angles of reflection of the Solar illumination he found that a surface of a porous ash or pumice-like description is indicated. This was confirmed later by Wright and others at Lick Observatory.

The earlier investigations of R. W. Wood who made photographs of the Moon with coloured screens using special plates sensitive to different regions of the spectrum, were also of great interest. The most striking of his results concerned a spot close to the crater Aristarchus. They were strongly suggestive of a sulphur coating to the surface at this point, as a series of photographs which he took in ultra-violet, violet and orange light gave much the same images as would be provided by a deposit of sulphur on a rock surface. Wood's "sulphur patch" has also been photographed by Wright, who took a series of photographs of the Moon in 1928 by ultra-violet and extreme red light that showed differences in brightness in a number of lunar areas and markings.

A considerable amount of speculation as to the origin of the lunar features, particularly the craters, marked the period. Beginning with R. A. Proctor in the latter part of the nineteenth century, many have advocated that the craters are the result of meteoric bombardment. At first it was considered that this must have been when the Moon was in a plastic condition thus accounting for the generally circular shape of the craters. In 1915, however, A. W. Bickerton pointed out that a meteor hitting a solid surface would, in the absence of slowing down from any sensible atmosphere, have such a "striking velocity" as to cause a violent explosion and a circular crater whatever the direction of impact. This explanation of the origin of the craters has not been universally accepted, but not many seem to have favoured the alternative of an ordinary volcanic origin. Statistical studies of the relative frequencies of crater diameters have been made by several investigators, notably T. L. MacDonald and J. Young. The latter concluded that his results, showing as they did abnormally great frequency at certain definite sizes and an unequal and unsymmetrical distribution of the various diameters over the Lunar surface, were generally not in favour of the meteoric impact hypothesis.[3] One explanation proposed by H. G. Tomkins in 1927 has met with some support. This attributes the craters to a consequence of the formation of what are known as "laccoliths" in terrestrial geology (*i.e.*, intrusions of molten material), under the solid crust of the Moon, upheaved by internal pressure into domes the crowns of which fractured and fell bit by bit into the lava of the centres where they melted and were deposited on crater floors, the crater walls being formed by the edges of the domes.

But the origin of the various lunar surface features provides a subject for interesting speculation that cannot be said to have resulted in any really convincing explanations as yet, except perhaps in the case of the mountain ranges and peaks, which were, in all probability, formed by the contraction and "wrinkling" of the lunar crust as the interior cooled and shrank; and cracking of the crust from local tensions may have formed the rills or clefts (see p. 137). The bright streaks or rays, the most prominent of which proceed like the spokes of an umbrella from or near the crater Tycho, are so far not

very satisfactorily accounted for, although radial splashes of fragmental material caused by meteoric impacts have been suggested.

The strongest advocate of lunar change was W. H. Pickering who was convinced from about 1902 onwards, as the result of a telescopic and photographic study of small selected regions, that some volcanic activity with emission of white vapours, and deposition of something like hoar frost, can be seen. He also considered that he had secured evidence of progressive changes suggesting vegetation. But few astronomers agree with his contentions, believing more evidence to be necessary before such changes can be said to be demonstrated, whatever might be their explanation if they ever were confirmed.

The works on the Moon produced during the period included an atlas of photographs taken with a long focus refractor by Pickering in Jamaica, and published with descriptive matter in 1903 with the title "The Moon" ; a work by P. Fauth, "The Moon in Modern Astronomy" (1907) ; and an atlas 77 inches to the Moon's diameter (1910) by W. Goodacre, for many years Director of Lunar Section of the British Astronomical Association, later followed by his book, "The Moon with a Description of its Surface Features" (1931), with a 60-inch reproduction of his atlas. Other publications of interest were "Named Lunar Formations : Catalogue and Maps," by Blagg and Müller, published by the Lunar Commission of the International Astronomical Union in 1935 and the Memoir of the British Astronomical Association "Who's Who in the Moon," published in 1938. The accurate determination from photographs of the positions on the Lunar surface of more than 3000 points, by Saunder, constituted a work of first class importance to Selenography ; while the altitudes above or depressions below a mean lunar sphere for 38 points, varying from 16,000 feet altitude to 9000 feet depression, obtained by Saunder from photographs, enabled him to conclude that the Moon is not perfectly spherical, the radius towards the Earth being rather more than half a mile longer than the polar radius.

SOLAR WORK

Solar research has been among the most active of all departments during the century. In 1908, with photographic

plates sensitive to wave-lengths at the red end of the spectrum, G. E. Hale found that the hydrogen clouds above sunspots, photographed with the spectroheliograph, appeared to be in vortical motion in opposite directions on opposite sides of the Solar equator; and he was able to show that the spectral lines from these clouds were modified (widened or doubled) in the same way as takes place between the poles of a powerful magnet (the "Zeeman effect"). This indicated that there is a strong magnetic field surrounding the spots.

When spots are in pairs (which happens for about 60 per cent) each member of a pair has opposite magnetic polarity, this relation of polarity being reversed in the southern Solar hemisphere from that found in the northern. In 1912 the high latitude spots of the new sunspot cycle were noted to have polarities opposite to those of the low latitude spots of the previous cycle (see Spörer's Law, p. 165) and this was found to occur in the subsequent cycle, high latitude spots of a new cycle evidently always appearing with polarities opposite to those of low latitude spots of the cycle which was ending. This led to the formulation in 1923 of a general polarity law covering the phenomena. As a consequence, although the sunspot period, as defined by numbers or total areas of sunspots, is 11·1 years, the period is twice as long if it is regarded as the interval between the appearances of spots of the same magnetic polarity. In this connection it may be noted that, beginning with the sunspot maximum of 1847, successive maxima have been high, low, high, low alternately, which gives a double period of 22 to 23 years.

A tentative theory of sunspots was advanced in the year 1912 by G. E. Hale. According to this a sunspot is a vortex of a funnel shape in the outer parts of the Sun. Inside it gases are ascending, spiralling outwards, and as there is rapid expansion there is considerable cooling to about 2000 degrees centigrade below the temperature of the surrounding bright surface of the Sun. This cooling is responsible for the relative darkness of the spot area, and also for the formation of chemical compounds like titanium oxide, discovered by A. Fowler in 1908 in sunspot spectra. In the same year J. Evershed discovered that the gases flow outwards over the Solar surface (the "Evershed effect"), and this was confirmed in 1913 by

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St. John at Mount Wilson, who also noted that the direction of flow is inwards at high levels.

The magnetic fields of sunspots were at first thought to be caused by the rotation of electrically charged particles accompanying the ascending and spirally rotating gases. This should have been marked by a close correspondence of the direction of the "whirl" with the polarity shown by the Zeeman effect. But in 1927 Hale found no relation between polarity and vortical structure for 51 spots, and in 1941 R. S. Richardson confirmed Hale's result. On the other hand, Richardson noted a predominance of clockwise vortical motions in the southern Solar hemisphere and the reverse in the northern which indicates that the direction of rotation of the vortices is governed by the differential speed of Solar rotation with change in latitude (see p. 165). A general magnetic field for the Sun as a whole, similar to that of the Earth, has been found by Hale and his collaborators, the magnetic poles being close to the poles of rotation.

In 1926, V. Bjerknes suggested that all the spots in a hemisphere during a particular Solar cycle are produced by a primary tubular vortex which may extend east and west below the Sun's surface, perhaps entirely round the Sun, occasionally bending upwards and breaking through the surface, often at two spots (a pair). This would explain the persistence of the outward flow of gases from double spots for weeks at a time, and it might also account for Spörer's Law of the appearance of spots working towards the equator from higher Solar latitudes.

More than 15,000 groups of spots have been recorded at Greenwich since they were first photographed and catalogued systematically there in 1873 (see p. 166), ranging in size from a few hundred miles in diameter to enormous groups up to 200,000 miles long. Of these groups 26 were 3000 million square miles or more in area (about 15 times the Earth's surface). The largest of the 26 groups (at its maximum) was that of February, 1946. It consisted of two main spots, the larger one of 4600 million square miles area. Another great group in March, 1947, maintained a greater area for a longer time than any of the others, and had an even larger spot in it.

No satisfactory theory accounting for the appearance of spots on the Sun has as yet been advanced. As long ago as

1612 Galileo thought that the planets might have something to do with it ; and a planetary influence on the spots has been attributed by R. Wolf (1859), Balfour Stewart (1864), D. Alter (1930) and others, but there has been no general acceptance of the ideas involved.

The appearance of the granulations on the Solar surface has been described earlier (pp. 140 and 166). They form and dissipate and continuously change their shapes. Within a few seconds they alter noticeably and individual granules lose their identities in a matter of minutes. This was first shown in 1905 by the Russian astronomer M. Hansky of Pulkowa, who found on enlargements of photographs that although in 25 seconds little change occurred, there were great modifications in one minute, and that after three minutes only a few of the granulations were recognisable. He measured their diameters as from 400 to 1200 miles. In 1942, P. C. Keenan photographed them with the 40-inch Yerkes refractor. He estimated their diameter to be one to two seconds of arc normally, *i.e.*, 450 to 900 miles, the biggest being about three seconds or 1350 miles. The total number on the entire Sun at one time exceeds $2\frac{1}{2}$ millions, and the average distance between their edges is about 900 miles, but the intervening space is not uniform in light as it probably contains granules which are forming or ascending and others that are cooler and disintegrating or descending. The general opinion seems to be that the granules are probably the tops of columns of hotter and brighter gas ascending from the lower layers of the Sun, the darker spaces between being occupied by descending cooler matter.

In a paper published in 1904, Maunder of Greenwich Observatory brought forward further evidence of connection between sunspots and the Earth's magnetism. Nineteen great disturbances of the intensity and force described as "Magnetic Storms," had occurred from 1875 to 1903, and all had been at a time when there were large groups of spots on the Solar disc. He demonstrated that the cause of the magnetic disturbance is associated with limited areas on the Sun and inclined to the opinion that streams of electrically charged particles driven from the Sun are responsible. In 1930 Hale, in a paper dealing with the work of the spectro-

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helioscope, stated that he considered that these widespread disturbances of terrestrial magnetism and also the Aurorae often visible at the time, are directly connected with certain phenomena he had observed with that instrument in the areas occupied by sunspots.

As the consequence of much detailed observational work it has since been found that the light gases such as hydrogen, helium and calcium which extend high above the Solar surface are greatly disturbed over spots. Clouds, sometimes as large as the spots themselves, formed of these elements, become exceedingly bright, and in a few minutes they "flare" up, fading usually after about twenty minutes or half-an-hour, although durations up to three or four hours have been observed. These occurrences have been given the name of "Solar Flares"; until about 1937 they were called "chromospheric eruptions." As this latter term suggests a vertical movement upwards of the bright clouds, which does not necessarily take place, the expression Flare has been preferred, combining the idea of a sudden appearance with great brilliancy and also suggesting variation in intensity.

The first recorded observations of the phenomenon were those of Carrington and Hodgson (see p. 165). That Flare is especially noteworthy as, besides being the first to be seen, it was one of the very few recorded as having been observed visually,* which suggests that it would have been seen to have had a continuous spectrum if the occurrence had been observed with a spectroscope. The others have been noted only in monochromatic light from glowing hydrogen, helium or calcium gas.

The Flares are now observed by means of the spectrohelioscope, and just at the time they occur there is a slight disturbance of the Earth's magnetic field by a small quick movement or "crotchet" on the traces showing the intensity and direction of the terrestrial magnetic forces which are always being automatically recorded. This is believed to be due to a "blast" of ultra-violet light that on striking the

*One was certainly seen on March 5th, 1946, at Geneva by du Martheray and two other observers, between two large sunspots. Others appear to have been observed by Secchi in 1872 and du Martheray in 1921 and 1928.

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Earth's atmosphere changes the "ionosphere" (the ionised region of the upper atmosphere responsible for the reflection and transmission of short-wave radio) so that it does not act, and a fade-out of high frequency radio results. About twenty-five hours later there may be a much more marked interference with the Earth's magnetism, that known as a "Magnetic Storm," and probably a great display of the Aurora. This is thought to be due to a stream of electrically charged particles issuing from the area of the Flare, taking much longer to reach the Earth than the ultra-violet light responsible for the crotchet.

It is interesting to note that out of nearly 600 Flares of all grades of size or intensity observed at Mount Wilson from 1917 to 1943 inclusive, five were found to have appeared where no spot existed at the time, although in several of these, small groups of spots had been seen near the positions one to three days before. In this connection it may be noted that in 1921 Hale found evidence by the Zeeman effect of the existence of areas with magnetic polarity where no spot was visible.

As continuous observation of the Sun as possible for the detection and observation of these Flares is carried out at a number of observatories such as Mount Wilson and Greenwich (where H. W. Newton has done valuable work), while there are several amateurs in this country and abroad who also participate in the work. Flares are found most commonly near sunspot maximum, and usually in the middle part of the life of a large spot. The great Magnetic Storms are generally connected with Flares, but the smaller storms seem possibly to be caused by a stream of particles expelled from the Sun, lasting for a period of weeks or months, partaking in the Sun's rotation, and passed through by the Earth after an interval of 27 days (the Solar rotation period modified by the Earth's orbital motion).

Above the photosphere or chief light producing layer* of the Sun, where the sunspots, faculae and granulations appear, are situated the reversing layer (see p. 168) several hundred miles deep, and then the chromosphere made up of glowing

*The photosphere is perhaps better described as a discontinuity between the gaseous body of the Sun and its more tenuous surroundings, the spectrum changing from a very bright continuous one to a fainter emission line type.

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gases especially of hydrogen, helium and calcium, 5000 to 10,000 miles deep (according to the particular element considered) and at rather lower temperatures than the reversing layer out of which it rises. From this chromosphere Prominences often rise to great distances. They were first reported by Vassanius, a Swede, at the solar eclipse of 1733. In 1737, 1748, 1778, 1806, 1820 and 1836 they were reported by Short, Ferrer, Van Swinden, Bessel and others. These accounts were not taken any particular notice of by astronomers; it was the 1842 eclipse observations which roused scientific interest in them (see p. 138), and they were studied only at eclipses for the next twenty-five years until in 1868 the method for observing them in daylight was introduced (see p. 167). Long series of observations have since then been accumulated at Rome from 1869 to 1911, and since 1922 at the Arcetri Observatory, Italy, while from 1890 onwards J. Evershed has observed them in India and later in England; and their observation at Mount Wilson has been continuous for a long period.

During the year 1870 a distinction in type was proposed independently by three spectroscopists, Lockyer, Zöllner and Respighi, into "Cloud Prominences" and "Flame Prominences." These became better known as the "Quiescent" and "Eruptive" types. The former are enormous cloud forms, often 30,000 to 60,000 miles in height, and of even greater horizontal dimensions, resting on the chromosphere directly or connected with it by stems or columns, and persisting for relatively long periods. They are not so bright as the other type, and usually show only bright lines of hydrogen, helium and calcium. They are formed at all parts of the Sun's circumference, but in greatest abundance at 25 degrees north and south of the Solar equator; with two other points where they are frequent at sunspot maximum moving gradually towards the poles between the spot maxima. They vary in number during the cycle but not so noticeably as the spots.

The Eruptive Prominences appear only in the belts of the Sun where spots are found, and are usually connected with spots, or rather, with the disturbed areas surrounding spots. Generally they are not very large but they may, however, attain enormous heights, one observed on June 4, 1946, at

Mount Wilson by Pettit, reaching to about a million miles above the Sun's surface. Their velocities of movement from the Sun appear to increase abruptly by stages, according to Pettit, each stage usually moving at a speed which is a simple multiple of the velocity during the previous one, the time occupied in change of velocity between stages being as short as a minute.* In the case of the large Prominence referred to above, the maximum velocity was nearly 180 miles per second, but speeds up to 250 miles per second have been noted.

Downward movements in Prominences are very common and interchange of parts between two of them are occasionally seen. In the former case the matter streaming downwards seems to be moving to a definite point on the Sun's surface.

A great amount of the information recently obtained has been by means of the moving-picture technique referred to earlier (see p. 201). The observational data shown by the McMath-Hulbert motion pictures and interpreted by McMath and Pettit, reveal very complicated movements. Sometimes Prominences appear in the normally expected style, with eruptions and connecting stems, etc. But on other occasions the material moves down from what is apparently a source high above the chromosphere. Very often there is no obvious replenishment of the source. Occasionally the motions are so sudden as to suggest a lightning flash, while at other times the great clouds of gas stay unchanged in shape for days suspended at considerable heights.

Eruptive Prominences, being evidently connected with sunspot disturbances, would be expected to vary in frequency of appearance in the eleven years cycle. Pettit has found the numbers to range from between twenty-five and fifty at minimum to about 400 at sunspot maximum, per year.

The Quiescent type are evidently supported above the Sun's surface by some repellent force which has been thought to be the pressure of radiation.† The same force has been

*But later work (1946) seems to throw doubt on these views of Pettit, indicating that the changes in velocity are not really sudden and the relationship between initial and final velocities not simple.

†Radiation carries momentum as well as energy, and thus exerts a pressure capable of supporting atoms against gravity. Only the radiation of a very small part of the Solar spectrum is intercepted by an atom ; but on the other hand, the mass of an atom is very small.

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suggested as the explanation for the Eruptive kind, but this does not satisfactorily account for the fact that different gases such as hydrogen and calcium keep together, nor for the existence of horizontal and downward motion.

A phenomenon recently discovered by W. O. Roberts at Climax in Colarodo is specially interesting. Thousands of very small projections appear on the Sun's limb out of the chromosphere. They first appear as bubbles and in a minute or two they burst and shoot upwards small luminous extensions, the lifetimes of the appearance ranging from about two to twelve minutes. It is thought possible that they may form a direct connection between the granulation of the disc and the fine streamers of the Corona.

The luminous cloud and streamers seen surrounding the Sun in total eclipses, known as the Corona, had been observed for an aggregate time of only two to three hours since 1842 until Lyot made it possible with the Coronagraph (see p. 201) to observe at least its inner parts without an eclipse. Its total luminosity changes very little judging by measurements made at several eclipses, and is about equal to half the light of a Full Moon or about a millionth of the Sun's light. Its inner regions are yellower than the outer, but the average quality of the light is not unlike the Sun's. There is rather a sharp transition from the greater brightness of the inner to that of the outer Corona ; the latter is pearly white and consists of filaments or rays mostly radially disposed, with long pointed streamers which may extend far from the Sun's surface, one photographed in 1898 by Mrs. Maunder being over 6 million miles long. The density is exceedingly small, several comets having passed through the inner part without observable effects on their motion. The discovery of its change of shape in the period of the sunspot cycle has already been mentioned (see p. 167).

Observations at the Coronagraph Station at Climax, Colorado, show that the Corona does not change rapidly, and Lyot has also found that alterations in its formation appear rather to be fluctuations of intensity of illumination than movements of matter. It rotates with the Sun and observations at Climax and the Alpine station at Arosa show that

there is a connection between its activity and disturbances of the Earth's magnetism, and that a change in the ionosphere is to be expected, after an appropriate interval, when an excited Coronal region has been seen on the eastern limb of the Sun.

The spectrum is composite—continuous, with a number of bright lines superposed. None of these bright lines, more than thirty in number, had been identified until the work of B. Edlèn (following a suggestion by W. Grotrian) some results of which were published in 1941. He showed that a number of lines are due to atoms of iron, nickel, calcium and argon, ionised to the extent of having from nine to fifteen electrons removed from their atoms. All but about 3 per cent of the total intensity of that part of the light of the Corona derived from the bright lines was thus accounted for. His interpretation of the Coronal spectrum requires that there should be temperatures of hundreds of thousands of degrees to account for the removal of so many electrons from the atom by high energy particles. He attributed these particles to the prominences and ejections from the chromosphere, the total energy necessary to maintain the effect being small as radiation losses from the Corona are slight owing to its very low density.

The conclusions which Waldemeier, the Director of the Arosa station, drew in 1942 from his observations were that the inner Corona consists chiefly of a spherical shell of electrons, atoms and ionised atoms. The electrons produce the continuous spectrum by scattering the light from the photosphere. Above the spot zones and specially excited regions of the Corona, extreme ultra-violet radiation, and also corpuscular radiation from the Sun, ionise the atoms to a very high degree, putting them into states from which emission of the bright lines of the Coronal spectrum is possible according to Edlèn's results. In 1945, the Indian physicist, M. N. Saha, suggested that there may be some process near the Sun's surface similar to uranium fission (atomic bomb type) and remarked that no ordinary thermal or photo-electric effect can account for the observed phenomena, which at first appear irreconcilable with the known much lower temperature of the Sun's outer layers.

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SOLAR TEMPERATURE

Estimates of the effective temperature of the Sun were made by Newton in 1680, and by many others since the beginning of the nineteenth century. The results were extremely conflicting because of the very different arbitrarily-chosen principles and laws of radiation. These ranged from Newton's assumption that radiation and temperature advance at the same rate—twice the radiation twice the temperature and so on—to the generalized experimental results of Dulong and Petit in 1817 (on temperatures too low to be significant) that while temperature advances by arithmetical, radiation increases by geometrical progression. The consequent temperatures for the Sun had very diverse values, from ten million degrees Centigrade to less than two thousand degrees. The outcome of it all was that at the beginning of the present century the Sun's temperature was considered somewhat vaguely to be between about 6000 and 12,000 degrees C.

In 1879, J. Stéfan of Vienna had proposed a law that radiation increases as the fourth power of the temperature, and this was shown by Boltzmann many years later to have a good theoretical basis. Further researches, by W. Wien, defining the relation between the wave-length of maximum radiation and temperature, and by M. Planck giving the amount of energy at all wave-lengths radiated at a given absolute temperature, also became available in the early part of this century. From all three the effective temperature of the Sun turns out to be close to 6000° centigrade absolute (6000°K). As radiation from the centre of the Sun's disc comes from deeper layers, the rays from the limbs starting from higher levels (those below being absorbed more strongly by the greater thickness of gases to be tangentially passed through), 6300°K is considered to be the temperature for the centre and about 5000°K for the limbs. The temperature of a sunspot is about 4000°K.

The effective temperature of a star like the Sun has been defined by Eddington as a "conventional measure specifying the rate of outflow of radiant heat per unit area; it is not to be regarded as the temperature at any particularly significant level in the star." Nevertheless the degree of heat at about

the Solar photosphere is probably fairly closely measured by the value 6000°K. The temperatures lower down are believed to increase to a central value of millions of degrees, as the result of researches beginning with J. Homer Lane in 1870 and continued by many among whom may be mentioned Ritter, Kelvin, Emden, Schwartzschild, Eddington, Jeans and Milne (see p. 282).

Lane's pioneer work determined the tremendous gravitational pressures on the gases in the Sun's interior, and he calculated the temperatures and densities necessary to provide the expansive forces balancing these pressures. A. Ritter greatly extended Lane's results but, like his predecessor, he did not allow for factors which later research showed to be highly important, and A. S. Eddington in 1916, and other investigators, endeavoured to take them into account. One of these is radiation pressure which, varying as the fourth power of the temperature, may reach great values inside the Sun where the temperatures are so high; another is that the electrons are stripped from their atoms and assist, like the particles of a perfect gas, in withstanding the pressures of the overlying masses. According to Eddington and others the temperature at the centre of the Sun must be about 20 million degrees centigrade in order to balance the enormous pressure there.

SOLAR RADIATION

The quantity of energy that falls on unit time on unit area of a surface placed at right angles to the Sun's rays just outside the Earth's atmosphere is known as the "Solar Constant." It has been studied systematically since 1905 by C. G. Abbot and his collaborators. Abbot's average value for it is about 1·94 calories per minute per square centimetre, a calorie being the amount of heat necessary to raise the temperature of a gram of water one degree centigrade. Although termed a constant it is variable to the extent of two or three per cent and has been found to be greater when sunspots are numerous. The instrument used for its determination is the pyrheliometer, the measurement being made by permitting a beam of sunlight of known cross section to be completely absorbed by a body of known heat capacity (for example, a blackened silver disc),

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and noting accurately the consequent rate of rise of temperature. Corrections are made for any heat lost by the receiving body or for any received by it from other sources, also for atmospheric loss (30 per cent.)

Stations at which the day by day observations have been taken are at Mount Montezuma in Chile, Burro Mountain, New Mexico; Table Mountain, California. Earlier stations were at Mount Wilson, California; Mount Whitney, California and Bassour, Algeria. Abbot's conclusions after many years of observation and statistical analysis have been generally summarised as follows: The variation of the Sun's output of radiation, though small in percentage, appears to be a factor of importance in weather. By extensive statistical studies of the quarter century of nearly daily Solar constant values now available from Mount Mountezuma and the other stations it has been shown that the short interval fluctuations of three to five days in length seem to affect temperatures for at least two weeks following. The temperature changes are large, 10° to 20° Fah. It has also been shown that the Solar radiation varies in no less than 16 periods, about 7 months to about 23 years long, all nearly integral fractions of 273 months. These periodicities are associated with fluctuations of temperature and rainfall. The correlation at some stations is close enough to serve as a basis for long-range weather forecast. But all the foregoing needs verification.

In view of the Flare and Coronal phenomena, described earlier, it is interesting to note that the variation of Solar radiation is much greater at the violet end of the spectrum than at longer wave-lengths; Pettit's observations during 1924 to 1927 showed a fifty per cent range in the ultra-violet.

THE SOURCE OF SOLAR AND STELLAR ENERGY

The source of the Sun's energy has been attributed to various causes by theorists during the past hundred years. In 1848, J. R. Mayer of Heilbronn remarked that if the Sun had been composed of coal and set alight five thousand years ago, it would by now have completely burnt out. He suggested that instead of combustion, the energy of meteorites falling on the Sun was the effective cause. This was for a

time accepted by other physicists such as Lord Kelvin (1824-1907), but examination soon showed its inadequacy, and also that if it were adequate the Earth itself would be red hot from meteoric impacts.

The next hypothesis proposed was that of H. von Helmholtz (1821-1894), advanced in 1854. He developed a theory that the radiation from the Sun is due to its contraction. It possesses an immense store of energy in the form of the mutual gravitation of its parts ; if from any cause it shrinks, a certain amount of gravitational energy is necessarily lost and takes some other form, heat and radiation ; and a shrinkage of a few hundred feet annually, imperceptible for centuries, would be sufficient. Until the beginning of the twentieth century most scientists accepted this theory as correct. It was simple ; and computations of the possible age of the Sun were easy to make, giving about 20 to 30 million years for a past duration and about 10 millions for its future.

Among the first to question the adequacy of the "contraction theory" were the geologists. Later, their estimates of the age of the Earth from rates of deposition of strata were much reinforced by a method depending on the uranium-lead ratio of the rocks. The heavy elements uranium and thorium disintegrate spontaneously but gradually, their atoms changing into atoms of quite different sorts but ultimately becoming atoms of lead. The lead from uranium has atomic weight 206, and from thorium 208, and both can be distinguished, by careful chemical analysis, from ordinary lead, atomic weight 207. When lead of either sort is found in a mineral, it is taken to be certain that the lead has been formed by a radioactive change since the mineral crystallised from melted rock. One per cent of uranium is transformed in 66 million years. In this way the ages of minerals in the oldest geological formations are found to be about 1800 million years and the Earth's crust must be somewhat older[4] ; geologists have thus arrived at values of 2000 to 3000 million years, or about a hundred times as long as by Helmholtz's theory. A much more powerful source of energy is obviously necessary for this longer period than is provided by contraction, as the Sun is believed to be at least as old as the Earth.

It thus appeared more and more certain that the Sun, and

also the stars generally, contain within themselves the more powerful energy which is to last their lifetimes. Even as early as 1899, T. C. Chamberlin (a geologist) wrote the following astonishingly prophetic passage: "Is it safe to assume that the Helmholtzian hypothesis of the heat of the Sun is a complete theory? Is present knowledge relative to the behaviour of matter under such extraordinary conditions as obtain in the interior of the Sun sufficiently to warrant the assertion that no unrecognised sources of heat reside there? What the internal constitution of the atoms may be is as yet an open question. It is not improbable that they are complex organisations and the seats of enormous energies."[5]

It was thus generally realized that the source of energy must be sub-atomic, that is to say related to the behaviour of atoms and electrons. Three hypotheses have been suggested:—

(1) Radio-activity or the breaking down of more complex atoms into simpler elements.
(2) Mutual cancellation of protons and electrons, *i.e.*, annihilation of matter.
(3) The building up from simple elements of some of more complex atomic structure.

The first of these three involves the postulating of elements of higher radio-activity and greater weights than those known to terrestrial experience. Even if the Sun were made of pure uranium the radiation would be only about half what is observed and its life only a fraction of what seems likely to have been the case.

The second hypothesis, unlike the other two, depends on the Sun's interior temperature and pressure. This is because it involves the idea of collision between a proton and an electron so that their electric charges cancel each other and nothing is left but radiation of extremely short wave length which soon becomes transformed into longer waves of heat. Such collisions will be more frequent at high temperature and densities. It was contended, however, that stars under such circumstances would be explosively unstable and that the hypothesis was therefore unsound. It was also urged that the interior temperatures, high as they are, are much too low to cause mutual annihilation of matter and production of radiant energy.

In the case of (3) what is involved may best be illustrated by the transmutation of hydrogen into helium that may take place inside a star. The atomic weight of the hydrogen atom, which is composed of a proton and one electron, is 1·008 (oxygen's atomic weight being taken as 16·00). When a helium atom is formed from four hydrogen atoms, an atom consisting of a nucleus of two protons and two neutrons, and having two electrons revolving round it, is built up. The atomic weight is 4·004, not $4 \times 1{\cdot}008$ or 4·032, as might be expected since the mass of both hydrogen and helium atoms is practically all contained in the nuclei. There has thus been a loss of 0·028 or 0·7 per cent of the mass; the energy corresponding to this mass must have been set free during the process of combination.

This third hypothesis has received what is possibly decisive support, from the work during the past ten or twelve years of H. A. Bethe, C. F. von Weizsäcker, G. Gamov and others. Although details may require considerable revision, a probable source of the energy seems to have been revealed by an intensive study of the physics of atomic nuclei. The following brief account[6] gives in outline the theory as it now stands.

Lord Rutherford's experiments showed that the nuclei of the atoms of light elements could be transmuted into nuclei of different chemical properties by bombardment with α-particles. At stellar interior temperatures, α-particles, which are merely the nuclei of helium atoms, are probably quite common, heat having completely removed the orbital electrons.

Atomic nuclei and the atoms generally are bombarded by α-particles and other swiftly-moving projectiles—protons, neutrons and the like. These reactions are called thermonuclear, and they are mostly very sensitive to changes of temperature, so that each reaction may be associated with a definite critical temperature below which it occurs only very slightly, but above which is torrential amounts. This critical temperature depends greatly on the complexity of the nuclei involved and it is consequently lowest—500,000° to 1,000,000° —for the simple reaction of one proton with another. In this a deuteron (positive electron and two neutrons, the nucleus of a "heavy hydrogen" atom) is liberated. The ejected positron then encounters an ordinary negative electron with

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mutual cancellation and the production of radiation; and the deuterons also react with protons producing helium of mass 3 and radiation.

When these processes (which take place at the first stage of a star's life following that of initial gravitational contraction) have exhausted their possibilities, the star contracts owing to the lessening of energy necessary to keep it at its former size. This produces central temperatures of 2 to 9 million degrees, when protons may react with the nuclei of the light elements lithium, beryllium and boron; and later, there is further contraction and increase in temperatures. When temperatures at the centre, of the order of 20,000,000° are reached, there is the reaction of a proton with a carbon nucleus of mass 12. The combination forms a nitrogen nucleus of mass 13, but this is only the first of a series of processes, of what is termed the "carbon cycle." The nitrogen nucleus may capture a second proton, becoming an ordinary nitrogen nucleus of mass 14, and then a third proton, becoming a nitrogen nucleus of mass 15. This may capture yet a fourth proton, but the result is not nitrogen of mass 16; it is usually a carbon nucleus of mass 12, together with a helium nucleus of mass 4.

The foregoing description of the carbon cycle has omitted some minor processes which have no effect on the final result. The main events consist of the carbon nucleus absorbing four protons one after the other, and thereby being moved along the sequence of nitrogen isotopes* until this road comes to an end. In this way four protons are bound together to form a helium nucleus; all the other nuclei are unaltered. The carbon has acted as a sort of catalyst.

This transmutation may not appear to have any relation to the supply of energy for the star's radiation. But, as has been referred to above for hydrogen, there is a loss of 0·7 per cent of mass which goes off as radiation of short wave ("soft X-ray") to be absorbed and re-emitted by the interior material as longer wave "temperature" radiation constituting the star's luminosity.

*Practically all elements have varieties of their atoms identical in chemical properties, but differing in atomic weight owing to additional neutrons in their nuclei; these are termed "isotopes."

The various stages described are considered probably to provide the causes of luminosity of the stars as they increase in temperature.

A number of difficulties have yet to be overcome in connection with this attractive hypothesis and there will probably be modifications, some of them important, particularly in the case of the red giant stars and hot temperature high luminosity stars. And although there appears to be little doubt of the essential correctness of the basic conceptions of the hypothesis it may be that it will require modification if the hydrogen content of the stars generally is much greater than what has been assumed. In 1947, F. Hoyle gave reasons for believing the fraction to be as high as 99 per cent.

AGE OF THE SUN

From the foregoing a theoretical maximum age for the Sun may be derived. Assuming the proportion of hydrogen in the Sun to be a third (about what is usually assumed although as stated above a much larger proportion has recently been advocated), then 0·7 per cent, or seven thousandths of this mass of hydrogen will be radiated away as energy (energy possessing mass (see p. 205)). As the Sun's radiation is equivalent to 4,200,000 tons per second, the life found for the Sun is 35 thousand million years, if the present rate of radiation can be taken to be the average for its lifetime.*

$$\text{*Lifetime in years} = \frac{\text{Sun's mass} \times 7}{3 \times 1000 \times 4{,}200{,}000 \times \text{seconds in year}}$$

$$= \frac{2 \times 10^{27} \times 7}{3 \times 10 \times 4{\cdot}2 \times 10^{6} \times 3{\cdot}16 \times 10^{7}} = 3{\cdot}5 \times 10^{10}$$

References.

[1]See 'Telescopes and Accessories," Dimitroff and Baker, pp. 179-185 (cell photometry) ; and p. 272 (coating of lenses) (1945).

[2] *Journal of the British Astronomical Association*, vol. 55, pp. 143, 158 (1945) ; and vol. 57, p. 174 (1947).

[3] *Ibid*, vol. 50, p. 309 (1940).

[4] See "Astronomy," Russell, Dugan, Stewart, vol. i, p. 133 (1945).

[5] *Science*, June 30, 1899, p. 889.

* It is based on the writer's "Outline of Stellar Astronomy," pp. 80-82 (1947).

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CHAPTER XV

TWENTIETH CENTURY (*Continued*)

SOLAR PARALLAX

ATTEMPTS to obtain a more accurate value of the Solar parallax continued. Astronomers have always realized that this is one of the most important, if not indeed the most important, of the constants in the science. The scale of the whole universe, not only that of the Solar system, is fixed by it, and calculations of distances, masses, sizes and densities of planets or their satellites and of the stars themselves, depend on it. As a consequence enormous labour has been expended on its determination by astronomers.

The first opposition of the minor planet Eros to be used for the purpose was that of 1901 when its distance from the Earth was about 30 million miles. In the year 1900 it was agreed at an international conference held in Paris that observations of the planet would be undertaken at a large number of observatories. From the great quantity of data thus secured, A. R. Hinks published in 1910 a parallax of 8·806″, corresponding closely to the 8·80″ adopted internationally in 1900 to which a distance of 92,900,000 miles applies. Very similar results were obtained from the method involving line-of-sight velocities of stars (see p. 170) by Hough in 1912, and from gravitational methods by Noteboom in 1921 and H. S. Jones in 1924 (see p. 158).

But much the most accurate value as yet reached was published in 1941 by the Astronomer Royal, Sir H. Spencer Jones. This was derived from the opposition of Eros (see p. 175) in 1931 when its least distance from the Earth was only 16,200,000 miles. The close distance, the improvements in application of photography to precise measurements, and the unprecedentedly great number of high-class observations, resulted in a parallax of 8·79″, and a distance of 93,005,000 miles which is considered to be probably accurate within less than 10,000 miles either way. As the Astronomer Royal

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puts it : "One hundred years ago the distance of the Sun was uncertain to one part in twenty ; gradually the uncertainty was narrowed to one part in a hundred, and then to one part in a thousand ; now it has been reduced to one part in ten thousand." The still greater accuracy which will in time be got by radar methods (see p. 203) may hardly be necessary for any dependent astronomical quantity. The extraordinary accuracy of the latest determination will be evident from the fact that the total parallax angle itself (8·79″) is only about the difference in direction of an object nearly a mile away when looked at first with one eye and then with the other.

MERCURY

In planetary Astronomy the rotation period for Mercury of 88 days derived by Schiaparelli (see p. 171) has been supported by E. M. Antoniadi's observations with the 33-inch refractor at Meudon Observatory. He found markings on the disc similar in shape and position to those described and drawn by the Italian observer ; and in 1928 he published results in agreement with those arrived at nearly fifty years before. He also concurred with Schiaparelli's opinion that the atmosphere of Mercury is not considerable, and he noted markings which he thought meant the presence of obscuring dust clouds occasionally. In 1937 B. Lyot saw Mercury pass in front of the Solar Corona, with the coronagraph at Pic du Midi. On photographs taken at the time no traces of illumination at the edge of Mercury's disc could be seen. Lyot concluded that if there is any atmosphere it produces a refraction less than 1/30th of that in the Earth's atmosphere. The idea of tenuity of the atmosphere is supported by the spectroscope which shows an unmodified Solar spectrum. The temperature of the sunlit side of Mercury, permanently turned to the Sun's rays, was found by Nicholson and Pettit to be about 380° centigrade, or hot enough to melt lead. The surface of the planet has been found (1929) by B. Lyot at Meudon to reflect light which when polarized resembles that from a mixture of volcanic ashes. Mercury's "other side" must be intensely cold, but owing to variation in motion due to the comparatively great ellipticity of orbit there is a part

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of the surface at each side of the planet, where sunrises and sunsets occur, to which calculation gives breadths of 23½ degrees of Mercurian longitude. If Mercury does have an atmosphere of any considerable amount these two spaces must be areas of much disturbance because of great temperature variation. But Lowell believed the planet to be without atmosphere of any kind.

VENUS

In the case of Venus there is still uncertainty as to its rotation period. Schiaparelli and others (see p. 172) considered that they had found Venus to rotate in the same time as it revolves round the Sun, 225 days. Others favoured a period about the same as the Earth's, and W. H. Pickering announced in 1921 that he had found indications that it turns on an axis very nearly in the plane of its orbit in 68 hours. The latest evidence from the spectroscope is in accordance with a period of at least several weeks, as the limb velocity of any shorter period would cause displacement of lines greater than has been observed.

The markings on the surface are dusky spots very difficult to represent satisfactorily in drawings, giving the impression to most observers that they are not permanent. In 1927, F. E. Ross photographed the planet with the 60 and 100-inch reflectors at Mount Wilson, using ultra-violet light; he found that the markings thus photographed are much more conspicuous than to visual observation. Later in the same year Wright confirmed this with the 36-inch Crossley reflector at Lick, and found that the markings are practically absent in photographs taken in infra-red light. These ultra-violet photographs are inconsistent with a short rotation period; the markings tend to reappear in the same parts of the planet's surface as clouds do in certain regions of the Earth. Several observers' drawings in the past have shown bright caps at the north or south ends of the disc or crescent (the "cusps"). It is interesting to note that there is a persistent cap near the south cusp in the ultra-violet photographs. If this is a polar feature the inclination of the axis of rotation is small.

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Spectroscopic observations have led to the conclusion that if there is any oxygen in Venus's atmosphere, it must be less than a thousandth of what exists over an equal area of the Earth's surface, where the contained oxygen is equivalent to a layer, at standard temperature and pressure, more than a mile in thickness; and water vapour has not been detected. But in 1932 Adams and Dunham at Mount Wilson discovered three well-defined dark bands in photographs of the infra-red region of the spectrum which were later shown by Dunham and others to be due to carbon dioxide. The quantity indicated is very great, being equivalent to a layer two miles thick at standard terrestrial atmospheric pressure and temperature, 320 times as much as in terrestrial air. These quantities for Venus refer to an atmosphere which is probably that part above the reflecting cloud surface; below this the constitution of the atmosphere cannot be directly studied by us. Lyot's researches published in 1929 indicated that polarized light from the planet is like that from suspended small drops of liquid and is, therefore, reflected from an atmosphere.

Nicholson and Pettit found by thermocouple measurements of the radiation from the dark side that a considerable heat comes from there, indicating a temperature which although a low one (−25° centigrade), is not nearly so cold as would be the case if the planet had always one side turned from the Sun, and this determination consequently favours a period shorter than the 225 days of Schiaparelli. Earlier measurements of the radiant energy from the illuminated side, made by Coblentz and Lampland, suggested a temperature of 50° centigrade; but it does not appear that observations have been sufficiently undertaken to separate with certainty the part merely due to reflected sunlight and that coming from the warmed up surface and atmosphere. Calculations, taking into account the influence of the kind of atmosphere concerned, make it appear probable that that part of Venus's surface where the Sun is in the zenith has a temperature as high as 100° centigrade.

MARS

During the century much work has been expended on Mars, probably the most interesting of the planets. This

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has been carried out to some extent by amateurs, but professional investigations have also been active. At the Lowell, Lick, Mount Wilson and Meudon Observatories and other large establishments, telescopes of the greatest power have been used visually and photographically, so that a great amount of information has been obtained about the surface and the physical conditions of the planet. To give an account in detail would occupy more space than is available, but a summary of the position reached will be useful.

The Polar Caps are now believed to be composed of snow, but thin and containing very little water, and it has also been considered that they may be, at least partly, composed of clouds over the region during the winter season. Solid carbon dioxide ("Dry Ice") was suggested at one time but the temperatures measured do not suit for this substance. There are occasional patches of a cloudy nature seen (or photographed) on the disc, suggesting the presence of clouds, fogs, or haze, and projections beyond the illuminated edge (the "terminator") of the planet, evidently high-lying clouds, have been observed (Antoniadi, 1924, and others). Photographs in violet and in infra-red light were taken by Wright in 1924 at Lick Observatory. A comparison of these with photographs of a mountain range about 13 miles distant, through atmospheric haze which obscures the distant details of the picture in a violet photograph but has no such effect in a red photograph, suggests that for Mars red light penetrates to its surface. It is probably also indicative of an atmosphere of considerable depth that the violet picture of Mars is greater in radius than the red one by an amount corresponding to 60 miles. In 1937, however, E. C. Slipher took pictures of Mars, with blue and yellow light, and found surface details conspicuous for a few days with blue light, those taken in yellow light showing little alteration; this indicates a temporary clearing of the usual haze.

The latest evidence from the spectroscope shows that oxygen and water are not present in notable quantity in Mars's atmosphere, Adams and Dunham, in 1934, 1937 and 1943, finding that the amount of oxygen per square mile above the equatorial region does not exceed a thousandth of that on the Earth, the proportion for water vapour being a hundredth.

But carbon dioxide of an amount probably greater than that in the Earth's atmosphere is indicated by the presence of bands in the infra-red part of the spectrum, as found in 1947 by G. P. Kuiper with the 82-inch reflector at the McDonald Observatory in Texas. But the absence of bands due to chlorophyll in the spectrum of the dark areas seems to rule out anything like terrestrial green foliage, although not lichens or mosses.

The surface temperatures have been radiometrically determined both at Flagstaff (1924) and Mount Wilson later. In the equatorial regions at noon it is not unlike that of a cool bright day on the Earth and may reach to 10° or 20° centigrade (50° to 68° Fah.). The dark areas appear to be somewhat warmer than the lighter ones ; at the equator the temperature is below freezing at sunrise and sunset, and the nights must be cold. The Polar Caps go to as low as minus 70° centigrade but when they disappear in late Martian summer the surface there becomes about as warm as at the equator.

As has been shown by Lowell and others the apparent seasonal changes of the dark areas are closely related to those of the Polar Caps, and in spite of the apparently small amount of water in the atmosphere, these dark regions may be regarded as possibly vegetation. Colour changes in them, towards green when the Caps are melting, and brown or grey at other times, support this. The other areas may be desert. As Lyot finds that polarized light from Mars is similar to that from a volcanic cindery surface, it seems possible that those lighter parts, which are considerably more than half the area of the planet's surface, and, being brighter, give much more than half the light, are similar to volcanic cinders in their composition.

As for the canals, opinions still differ considerably. The existence of markings of the kind is now universally agreed to ; but on their number and nature there is still much diversity of opinion. Some believe that they are very numerous, and narrow, more or less straight, lines. Others think they are not so plentiful, and that they are really diffuse, with fine detail to be glimpsed even with the largest instruments only at fleeting moments of very good definition. As to the question of life on Mars, the verdict would still appear to be

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"not proven," and perhaps one may add "not likely," with regard to theories such as that of Lowell (see p. 174).

Two papers on the canals of Mars by Edison Pettit have recently been published.[1] The first gives his experience of observation of the planet with a 6-inch refractor in Pasadena, California, and a 20-inch reflector from the top of Mount Wilson. At first he could see no canals, but the outlines of the light and dark areas were obvious. Gradually several canals became visible one after the other at moments of finest definition, and more and more were noted, many in the positions of Schiaparelli's map, although Pettit's acquaintance with the canals of Mars was confined to the information "that they were supposed to exist." Pettit's drawings show them straight and narrow, and he states that at best moments he could see that they are green in colour.

In his second paper he examines the position regarding photographs of the planet taken from 1905 to 1939 with instruments ranging from the Lowell 24-inch refractor to the 100-inch reflector at Mount Wilson. In his earlier paper he had remarked that "although interesting pictures have been obtained, no photograph has shown the fine details described by visual observers. With the increased optical power and improved photographic techniques now available it may seem strange better photographs have not been made." The chief difficulty is the securing and utilization of the very best seeing conditions. Taking the picture exactly when these occur which, at the utmost, is only for a few seconds at a time, is essential; and large instruments, so that exposures can be very short, are absolutely necessary. With moderate definition and atmospheric steadiness, canals are visible only one or two at a time. To see the canals as a pattern covering the planet's disc the very greatest clearness and steadiness have to be awaited.

Pettit suggests that a fairly large instrument of 20 or 30 inches aperture be placed near one of the largest reflectors as a pilot telescope from which the man in charge of the photography with the big reflector would be informed by an assistant of approaching moments of "superseeing." He estimates that the exposures necessary with a 60-inch reflector for an image about a third of an inch in diameter would be about

a fifth of a second (yellow light) to 1·1 seconds (red light), these figures being reduced for the 100-inch and 200-inch reflectors to a sixteenth and four-tenths, and a sixtieth and one-tenth respectively. A series of pictures should be taken using a motion-picture camera with electric drive at the focus of the large instrument, so that the greatest possible number of the best moments could be captured.

Unfortunately, as the canals cannot be expected to be well seen or photographed until the apparent diameter of Mars is at least 20 seconds of arc (its maximum is 25″·1), the next occasions useful for the purpose will not be until 1954 and 1956. In these years, owing to the effect of the eccentricities of their orbits, Mars and the Earth will come fairly close to each other. It is much to be hoped that one of the largest instruments can be made available for the purpose even if the outcome only settles the positions, approximate numbers and apparent widths of the canals; although more may be possible, including a determination of their development (if any) with the progress of Mars's seasons, and even perhaps their colour.

From statements by E. Hubble in a lecture delivered in 1947 in Pasadena, California, it appears likely that the 200-inch itself will be applied to the investigation. It may interest readers to know that the diameter of the image at the focus of this instrument will be only about a thirteenth of an inch. This will probably be enlarged several times by a suitable system of lenses.

MINOR PLANETS

By 1940 the number of asteroids or minor planets for which observations had been sufficient to provide data for reliable orbits reached 1500. The number is only rather more than half what had been actually discovered until then, and this fraction with orbits determined is becoming less. For example in the ten years prior to 1940 less than a quarter of the 1537 discovered were provided with orbits. Those which are now being found are only of the brightness of stars of the thirteenth to fifteenth magnitude, that is to say they are roughly a three hundredth to a five thousandth as bright as those which were first dis-

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covered. But many more no doubt remain to be found. W. Baade and E. Hubble have independently estimated, from studies of the numbers found on photographs taken for other purposes with the Mount Wilson 100-inch reflector, that more than 30,000 could be photographed with that instrument if it were systematically used for the purpose.

Eros has been referred to as a minor planet that comes close to the Earth (within 14 million miles) ; but several others have been found within recent years that come much nearer. For instance, Apollo, found in 1932, is 6½ million miles from the Earth at its closest, Adonis, 1936, 1½ million miles, and Hermes, 1937, only half-a-million miles. These three were unfortunately lost to observation in a few weeks after discovery and may not be found again for a considerable time. At the best, however, their use in determination of the Solar parallax would be very difficult owing to their faintness and the short periods of time they spend near the Earth.

Several of the minor planets show periodic variation in brightness that is probably due to rotation of an irregular figure. For instance Eros varies in about 5¼ hours,* but the variation is sometimes more conspicuous than at other times. In 1937, F. G. Watson suggested that the light changes observed could be explained by assuming that Eros is shaped like a cylinder 22 miles long and a third of that in diameter, rotating about an axis at right angles to its length. Irregularity in shape is certainly suggested, and in fact this was seen in 1937 by van den Bos with the 26-inch refractor at Johannesburg. It is very probable that the smaller minor planets are irregular in form as their gravitational power may be insufficient to have compelled a spherical shape. In size the range of diameter for those discovered is from about 480 miles (Ceres) to a mile or two.

Twelve remarkable minor planets found during the present century are in the Trojan group, which are all named after Homeric heroes. Their mean distances from the Sun are nearly equal to Jupiter's. Each moves in such a way in its orbit as always to be relatively near to the corner of an equilateral triangle with the Sun and Jupiter at the other two

*Other minor planets which vary in light are Iris 6⅓ hours, Eunomia 3 hours, Sirona 6⅔ hours, Tercidina 8¾ hours.

corners. They therefore always remain at about the same distance from Jupiter. Seven of them precede and five follow that planet. They provide examples of a particular solution of the problem of three bodies under gravitation, announced by Lagrange in 1772.

The Japanese astronomer, K. Hirayama, showed that there are at least five families of minor planets for the members in each of which the mean distances and inclinations of orbits are similar, the centres of their orbits being equidistant from a certain definite point in the line joining the Sun with the centre of Jupiter's orbit. These features of the orbits in a particular family are what would exist, after gravitational disturbances by Jupiter, for bodies revolving round the Sun that are fragments of a mass which has exploded. There is great difficulty in conceiving the succession of explosions in different bodies necessary for this ; but the existence of these families, which comprise 133 members, seems to favour the old idea of the origin of minor planets by explosion of a large body (see p. 142) rather than that they are the unused material for a larger planet which has never formed.

In 1927, Leuschner made the suggestion that short-period comets may become minor planets after losing their gaseous material (driven off by Solar radiation pressure and electrical repulsion) and sometimes breaking into fragments, thus forming minor planet families.

JUPITER

The varying rate of rotation for Jupiter found from markings on its disc, those at the equator giving a shorter period than others in higher latitudes, has been known for a considerable time (see p. 176). This variation is however not of the regular type seen in the Sun, and appears to be more in the nature of changing currents in the belts. These belts and the zones between them are named, using terrestrial nomenclature, Equatorial, Tropical, Temperate and Polar ; but they are not quite constant in position or in the rotational speeds derived from them. These mean rotation periods range from about 9 hours 50½ minutes for the equatorial zone to 9 hours 55½ minutes just north and south of it, but the

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transition is rather abrupt from the value for the equatorial zone, and the mean periods to the north and south of it are very similar in the higher latitudes. Periods got from individual spots or markings range, however, from 9 hours 48 minutes to 9 hours 56½ minutes.

Systematic observation of the various motions in the different Jovian latitudes was practically originated by the English amateur, A. Stanley Williams, who published in 1896 the results of a long series of observations showing the existence of eleven surface currents. The spots belonging to them were usually changeable in form and transient, all being clearly atmospheric phenomena. Several markings are long lived and have been observed for many years. The chief of these is the Great Red Spot (see p. 176) about 30,000 miles long by 7000 broad, at some times very red and conspicuous, at others losing its colour and only showing, by a large hollow in the south Equatorial Belt, the place it usually occupies. Another marking has persisted since 1901 and is referred to as the South Tropical Disturbance. The rotation period derived from it is irregular but on the average it is shorter than that from the Red Spot, so that it overtakes the latter in from two to four years intervals. Although the Red Spot itself has been so long lived, it cannot now be part of any solid body of Jupiter as the rotation period derived from it is also not constant, having been 9 hours 55⅔ minutes between 1883 and 1910, but five seconds shorter during the subsequent fifteen years. As regards permanence of markings, it is seldom that one is seen at more than two consecutive oppositions of the planet, but several have been recently noted lasting longer than usual, all of them in the belt or zone just south of the latitude of the Red Spot.

Most of the work in determination of the velocities of currents and changes in the surface markings has been performed by the amateurs who have formed the Jupiter Section of the British Astronomical Association, with the late T. E. R. Phillips as their Director and chief inspirer. Greatly improved photographs of the planet have been secured in recent years by E. C. Slipher of Lowell Observatory, W. H. Wright at Lick, B. Lyot at the Pic du Midi, and at Mount Wilson, sometimes in light of different wave-lengths. If these can be systemat-

ically continued in conjunction with careful visual work, much valuable information of the surface features, perhaps including some idea of their heights in the Jovian atmosphere, should be obtained.

The details of the spectra of the four giant outer planets were first satisfactorily shown by photographs on red-sensitive plates taken by V. M. Slipher at the Lowell Observatoty, Flagstaff, in 1907; and the interpretation of the spectrum of Jupiter with its bands in the orange and red (which are progressively stronger in Saturn, Uranus and Neptune) was given twenty-five years later by R. Wildt. He showed that these bands are produced by methane (Marsh Gas or Fire Damp) and ammonia, the amounts present being respectively equal to layers of about half-a-mile and thirty feet thicknesses at standard temperature and pressure. Radiometric work by Coblentz in 1914 and 1922 gave 140°C. below zero as the average temperature of the surface, not much different from what would be expected if the surface is warmed only by the Sun's radiation. Such a low temperature is in accordance with the theoretical results of H. Jeffreys in 1923.

It is now thought that the atmosphere of Jupiter probably contains frozen crystals of ammonia with gaseous methane and some ammonia gas (sublimed from the crystals). "We may conclude that hydrogen is so abundant that it combines with all the carbon and nitrogen present [making methane (CH_4) and ammonia (NH_3)]. Probably it has combined also with the available oxygen to form water (H_2O) but the water has frozen and sunk to the bottom of the atmosphere out of sight. . . . Free hydrogen gas probably constitutes the major part of the atmosphere, but it is undetected in the spectroscope; hydrogen gas, when cool absorbs light only in the extreme ultra-violet, where our atmospheric shield is completely effective."[2] The results of polariscope observation by Lyot, published in 1929, are in agreement with the presence of a cloudy atmosphere. The causes of the red, brown, bluish and even greenish colours observed in the belts and markings are not known, but Wildt has suggested that sodium or potassium in combination with ammonia may be responsible. Variation of the colours of the North and South Equatorial belts in a period of 12 years so that one belt is most coloured

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when the other is least so, has been found by Stanley Williams (1899) but this has not yet been confirmed.

According to Wildt the internal constitution of the planet is a rocky metallic core about 44,000 miles in diameter, with a layer of ice above it about 16,000 miles thick and above that again a layer 6000 miles thick composed of compressed hydrogen (chiefly) and helium, etc. But this is of course very speculative as the behaviour of substances under the enormous pressures concerned is not known.

The general view now is that the surface markings are produced by eruptions from below consisting of dense viscous gases, the difference in rotation periods derived from the various spots and markings being caused by currents in the Jovian atmosphere. One of these currents, a great one in the equatorial zone, moves to the east at 250 miles per hour and other eastward movements about half as fast have been noted. R. Wildt has suggested (1939) that the Red Spot, taking into consideration its evident movement on Jupiter's surface, may be a solid body floating in an "ocean" of denser permanent gases, and B. M. Peek considers that it may be a detached part of Wildt's lower frozen layer mentioned above.

JUPITER'S SATELLITES

Jupiter's known satellites now number eleven, six having been found, all by photography, since Barnard discovered Satellite V in 1892 visually with the 36-inch refractor at Lick Observatory. VI and VII were discovered by C. D. Perrine in 1904, VIII by P. J. Melotte in 1908, IX by S. B. Nicholson in 1914, X and XI by Nicholson in 1938, VI, VII and IX were found with the 36-inch Crossley reflector at Lick, VIII with the 30-inch reflector at Greenwich, and X and XI with the 100-inch at Mount Wilson. The direction of orbital motion for VIII, IX and XI is opposite to that of the others which all revolve in the direction usual for planets and satellites in the Solar system. All the Galilean satellites, I, II, III, and IV, show markings on their surfaces to observers with large telescopes, and from these the rotation periods for III and IV (the two largest) have been ascertained. All four have been found by Guthnick (1914) and Stebbins (1926) to vary regularly

in brightness with position in their orbits just as would be the case if their surfaces are of a spotty nature and their revolution and rotation periods the same as with our Moon and the planet Mercury. This relationship is attributed to the powerful tidal action of Jupiter which has slowed down or quickened any original rotation period until it has become equal to the time taken in the orbital revolution. The diameters of the Galilean satellites are large enough for measurement micrometrically with large telescopes. This has been done by W. Struve, A. Secchi, Engelmann, E. E. Barnard and others; the values found average about: I, 2400 miles; II, 2000 miles; III, 3500 miles; and IV, 3300 miles. All eleven of the satellites have been observed visually except X. Estimates of the diameters of the seven smaller moons give from about 100 miles for V down to about 10 miles for the faintest, X; these figures are based on brightness and an assumed surface reflectivity. The distances of VIII, IX and XI from Jupiter are so great that, through perturbations by the Sun, the orbits are not even approximately elliptical.

SATURN

As compared with Jupiter, the study of the surface phenomena of Saturn is handicapped by greater distance and considerably smaller solar illumination, roughly twice as far and a third as bright. Consequently no great detail has been noted in Saturn's belts although variation in position, number and width has been always a feature, and spots have been occasionally observed. Drawings made by skilled observers using large telescopes, and photographs with the giant refractors and reflectors, agree in the general outlines and show these changes. For example on a drawing made by Barnard in 1898 with the Yerkes 40-inch refractor there are in addition to a dark Polar Cap, no less than five dark belts, four of which are north of the planet's equator (only one belt south of the equator being visible owing to the position of the rings), while another picture by the same observer and telescope shows, in 1907 when rings were edgewise to the Earth, only two dark belts on each side of the equator; and even a smaller number of belts have been noted with powerful instruments.

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The appearance of short-lived spots, and the rotation periods derived have already been mentioned (see p. 177). Since 1900 others have been noted, one in 1903 by Barnard at 36° north latitude giving a period of about $10\frac{2}{3}$ hours, while Phillips found a similar value from a spot in the same latitude south in 1910. Another was discovered by W. T. Hay in London, 1933, which confirmed the shorter equatorial period of $10\frac{1}{4}$ hours. In 1939 spectrographic observations by J. H. Moore at Lick demonstrated that the rotation periods increase steadily (unlike in Jupiter) from about $10\frac{1}{4}$ hours at the equator to nearly an hour longer at 57° latitude, so that if the period at 36° is taken as the standard, an eastward current of nearly 900 miles per hour exists at Saturn's equator, more than three-and-a-half-times as great as that of the equatorial zone current in Jupiter !

The photographs taken at Mount Wilson, Lick, Flagstaff and elsewhere show the Polar Caps, belts, Cassini's division between the rings, the greater brightness of the inner ring, the shadow of the ball on the rings, and the Crape ring where it crosses the ball. In 1915, R. W. Wood found that on photographs with red light, markings on the ball were almost absent, with yellow light more appeared, and with violet or ultra-violet the belt markings and Polar Caps became conspicuous. The darkening at the edges of the ball was, however, less noticeable in the violet, and the rings were faint in red and brighter in the violet.

Radiometric observations by Coblentz gave a temperature about 150° centigrade below zero. At this low level nearly all the ammonia should be frozen out of the atmosphere of the planet, leaving methane more prominent in the bands of the spectrum than in Jupiter, and this has been found to be the case. Wildt considers that the interior structure and constitution must be similar to Jupiter's with a smaller rocky metallic core, the ice layer less than half as thick, but the compressed hydrogen, etc., layer more than three times as deep, relatively to those features of Jupiter's interior, the outermost atmosphere being constituted as in the larger planet but colder and with less gaseous ammonia.

Lyot's polarization researches, published in 1929, showed that the light from the ball is consistent with a cloudy atmos-

phere, but that the rings gave quite a different result, the inner bright ring indicating polarization similar to that from the commonest terrestrial rocks, the outer showing a polarization unlike that of any of the terrestrial materials he examined.

As regards the constitution of the rings, the innermost or Crape ring is transparent, the outlines of the ball being always visible through it ; and the outer ring can also be seen through, as the light of a seventh magnitude star was visible through it to M. A. Ainslie and J. Knight in 1917. The bright ring has also been found to be to a lesser degree transparent, W. Reid having seen a star pass behind it in 1920 without ever becoming entirely obliterated. That the thickness of the ring system must be extremely small compared with its width, has been proved by its disappearance when edgewise to the Earth, even with large telescopes, as was observed in the past, and again in 1907 and 1921 for several days. It is considered that the rings cannot be as much as 50 miles thick and that they are probably much thinner. A number of divisions in the rings in addition to Cassini's have been reported by several observers. The best known and oftenest seen, is that in the outer ring named after Encke, but it does not appear to be clearly a division, being possibly a thinning out among the particles and a variable feature ; and this may be the explanation of the others visible from time to time.

SATURN'S SATELLITES

The number of Saturn's satellites has not been added to since Pickering found the ninth in 1898 ; but the same astronomer reported that he had found another faint one in 1905, the discovery of which has not been confirmed. The largest, Titan, has according to G. Struve (1933), a mass about equal to twice that of our Moon. Two of the others have been calculated by the same astronomer to have masses of the order of a thousandth and a two thousandth that of the Moon ; these are the smallest celestial masses ever measured by gravitational attractions and mutual perturbations. Barnard and Lowell were able to measure Titan's diameter micrometrically ; it is about 2600 miles. The diameters of the others are, from their brightness, estimated to range from

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about 1000 to 200 miles. Several besides Iapetus (see p. 74) vary in their light and it is considered to be likely that at least the inner ones revolve and rotate in the same periods as Jupiter's moons do.

In 1944, G. Kuiper of the McDonald Observatory, Texas using the 82-inch reflector there, found that the spectrum of Titan is rather like that of Saturn itself, the bands of methane being certain and those of ammonia suspected. No trace of the methane band was found in the spectra of Jupiter's large satellites, however.

URANUS

The observations of Uranus referred to in Chapter XIII (see p. 178) were apparently trials of what the new large refractors could show. Few similar observations seem to have been made since then, although W. H. Steavenson and R. L. Waterfield in 1915 and 1916 with a 10-inch refractor saw a broad white equatorial zone and a dusky belt on each side of it which appeared to change their angle of position on the disc. No definite markings for direct measurement of rotation period have been recently noted, but the approximate period of 10 hours from observation of a bright spot with the Nice 30-inch refractor in 1884 has been confirmed by the spectrographic observations of Lowell and Slipher at Flagstaff in 1912. They found $10\frac{3}{4}$ hours, or a few minutes shorter than derived from L. Campbell's photometric observations in 1917, when periodic light variation of a seventh of a magnitude was observed. Up to 1948 no satellites additional to the four discovered by W. Herschel and Lassell had been found. But in February of that year a faint satellite of 17th stellar magnitude was found by photography with the 82-inch reflector at the McDonald Observatory, Texas. It is at rather less than two-thirds of the distance from its primary of Ariel, the closest of the four larger moons, and has a period of revolution of about 34 hours. The four bigger ones, Ariel, Umbriel, Titania and Oberon are fairly large bodies ranging from about 400 to 1100 miles in diameter judging by their brightnesses.

The spectrum has heavy bands in the red, orange and green similar to, but stronger than those of Jupiter and Saturn. All these bands are due to methane, and they are intense enough

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to be equal in effect to about four miles thickness of that gas at standard temperature and pressure. The internal constitution is no doubt similar to that of Jupiter and Saturn.

NEPTUNE

Neptune's greenish disc has shown no definite markings to provide a determination of rotation period. But this has been spectrographically found by J. H. Moore and D. Menzel at Lick to be 15¾ hours. Maxwell Hall's observation (see p. 179) of variability in a period of slightly less than 8 hours was confirmed by Öpik and Livlander at Tartu Observatory in 1922-3; it is probably due to the existence of two brighter parts of the planet's surface situated on opposite sides.

In Neptune's spectrum there are exceedingly strong bands of methane. So much of the red and orange of the spectrum are obliterated by them as to account for the greenish colour of the planet. It is estimated that their absorbing effect is equivalent to that of a layer of the gas at standard temperature and pressure no less than 25 miles thick. The presence of these bands indicates a temperature which must be about 45 degrees higher than that due to the Solar radiation only (about 210°C. below zero), so that some internal heat, even from the low temperature layers of the interior, is evidently escaping.

Neptune's single known satellite,[3] Triton, is comparable with the largest of Jupiter's satellites, or with Saturn's Titan, in size and mass. Alden has found (1942), from its gravitational effect on Neptune itself, a mass about the same as Titan's or twice that of the Moon; and Kuiper, using the 82-inch reflector of the McDonald Observatory, Texas, suspects bands of methane in its spectrum.

PLUTO

After the discovery of Neptune by Adams and Leverrier in 1846 many astronomers attempted to find still more distant planets. Two methods were tried: one by means of irregularities in the motion of Uranus still apparently outstanding (Neptune having as yet moved through too small a part of its orbit since discovery to be useful for the purpose); another,

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by the possible effects of a trans-Neptunian planet on comets' orbits. In the latter it was thought that there was evidence of the existence of "families" of comets having their aphelia in a position suggesting outer planets with the same relation as Jupiter has to its comet family (see p. 181). In 1880, G. Forbes suggested that two such families with aphelia at 100 and 300 times the distance of the Earth from the Sun, owed their existence to planets with periods of revolution of 1000 and 5000 years respectively; but searches by photography for the nearer of these were not successful. The most thorough investigation by the first method was made by Lowell who published his results in 1915. Preliminary photographic work and some search activities were carried out at Flagstaff Observatory from 1905 until Lowell's death in 1916 when the search was discontinued for thirteen years.

This was resumed in 1929 with a specially constructed star camera having a doublet lens of 13-inches aperture. The task of examining the very many thousands of star images would have been impracticable without the aid of an instrument, the "blink microscope," with which two photographs of the same part of the sky taken at different times are rapidly viewed alternately by movement of a lever, when any motion or change in brightness of an image becomes obvious. About two million images had been examined in this way by C. W. Tombaugh when he found, on February 18, 1930, a faint object that had changed its position in six days by about the amount a planet at the predicted distance should have moved. After further photographs and examination for two months the discovery of a trans-Neptunian planet was announced, and the name Pluto (suggested by H. H. Turner of Oxford) was given to the new object.

By means of positions from earlier photographs of the planet found on plates some of which had been taken as far back as 1914, an orbit was calculated giving a period of 248 years and a mean distance from the Sun 39½ times that of the Earth. The eccentricity of the orbit and its inclination to the Ecliptic are greater than for any known major planet.

Although this orbit is remarkably similar to that predicted by Lowell's calculations and also to one worked out by W. H. Pickering by simpler but less rigorous mathematical methods,

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it has been shown that the new body could not produce sufficiently large perturbations of Uranus to have been discovered by Lowell's calculations, essentially correct as they were. The agreement seems to have been no more than a most extraordinary coincidence, but the discovery of the planet was nevertheless even then undoubtedly due to the theoretical research of Lowell and to the observational work of the Observatory he founded.

Pluto is seen as a star of about 14½ magnitude or nearly the limit visually of a twelve inch aperture telescope. It is yellowish in colour and therefore nearly a magnitude fainter photographically. In 1944, D. Brouwer and L. R. Wylie were able to find a value for its mass through its action in pulling Neptune slightly out of its orbital plane. This mass is about eight-tenths the Earth's, or only about an eighth of that found by Lowell. With reasonable assumptions as to density, its diameter is estimated to be not much different from that of the Earth ; and its angular diameter in that case should not be much less than half a second of arc which, because of faintness of solar illumination (only 1/1500th of the Earth's) would be discernible only with the very largest instruments under perfect conditions of seeing. In this connection it may be noted that V. M. Slipher found (1930) at the Lowell Observatory, Flagstaff, by experiment, on a distant artificial target of graduated illumination, made with a 24-inch refractor, that a disc illuminated like Pluto's would not be visible with that aperture, under good conditions of seeing, even if six-tenths of a second in diameter. All that is so far known seems to suggest that Pluto resembles the terrestrial planets in its constitution rather than the group of giant planets.

COMETS

During the first forty years of the century about 200 Comets were detected, four of which were classed as "bright." In 1902 the objective prism (see p. 187) was used for the first time in study of the spectra of comet's tails, by de la Baume Pluvinel. He was not successful in identifying any of the elements concerned, but Fowler and also Baldet produced spectra in the laboratory which showed that the chief bands

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of the tail spectra are from carbon monoxide molecules under very low pressure and nitrogen molecules, both having lost an electron (ionised). Other identifications were made later by Nicolet (1938) who found a compound of hydrogen and carbon (CH). This was confirmed in 1940 on spectrograms taken at the MacDonald Observatory, Texas, where other bands due to compounds of oxygen and hydrogen (OH) and nitrogen and hydrogen (NH) were also detected. Later a number of additional identifications by Swings, Herzberg, McKellar and Minkowski were made. Our knowledge of the chemical composition of comets may be summed up as follows : one atom, sodium, has been found, and 10 or eleven molecules the elements in which are hydrogen, carbon, nitrogen and oxygen. All have been found in the heads of comets, and several only near the nucleus. The three molecules which are ionised predominate in the tails where the density is exceedingly small. All the molecules found are those that would be expected from laboratory study of the gases contained in meteorites.

It is the heat of the Sun which causes gases to leave the solid masses or particles in the head of the comet ; these gases, along with dust particles borne by them, diffuse outwards and form the coma and tail. As comets go near to the Sun the tails are formed from the smaller particles, which are repelled and lost to the comet. The result is that comets of short period, being frequently subject to the Sun's action, gradually lose mass and brightness.

It is now considered that the luminosity of a comet is, apart from Solar light reflected or scattered, due to fluorescence, produced by Solar radiation. When ultra-violet light shines on certain substances they give off visible light. The energy of activation is released as a short-lived whitish glow, the wave-length of which is characteristic of the substance and longer than that of the exciting radiation. The brightness of a comet varies with two factors, its distances from the Earth and from the Sun. By the former its light to us varies inversely as the square of the distance ; but by the latter, on which the effective strength of Solar excitation depends, it is inversely as from the third to the fifth or even higher power. Comets therefore behave in regard to brightness

with irregularity during their appearances, and also as between one comet and another. Some attempt is now being made to relate sudden increases of brightness to the outbursts of Solar ultra-violet radiation. It is interesting to note that von Zach humourously advised the comet hunter Pons that comets would be found when sunspots are numerous. Pons, a simple-minded man, took the advice seriously and with success! Without attributing this success to the spots, it may perhaps be the case that more comets may be made visible when sunspots and the accompanying ultra-violet radiation are abnormally great. In this connection it may be significant that a relation between number of sunspots and the brightness of Encke's comet from 1822 to 1908 appears to have been found by M. J. Bosler.

The chief, if not the only force producing a comet's tail, appears to be radiation pressure as suggested by Kepler hundreds of years ago. This idea was revived by the Swedish physicist Arrhenius at the beginning of the century. Generally for bodies of normal size, radiation pressure is negligible compared with gravitation, but when the diameters of the particles are of the same order of size as the wave-length of light it becomes important. It must exert a repulsive force more than ten times the Sun's attraction on particles about one hundred thousandth of an inch in diameter and the pressure is at its greatest when the diameters of the particles are about a third of the wave-length of the radiation. This maximum agrees well with Bredichin's repulsive force for the straightest tails (see p. 179). But application of these theoretical results to the known materials in comets' tails does not always give accordant results, a larger force of repulsion being sometimes indicated. It is therefore not certain that formation of the tails is due solely to radiation pressure.

Masses of comets are now known to be relatively very small. By a method based on the amount of reflected sunlight indicated by the observed continuous spectrum, and by another founded on the necessary gravitational power for them to hold together, rough estimates have been made. Using the first of these, reinforced by calculation of the reflecting surface indicated by the rate of evaporation of gases from the nucleus as shown by the bright line spectrum, the

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Russian astronomer Vorontsov-Volyaminov found in 1946 a mass of about 1/200,000,000th that of the Earth for the nucleus (*i.e.*, most of the mass) of Halley's comet. By the second method Russell found about one twenty-fifth of that value as the *minimum* mass to hold the comet together. The calculations are admittedly very rough but they seem to demonstrate the extremely small mass of a comet as a gravitating member of the Solar System.

The problem of the origin of comets is still awaiting even a plausible attempt at a solution, but it can be said that all known comets are quite probably members of the Solar System and that they always have been. The evidence for this is as summarized by S. E. Strömgren (1945) : "(1) The great majority of comets have pronounced elliptical motion ; (2) Through calculations of the original orbits of the suitable comets, with orbits very close to a parabola, it has been found [allowing for planetary perturbations] that the majority were originally elliptic. As matters now stand, not a single case of certain hyperbolic motion remains. It cannot be guaranteed, of course, that some time in the future a comet will not move into our Solar System from interstellar space. We can say, however, that the investigations indicated above without a doubt point in the direction that the system of comets known at present actually belongs to our Solar System."

During the present century photographic methods of observation have developed with great increase of accuracy in recording cometary details. A number of remarkable comets appeared, more than forty of which were recorded as visible to the naked eye, and of these seven were striking. Four were abnormally fine objects : one in 1901 (visible only in the southern hemisphere), another in 1910 (known as the great "daylight comet"), Halley's in 1910 (much better seen in southern latitudes, the Earth grazed its tail on May 21), and one in December, 1927. Another comet remarkable for a different reason was that discovered by Schwassmann and Wachmann in 1927. It has an orbit of small eccentricity, revolves round the Sun in 16·3 years, and at its nearest is actually further from the Sun than Jupiter ; it can be photographed in every part of its orbit and must be large for a comet to be thus observable. Although always relatively distant

from the Sun it has large and irregular light fluctuations. Morehouse's Comet of 1908 may also be mentioned for the extraordinary changes in its tail even from hour to hour, suggesting almost explosive ejection of gases from the head, possibly the result of fragmentation of molecules by Solar radiation, as radiation pressure is probably much more effective on isolated luminous molecules of gas which are absorbing energy from sunlight.

Before leaving the subject of comets some reference should be made to the great progress in the accuracy of orbital calculations for periodic comets and in the allowances necessary for disturbing effects of the planets. To illustrate this the predictions of the return of Halley's comet, period (average) about 76 years, to its perihelion passage may be instanced. Since Halley himself, in 1705, made the first prediction of its return at the end of 1758 or beginning of 1759, which was several months too soon, the best prediction at each return has been : 1759, Clairaut, 32 days late ; 1835, Rosenberger, 5 days early ; 1910, Cowell and Crommelin, $2\frac{3}{4}$ days early. And the two last-named computers considered that the $2\frac{3}{4}$ days discrepancy was not due to errors in calculation or in the assumed masses of the planets, but to "some small disturbing cause at work whose character is not yet recognized." That it could not have been due to the later-discovered Pluto has been shown by R. S. Richardson. These $2\frac{3}{4}$ days correspond to about 7 seconds of the time taken for a continuous express train journey from London to Edinburgh and back !

METEORS

In 1899 W. F. Denning published a catalogue of 4637 meteor radiant points. The number has, however, been much reduced by later observers so that a list by C. P. Olivier, up to 1920, gave only 1200 in half of which he had confidence. These showers from radiants are exclusive of the enormous numbers of isolated sporadic meteors seen at all times of the year. Study of the paths, positions of radiants, and dates of occurrence, and heights and velocities, is usually by amateurs, members of such societies as the American Meteoritical Society, the Meteor Section of the British Astronomical Association, and other organisations in Russia and Germany.

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But in 1931-33 an expedition to Arizona was organised by Harvard and Cornell Universities which made observations of nearly 22,000 meteors using a specially designed instrumental technique to obtain greater accuracy than formerly. The heights of more than 3500 meteors were determined, the average at appearance for all brightnesses being about 60 miles, and at disappearance from 40 to 52 miles according to brightness, the fainter meteors dying out at a higher level. According to Öpik, who dealt with the collected results, the velocities of two-thirds of more than 1400 meteors exceeded the maximum which can belong to a body moving round the Sun at the Earth's distance, indicating that they are visitors from outside space; the other third are members of the Solar System.

But these results are not universally accepted as correct. In 1945, J. G. Porter pointed out that Öpik's material was so inaccurate that he had to make assumptions in order to obtain statistical heights from the Arizona results. Porter investigated many meteor observations from which about 800 accurate paths were got, and found no reliable evidence of the existence of velocities corresponding to extraneous meteors. He also found that the heights, with which velocities are strongly correlated, are the same generally for shower and sporadic meteors, and remarked that the Arizona velocities, obtained with some form of rotating device, are roughly three times those got by ordinary eye observation and that he could only conclude that there is an anomaly here requiring investigation.

It would appear that photographic methods may be necessary to decide; so far, accurate velocities have been thus obtained for only a few meteors, all of which belonged to orbital movement of meteors in short periods of revolution round the Sun.* *A priori* it does seem rather unlikely that two out of three meteors are from outside the Solar System, when at the same time it appears probable that there are no known comets (bodies evidently containing large numbers of meteoritic particles) which have come from outside space.

*Among 45 meteor orbits determined up to 1948 at Harvard College Observatory from photographs taken at two stations, none with the hyperbolic velocity of an entrant from space outside the Solar System was found.

F. L. Whipple, of Harvard, considers that there is no significant difference between the nature of the orbits of sporadic meteors, of those from recognised showers, and of those for short period comets. He has proposed (1948) a working hypothesis that the brighter meteors seen visually and those so far photographed (which are necessarily also the brighter ones) may be from cometary débris; but he thinks that entrants from outside the Solar System may be present among the fainter meteors. On the other hand *meteorites* may be from the débris of a disintegrated planet.

The spectra of about fifty meteors have been accidentally obtained during objective prism photography of the stars. Bright lines of iron, nickel, calcium, magnesium, etc., have been identified, calcium predominating in some and iron in others, probably corresponding to the division of meteorites (fallen meteors) into the stony and iron classes.

The sites and the probable accompanying phenomena of falls of large meteorites such as the Great Siberian one in 1908 and others in Arizona, Texas, Central Australia, Arabia and the Argentine, at dates hundreds to thousands of years ago, have received much attention and study. Records of falls of all meteorites for the past 150 years have been investigated and the numbers have been found by J. A. Russell (1947) to increase from about seven per decade (1790-1799) to ten times as many (1930 to 1939), the largest numbers per year being for 1868 (11) and for 1933 (15). This increase is, however, attributed to the general improvement in the human factors influencing the number of *observed* meteoritic falls. Chemical analysis of meteorites has shown the presence of about 75 of the known elements; the relative abundance is much the same as in the Earth's crust.

ORIGIN OF SOLAR SYSTEM

Reference has already been made to the ideas of Kant and Swedenborg; also to the "Nebular Hypothesis" of Laplace and to its defects (see p. 94). A complete history and description of the speculation on the origin of the Solar System would take much more space to deal with satisfactorily than is here available. But several important ideas of a different

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type have been advanced and these will now be briefly described.

The first person to put forward the suggestion that the Solar System had been generated as the result of the Sun's disruption by another body was the Comte de Buffon in his "Natural History" (1750). He realised that the common direction of revolution among the planets seems to mean a common origin, and he suggested that a comet, of much greater mass than usual, had grazed the Sun causing an ejection of matter revolving round the Sun in the direction of the comet's movement, this matter condensing into the planets. Laplace very briefly criticized the theory in his "Système du Monde," pointing out that there was no explanation provided for the small eccentricities of the planetory orbits. In 1880 a similar theory was advanced by A. W. Bickerton who introduced a star in place of the comet of Buffon. He also introduced the idea of the formation, by the impact, of a nebula surrounding the Sun, and referred to the effect of the resistance of this nebula in producing the small eccentricities of the orbits. In addition, he mentioned a process of gradual collection of smaller ejected bodies by the gravitational attraction of the larger masses, thus producing very massive bodies such as Jupiter and the other major planets.

In 1870 R. A. Proctor outlined a hypothesis of the growth of a planetary system from innumerable meteor streams revolving round a central body. The existence of these streams was established, he considered, by the large number of observed periodic meteor showers, a number clearly very small compared with the actual as it only comprises these meteors in the space included by a few thousand miles around the line of the Earth's orbit, and very much smaller than the number which would have existed before their capture by the planets. As will be noted Bickerton had adopted the idea of aggregation as part of his hypothesis. Both this idea and that of the previous action of an encountering body, acting by disruption instead of by collision, were part of the Planetesimal Hypothesis of T. C. Chamberlin and F. Moulton originally proposed in 1901. This hypothesis involved an approach to and disruption of the Sun by another star with ejection of a relatively small mass of solar material. In consequence of a

lateral motion given to it by the intruding star this material revolved round the Sun in the same direction and in much the same planes, and mostly condensed into small bodies, Planetesimals, that aggregated into the planets, possibly using as nuclei some of the larger fragments which had remained partly gaseous or liquid.

Jeans (1916) and Jeffreys, independently developed hypotheses of tidal disruption, by a passing star, a filament of matter having been drawn from the Sun that condensed into the planets; and in 1929 Jeffreys revived Buffon's idea of a grazing collision between the bodies to account better for the rotations of the planets. It may be noted that in all these hypotheses it is assumed that some of the ejected material fell back into the Sun with the effect of giving that body a quicker rotation at its equator. In 1936, R. A. Lyttleton advanced a hypothesis which assumed that the Sun was once a double star and that an intruding star encountered the Sun's companion, leaving at least part of a filament between them, within the Sun's control, from which the planets were formed.

Objections and difficulties have been advanced to all of these ideas and the question of the origin of the Solar System has been left just as much a puzzle as ever. None of the proposals account for the comets, with their different directions of orbital movement and variously inclined orbits. And Lyman Spitzer (1939) has made calculations which, it is claimed, show that the planets cannot have been formed from material violently torn from the Sun. At a depth below the Solar surface above which there would be sufficient material to provide for the planets' origin by either tidal disruption or collision, the temperatures are of the order of millions of degrees. Gases at this temperature, quickly set free from the constraint of Solar gravity, would within a few minutes of their release actually explode into space long before they would condense by cooling. Spitzer maintains that even a mass twice that of Jupiter would not suffice to hold gases together to let them condense, and that therefore, planets could not be formed directly out of matter forcibly drawn from the Sun.

There have been a number of other hypotheses recently advanced not involving the approach to or collision of another

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star with the Sun. Among these may be mentioned the idea advanced by Hoyle that the companion of a double star, the other component of which has burst out as a supernova, had captured some of the ejected matter and thus formed planets; Alfven's hypothesis of condensations, governed by electromagnetic forces, in the material of a nebula into which the Sun had entered; and one due to Weizsäcker, envisaging the formation of planets by condensation of matter in turbulent motion, near a star. Hypotheses involving the action of one star on another, but not of the ordinary tidal disruption or collision type, have also been proposed, such as Ross Gunn's idea that the Sun has caused perturbations in a neighbouring star already on the point of fission, capturing the fragments to form a planetary system, and the Indian astronomer Banerji's suggestion that a star by approaching a pulsating Cepheid variable star (see p. 290) has caused it to throw out matter which resulted in a planetary system.

With regard to the comets, Olivier has suggested that they may have arisen from matter expelled from the higher latitudes of the Sun, the planets being derived from its equatorial regions, and Bobrovnikoff considers that the Sun may have captured the comets while passing through a region in space with a relatively great amount of diffused matter in it like one of the obscuring nebulae. But there are objections to both of these ideas and also to one (due to Proctor) that some comets may have been ejected by the Sun or by the large planets.

REFERENCES, *etc.*

[1] *Publications of the Astronomical Society of the Pacific*, vol. 59, p. 5, and p. 125 (1947).

[2] E. L. Whipple, "Earth, Moon and Planets," p. 163 (1944). The ultra-violet part of the spectrum is blacked out by the ozone of the Earth's atmosphere.

[3] Kuiper of the McDonald Observatory, Texas, reported (May, 1949) the discovery of a very faint object near Neptune; as the motion was similar it was thought that the object might be a new satellite. This has been confirmed.

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CHAPTER XVI

TWENTIETH CENTURY (*Continued*)

STELLAR ASTRONOMY

PROGRESS in Stellar Astronomy since the beginning of the century has been relatively much greater than in other branches. That this is the case may be seen by comparison between text books published fifty years ago and those of the present time; the space devoted has grown to a substantially greater fraction.

The term Stellar Astronomy is here taken to cover dimensions, luminosities, and masses of the stars, their movements in space, numbers, and distribution; binary and multiple systems, variables (including novae); the constitution and evolution of stars and causes of stellar variability; the dimensions and structure of our Galactic system and its clusters and gaseous nebulae; and the external universes with their relations to the problems of space, matter, and time. The order of this statement provides a convenient basis for an historic account, although there will necessarily be many departures from strict chronological sequence.

In stellar spectroscopy the Harvard classification (see p. 187) maintained its position; R type characterized by absorption bands of carbon compounds, and S type with bands due to zirconium oxide, both orange or red coloured stars, are added classes.

GIANT AND DWARF STARS

But the oustanding discovery of the early part of the century was made by E. Hertzsprung (1905), and H. N. Russell (1913) independently, that the stars generally are divisible into two classes, one "giants," of high luminosities which are on the average of the same order, and the other "dwarfs," progressively fainter from white to red. As Russell put it :—

"The surface brightness of the stars diminishes rapidly with increasing redness. . . . The mean density of the stars of

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classes B and A is a little greater than 1/10th that of the Sun. The densities of the dwarf stars increase with increasing redness from this value through that of the Sun to a limit which cannot at present be exactly defined. This increase in density, together with the diminution of surface brightness, accounts for the rapid fall in luminosity with increasing redness among the stars. The mean densities of the giant stars diminish rapidly with increasing redness from 1/10th that of the Sun for class A to less than 1/20,000th of the Sun for class M. This counteracts the change in surface brightness and explains the approximate equality in luminosity of these stars."[1]

There was thus found to be a broad division of most of the stars into giants of small density and great diameter the bulk of which have luminosities about 50 or 100 times that of the Sun with diameters ranging from roughly sixty or seventy times the Sun's in the red M type down to about a tenth that in the white A type, and a much more numerous class of stars for most of which the luminosities decrease from roughly 50 times, in the hot white A type, to a hundredth that of the Sun in the cooler red M type, the diameters also decreasing from several times to less than half the Sun's. These smaller and denser stars were later given the name of the "Main Sequence," but the term, as will be seen later, does not necessarily indicate any order of growth or evolution. By study of binary systems and the mutual gravitation of their components, the masses of stars were found to be rather closely related to their luminosities, the giants being of the order of five to twenty times the Sun; while in the Main Sequence the average masses decrease from about twenty times to less than half the Sun, our luminary being placed about half-way down the Main Sequence. It should be noted that all these figures of luminosity, diameter, and mass are meant only to indicate what are about the commonest or typical values for the class of star mentioned.

As regards the temperatures of these different classes of star, the application of the methods for obtaining the Sun's effective temperature referred to earlier (see p. 219), and of others based on study of detail of spectrum, has led during the century to the ascertainment of values for the stars which range from 40,000 degrees in O type through 20,000 degrees

for B stars and 6000 for G to about 3000 in M type. From these, corresponding surface brightnesses could be calculated; also, by simple formulae due to Eddington and others, stellar diameters, once distances and luminosities were known.

In 1920, Michelson and Pease at Mount Wilson made the first actual measurement of the angular diameter of a star, using an "interferometer" attached to the 100-inch reflector. This instrument[2] depends on the principle of light interference, and by its means the apparent diameters of nine stars have been determined ranging from about a twentieth to a fiftieth of a second of arc which are the angles subtended by a halfpenny at 65 and 160 miles distance respectively. As the distances of these stars are approximately known it is possible to ascertain the actual dimensions, which range from about 700 million miles (α Herculis) to twenty million miles (α Bootis, Arcturus) all of the nine being giants or "supergiants" of yellow, orange, or red colour.

THE WHITE DWARFS

But a class of stars has been found the members of which do not conform to the mass-luminosity relationship referred to above. They are of very small luminosity for their masses, and the first of them to be identified was the companion of Sirius (see p. 190). The distance of this star being rather accurately known, its diameter could be estimated from its apparent stellar magnitude using the appropriate surface brightness for its spectral type (between A and F); and from this diameter and its mass, determined by the gravitational effects on its primary, an enormous mean density of the order of sixty thousand times that of water was derived.

This extraordinary density was confirmed by Adams at Mount Wilson in 1925 and later by Moore at Lick. According to the general theory of relativity a large mass acts on light emitted from it so as to increase its wave-length. The effect is not great enough to be observed in an ordinary star, but in one of considerable mass and relatively small diameter it should be measurable. For Sirius B the displacement should be that corresponding to 12 miles per second, and exactly that amount was found by Adams, and by Moore.

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The possibility of such great density was explained by Eddington (1924) as due to the stripping, by the thermal agitation of high internal temperatures, of the electrons from the atoms; nuclei and electrons were thus brought much closer together than otherwise possible. About a hundred of these dense stars are now known, and as they are difficult to find and identify owing to their low luminosity (of the order of a two hundredth of the Sun) they are probably really relatively numerous in space. W. J. Luyten has in fact recently suggested that perhaps one star in twenty or thirty will prove to have the characteristics of these "white dwarfs," a name which distinguishes them from the red M type dwarf of the Main Sequence.

STELLAR PARALLAXES

In an earlier chapter (see p. 185) the number of reliable stellar trigomometrical parallaxes known at the beginning of the twentieth century is given as 60. By 1915 nearly 200 had been determined, by 1925 close on 2000 and by 1949 nearly 10,000. It was on this increasing accumulation of data that the results of Hertzsprung, Russell and others were founded. And in 1914, W. S. Adams and A. Kohlschütter made the important discovery that the intensity of certain lines in stellar spectra varies with the luminosity of the star.* For example in the two K type stars Aldebaran (giant) and 61 Cygni (dwarf) two lines due to strontium are strong in Aldebaran and weak in 61 Cygni, the reverse being the case for a calcium line. By calibration of such relationships in stars for which distances and luminosities had been directly measured, it was found possible to determine definite relations between luminosity and intensity of lines. Thousands of distances of stars down to 8th magnitude have thus been obtained from their actual luminosities and apparent brightnesses. Theory shows that these variations of line intensity are due to the state of "ionisation" of atoms in the stellar atmospheres, being connected with the gravitational potential at the star's surface.

*A forecast of something of this nature was made by R. A. Proctor more than thirty years previously. (See "Mysteries of Time and Space, p. 410 (1892)).

A greater accuracy, which accompanied the increase in number of trigonometrical parallaxes, was very largely due to the work of F. Schlesinger of Yale, whose application of long focus telescope photography, and new methods of measurement and reduction of the plates, were adopted and improved upon by a number of observatories among which may be particularly mentioned Mount Wilson (van Maanen) and our own Greenwich establishment.

METHODS FOR DERIVING STELLAR DISTANCES

Other methods for determination of stellar distances were developed during the period. One, based on the assumption that the luminosity of a star is equal to the average for its spectral type, can have a fair degree of accuracy if the star can be classified by its spectrum as a giant or Main Sequence star; and parallaxes may be estimated for binary systems assuming appropriate masses for the component stars, and for eclipsing variables like Algol.[3] But the most important and powerful of these indirect methods is that founded on a relation between the length of period of variation of "Cepheid" variables (see p. 276) and their actual luminosities. This, the Leavitt-Shapley Period-Luminosity Law, was due initially to the work of Miss H. S. Leavitt, published in 1912, on the variables in the smaller Magellanic Cloud; and later mainly to H. Shapley who showed from distances derived by various methods (chiefly proper motions in the sky of the nearer stars of the class) that the relationship holds for Cepheids in all parts of the sky including these in globular clusters. The ability thus provided to estimate the distance of a star of this type, and therefore of any stellar aggregation to which it belongs, can be said to have revolutionized the study of great distances that are far beyond the possibility of measurement by the older methods. This is because there is no dependence on a parallactic shift (as in trigonometrical parallaxes), but only on periods of light variation and the apparent brightness of these stars, which are intrinsically of great luminosity ranging from about a hundred to seven thousand or more times the Sun with increase in length of period (see p. 276). Similar but not so reliable use can be made

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of long period variables and Novae the luminosities of which are relatively uniform at maximum.

And parallaxes of some value for classes of stars may be obtained from their displacements on the sky (proper motions) assuming these to be due to particular transverse velocities across the line of sight, or to be partly a reflex of the motion of the Sun in space with respect to its neighbouring stars (see pp. 120, 148, 263).

STELLAR MOVEMENTS

The study of stellar movements as regards displacements on the sky, *i.e.*, "proper motions," has always depended on the accurate measurement and cataloguing of positions. This class of observation constituted the main activity of the majority of observatories throughout the world during the century and a half following 1755 when Bradley made the first really accurate determinations of stellar positions (see p. 104). By 1900 there were very many catalogues of stars in existence containing hundreds of thousands of measurements of star places, some highly accurate and others of little value, but not one-tenth of these had been utilised for the problems of proper motion. It was largely owing to the foresight of one man L. Boss, Director of the Dudley Observatory, Albany, U.S.A., that work was undertaken destined to lead to the construction of a general catalogue of positions and proper motions making fruitful the preceeding century and a half of effort. A Department of Meridian Astronomy was founded in 1906 by the Carnegie Institution of Washington to carry out his plans, which operated under his supervision until his death in 1912, and after that date, under his son B. Boss.

The programme included the collection and reduction to a uniform system, of all extant observations of every star brighter than visual magnitude 7·0 and of many fainter stars that appeared to have larger proper motions than usual, the original plans being for 20,000 stars, to which more than 13,000 were added later; re-observation of all the stars with the same instrument, and as far as possible, the same observers; the reduction of nearly 200,000 new observations; and the formation of a General Catalogue. The great mass of data

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accumulated from 1755 to 1932 (the year chosen as a closure date) having been reduced to a system, definite positions for 1950 were derived, and in addition to these and proper motions, the magnitudes and spectral classes of most of the stars and numerous data regarding double and variable stars were given. This General Catalogue of Stellar Positions and Proper Motions was thus completed in 1939 through the joint efforts of the Carnegie Institution of Washington and the Dudley Observatory of Albany, and it contains the positions and proper motions of 33,342 stars. An additional value of the Catalogue is its provision of satisfactory positions of stars fairly uniformly distributed over the sky (one star to $1\frac{1}{4}$ square degrees) for use as comparison stars in determining positions of still fainter stars, proper motions for which will be necessary in the future in investigations of the dynamics of our Galactic system.

In this and other catalogues, many containing fainter stars in particular regions of the sky, something like 200,000 proper motions have now been listed. Of these only about 50 are greater than two seconds of arc per annum, about 200 more than one second and nearly 2500 larger than half a second. The motions of the stars may have any direction in space, but only that component at right angles to the line of sight is a cause of displacement on the sky relative to other stars. The proper motion is compounded of the angular displacement thus caused and that due to the motion of the Solar System in space. Large proper motion usually means proximity rather than great space velocity. On the average, the brightest stars, being nearer than the fainter ones, have large proper motions. But when individual stars are considered, it is found that the biggest motions belong to rather faint stars. For instance the greatest annual proper motion recorded is 10·3 seconds which is that of a 9·7 magnitude star known as Barnard's "Runaway star." The naked eye star of greatest proper motion (5·2 seconds) is 61 Cygni, which was the first star to have its distance accurately measured (see p. 147).

Stellar movements may be divided into two observational categories, proper motions as just described, and velocities in the line of sight (radial velocities) of which may thousands have been determined. A Catalogue of Radial Velocities

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was published by J. H. Moore in 1932 containing the radial velocities of 6739 stars, which includes practically all brighter than 5·5 magnitude with many fainter stars.[4] The Lick Observatory has been a pioneer in the increase of accuracy and accumulation of such data, and many other observatories throughout the world, notably the Dominion Astrophysical Observatory, Victoria, British Columbia, and the Mount Wilson Observatory, have contributed. Velocities of from 5 to 20 miles per second are common but any over 50 miles per second are scarce. The work of L. Boss, W. W. Campbell, J. S. Plaskett and others has shown that there is a general increase in proper motions and line-of-sight velocities as we pass from the white B or A type stars to the red M type, which is accompanied by a reduction in mass, as most of the stars concerned are in the Main Sequence along which there is a decrease of mass (see p. 257). It was suggested by Halm at an earlier date (1911) that such a correlation of mass and velocity might be expected whereby the energy of movement (kinetic energy) of the stellar system would be divided between the more massive and less massive members, the more massive moving more slowly with reference to the other stars.

SUN'S MOTION IN SPACE

The motion of the Sun relative to the stars in its vicinity (say within 1000 light years radius), first clearly demonstrated by W. Herschel (see pp. 120 and 148), continued to be the subject of many researches, and the accuracy of the results, particularly as regards the velocity of movement, was much improved by the use of radial velocities in the solution. The most recent determination gives a position for the Solar Apex near the 3·8 magnitude star o Herculis which is, as has been already remarked, within 10 degrees of Herschel's first determination. The velocity for the Sun indicated is $12\frac{3}{4}$ miles per second. But both the direction and the velocity found seem to depend on the magnitudes of the stars used in the computations and also on their spectral type and brightness; and this appears to indicate systematic motions among the various classes of stars themselves.[5]

MOVING STAR CLUSTERS

Group motions among the stars were first drawn attention to by R. A. Proctor in 1869, when he pointed out that five of the seven stars in the Plough had parallel and equal proper motions; and he also found similar common proper motion for certain stars in the constellation of Taurus (see p. 195). About forty years later L. Boss found that 39 stars in Taurus including most of the brighter stars of the Hyades cluster (but not Aldebaran) were moving in converging directions. Subsequent work by Eddington, Kapteyn, Hertzsprung, Rasmuson and others indicated the probability of connection in a number of other groups. The chief of these are a cluster in Perseus, a group in the constellations Scorpio and Centaurus, the stars in the constellation Coma Berenices, those in Orion, and a group of which 61 Cygni seems to be a member.

In the Taurus group there are nearly 80 stars ranging from about 3·5 to 6·0 magnitude, the proper motions all converging to a point in the sky about 6 degrees east of Betelgeuse. As the receding radial velocities of these stars had been measured spectroscopically, the true paths in space have been computed, assuming that they are really parallel and the convergence only an effect of perspective, and the distance of the cluster's centre found to be 130 light years and its diameter about 50 light years. The Ursa Major cluster was found by Hertzsprung to contain many stars in all parts of the sky as well as the five Plough stars of Proctor, including the brightest star in the sky, Sirius. More than 40 stars seem to belong to it, and it has a flat disc shape disposed perpendicularly to the plane of our Galaxy, 130 light years in its largest diameter, and moving as a whole parallel to the Galactic plane.

Of the other groups referred to above W. M. Smart has recently shown that their exact status is doubtful. Although those of Ursa Major and Taurus are undoubtedly real moving clusters, some are not certainly so; such as the ones in Perseus, Scorpio-Centaurus and Orion are perhaps composed only of stars with small individual motions. In any case it is known, however, that the great majority of the stars do not belong to moving clusters, but are apparently moving independently.

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But a remarkable phenomenon among stellar motions which seemed to contradict this was discovered by J. C. Kapteyn in 1904. This consisted of a systematic motion which suggested two streams of stars passing through each other, each stream going on the whole in a certain general direction, but with individual stars having movements relative to each other. This streaming apparent common motion was, however, later shown to be a consequence of differences in the orbits of stars revolving round the centre of the Galactic system.

ROTATION OF THE GALAXY

As might be anticipated, the flattened form of our Galactic system, suggested by the zone of the Milky Way, led to conjectures that it is in rotation in its own plane. Several have speculated on the possibility of the existence of a controlling central Sun of extraordinary mass. The philosopher Kant thought that this might be Sirius, and W. Herschel suggested the great globular cluster in Hercules, or the "compressed parts of the Milky Way," for the rôle. Mädler considered that the chief governing power is not that of any single mass, but that it is situated at the centre of gravity of the stellar system, probably near the Pleiades. All of these were ruled out except Herschel's second suggestion. For Sirius the mass would be inadequate, and the positions of the Hercules cluster and the Pleiades are too far out of the Galactic plane for the purpose.

Among the most remarkable of recent achievements in astronomy may be placed the demonstration of a Galactic rotation. Following on the development of a general theory of galactic rotation in 1926 by B. Lindblad, proper motions and line-of-sight velocities of high luminosity stars of various types, planetary nebulae, and gaseous matter between the stars (see p. 269), all objects with distances out to 2000 light years and more from the Sun, were studied by J. J. Oort, J. S. Plaskett, A. H. Joy and others.* The results have

*It may be noted that, more than fifty years previously in 1871, H. Gyldén found by investigation of proper motions that the Milky Way rotates about a centre apparently directly opposite to the one now accepted. This result was based on very small proper motions and did not find general favour. The centre derived was one of two possible antipodal positions : the nature of the solution did not rigidly decide between them.

demonstrated beyond question that our Galaxy is rotating in its plane with great velocity, the speeds of revolution of the stars decreasing from the centre outwards* with the direction of movement clockwise as seen from the northern side of the Galactic plane. The centre is about 30,000 light years away towards the constellation Sagittarius, being the same as is suggested by the distribution of objects in the Galactic system and by the disposition in space and radial velocities of the globular clusters (see p. 294) surrounding it. The velocity of revolution for the stars in the Sun's vicinity is about 150 or more miles per second ; their period of revolution round the centre is about 200 million years and the controlling mass necessary for this is something like two hundred thousand million times that of the Sun.

Most of the stars near the Sun appear to move in orbits round the centre of the Galaxy that are nearly circular in shape. Some stars of velocities, with respect to the Sun, of great amounts, seem to have long elliptical orbits crossing, in the Galactic plane, the line of the Sun's orbit at rather large angles on their way to or from their points of nearest approach to the Galactic centre. These stars therefore appear as a class to be moving rapidly backwards towards the constellation Argo, which is in a direction at right angles to the line from the Sun to the centre in Sagittarius. Those ahead of the Sun have, in general, approaching line-of-sight velocities and those behind have receding speeds. Their radial velocities are naturally great with respect to the Sun whose orbital speed is 150 miles or more per second. Such high velocity stars have been studied by Oort and others and the apparent high speed streaming motion is found to be such as would be expected. The Star Streams of Kapteyn, mentioned above, are the consequence of the movements of many stars in elliptical orbits which, in the Sun's vicinity, have an inward or outer trend as compared with the average of the more circular motions of others. When these outward and inward differences are considered, a preference is shown for a direction of move-

*i.e., their periods of revolution round the Galactic centre increase outwards. It is upon this fact that the demonstration of Galactic rotation rested. If the Galaxy rotated as one body there would be no systematic differential movements revealing the rotation.

ment towards or away from the Galactic centre, with a direction lying nearly in the plane of the Milky Way in a line joining the constellations Scutum and Orion.

NUMBERS OF THE STARS

The first important systematic investigations of the numbers of the stars were those of the Herschels (see pp. 118, 146), which were not to obtain numbers of different brightnesses but to get figures that, on an assumption of a general equal scattering of the stars, would provide an idea of the shape and perhaps also the size of our stellar system. Similar gaugings were carried out in the latter part of the nineteenth century by G. Celoria and H. Seeliger. But the most comprehensive counts of stars by their magnitudes were those of S. Chapman and P. J. Melotte, and of Kapteyn, followed by a very complete count by F. Seares and P. J. van Rhijn, all of which were based on photography.

Down to the tenth magnitude the numbers of stars in each successive magnitude grade are known, all such stars having been already catalogued. For the fainter stars it was not practicable to enumerate them over the entire sky and the numbers were therefore based on what was recorded in photographs taken with the most powerful instruments, including a number taken with the 100-inch reflector at Mount Wilson, in sample areas of the sky. These sample areas were according to a plan by Kapteyn (1906) whereby 206 "Selected Areas" distributed uniformly over the sky were chosen for intensive study of numbers and magnitudes, spectra, etc.; on these, general ideas of the whole sky might be based.

Briefly summarized, the results of Seares and van Rhijn give a total of about thirty thousand million stars. Down to sixth magnitude (about naked-eye limit) there are not quite 5000 stars, to the twelfth magnitude (the limit of a 3-inch telescope) about 2¼ millions, and to the sixteenth magnitude (the faintest visible in a telescope like the Lick 36-inch) about 70 millions. As the photographic and visual magnitudes of a star are not necessarily the same, corrections in the numbers counted from photographs have to be made to obtain such figures as are quoted above. These corrections

are based on the assumption that the photographic and visual magnitudes are equal for a white A type star ; the O and B type are then up to a magnitude brighter photographically, while the F. G. K and M types range to two magnitudes fainter photographically, or even more with the reddest stars.

A concentration of the stars towards the Milky Way zone has long been noted and was first brought out clearly by the gauges of the Herschels. Seares and van Rhijn's figures show this concentration to be rapidly increasing in the fainter stars. On the average there are about $3\frac{1}{2}$ times as many naked-eye stars in a given area of the Milky Way as there are in a direction at right angles to it, and the concentrations of stars down to the twelfth and sixteenth magnitudes, similarly considered, are about eight and sixteen times respectively.

ABSORPTION OF LIGHT IN SPACE

The rates of increase in the numbers down to successive magnitudes are progressively less than they should be if there is general equal scattering of the stars in space, or if there is no loss of light in traversing it. During the early years of the present century a number of investigations were undertaken on this possible loss of light, without much in the way of a definite result. But it has been demonstrated, in the past twenty years, that there is such a loss due to scattering and absorption by gas and dust, although it does not appear to be of a constant amount in different directions. A general absorption, which produces a reduction of a magnitude in the brightness of a star in a light journey of about 5000 light years on the average in the Sun's neighbourhood, was found by R. J. Trumpler in 1930 from a study of the brightnesses and diameters of distant clusters of stars in the Milky Way ("Galactic" clusters) ; and further work by other investigators (notably Joy, van Rhijn and E. Hubble) has given similar results. Study of the colours of the distant intrinsically bright B type stars and other objects bright enough for study at great distances, has shown that they appear redder than the normal star of the same types closer to us. The dust clouds responsible for the reddening seem to be contained within the general stratum of absorbing matter as concentrations in it ;

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obscuration by dust clouds is due chiefly to the smaller particles, and those smaller than about a thousandth of an inch in diameter are responsible; any of greater size simply obstruct light without changing its effective colour.

INTERSTELLAR GAS

Recent research has shown that in interstellar space there are atoms and molecules of gases which, although their aggregate mass is probably larger than that of the dust particles, have little dimming effect. This interstellar gas was detected by the spectroscope; lines due to calcium, sodium, potassium, titanium and iron atoms, and some produced by molecules of the hydrocarbon (CH), sodium hydride (NaH), and cyanogen (CN), have been noted; and with a specially-designed spectrograph *bright* lines of atoms of hydrogen and ionised oxygen have been detected in regions of the Milky Way. O. Struve (great-grandson of F. G. W. Struve), MacKellar and others have been active in this field of work. The strongest of these lines were first noted in the spectra of distant spectroscopic binary stars, being "stationary lines" which did not appreciably change their position in the spectra, as did the other lines because of orbital motion. The first observation of the kind was that of J. Hartmann in 1903; he found the phenomenon with the K line of calcium in the spectrum of δ Orionis. As a consequence of the great tenuity of the gas which produces the lines, they are fine and sharp, increasing in strength with the distance of the star through the added absorption of the greater amount of intervening gas. It is interesting to note that, during the past few years, doubling (or even trebling or quadrupling) of the calcium "stationary" lines has demonstrated the existence of more than one cloud, between a particular star and ourselves, moving at different line-of-sight speeds.

THE LUMINOSITY LAW

As has been related on a previous page (see p. 257), the stars differ very much in luminosity, and to ascertain the relative frequency of occurrence of these luminosities (the "Luminosity Law") has been the subject of laborious in-

vestigation by Kapteyn, van Rhijn, W. J. Luyten and others. Kapteyn's estimate (1922) gave numbers down to the twelfth or thirteenth absolute magnitude* (visual), which later investigation has shown to be much brighter than the faintest, and it is now considered that the greatest frequency is at about that magnitude, or even fainter, the least luminous being perhaps of the twentieth absolute magnitude, or about a millionth as luminous as the Sun. The present known extremes of luminosity are, on the one hand, the two components of a binary S Doradus, each 160,000 times as luminous as the Sun, and on the other, a very faint dwarf giving less than a hundred thousandth of the Sun's light, or a range of something like sixteen thousand millions to one!

The relative frequency of the different types of stars is not yet known with much accuracy, but recent results suggest something like the following proportions in the volume of space surrounding the Sun :—

Giants and Supergiants :	1 in 370
(All spectral types, including O, R, N and S).	
Main Sequence :	
Type B, ...	1 ,, 650
,, A, ...	1 ,, 100
,, F, ...	1 ,, 50
,, G, ...	1 ,, 25
,, K, ...	1 ,, 7
,, M, ...	3 ,, 4
White Dwarfs :	1 ,, 30 ?

Stars which are at least as luminous as the Sun do not seem to be more numerous than about one in fifteen.

GALACTIC DISTRIBUTION

Distribution of the stars with respect to the Galaxy has been found to differ for the various classes. That the B and A types are notably concentrated towards its plane was soon noticed when the first results of the compilation of data for the Henry Draper Catalogue of Stellar Spectra (see p. 187)

*The magnitude of a star as it would appear if situated at a distance (32·6 light years) where its parallax would be a tenth of a second. The Sun's is + 4·8 (visual).

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were examined. No marked tendency of the kind is found for the F and G types in the catalogue, but there is a slight preference for low Galactic latitudes in the case of the K and M stars. This distribution is, however, unreliable for the less luminous stars, as the proportion of them included is relatively much smaller than in space generally, since the Catalogue's limit (about 8·5 magnitude) is not faint enough to include more than a small fraction of the fainter stars of the Main Sequence that really are present in the volume of space dealt with for the more luminous types. The distribution along the approximate great circle of the sky marked by the Milky Way zone has been found to be marked by a concentration in the Sagittarius centre of more luminous stars such as Novae, O type, and Long Period variables, while the B type stars and M giants of bright apparent magnitudes show a tendency to concentration in the constellation Argo where a centre of a subordinate Local System in the Milky Way has been thought to exist. Among the notable contributors to knowledge of this kind have been E. C. Pickering, C. V. L. Charlier, H. Shapley and K. Lundmark.

DOUBLE AND MULTIPLE STARS

In 1906, S. W. Burnham published a catalogue of 13,655 double stars nearly all within 120 degrees of the north pole of the sky, and similar catalogues by R. T. A. Innes covered the remainder of the sky. R. G. Aitken's "New General Catalogue of Double Stars within 120 degrees of the North Pole," published in 1931, contains the results of the work of many observers on more than 17,000 pairs, practically all within limits of separation of the components which, as adopted by Aitken, range from 250″ for a pair with a combined magnitude of 2·0 or brighter, down to 2·5″ for eleventh magnitude and 1″ for all fainter pairs. Five-sixths of the total are separated by 2″ or less, five-eighths by 1″ or under, and between a quarter and a third are half a second or less apart. About one star in eighteen brighter than ninth magnitude is found to be a double star within the separating powers of the largest modern refractors. That this means real physical connection in most cases, is brought out by the fact that if the

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scattering of the stars brighter than the limit specified were at random over the sky, there would not be more than six or seven of them as close together as ten seconds of arc.

Double or multiple stars are more common in Milky Way regions, according to Aitken, than elsewhere; and this does not appear to be merely an effect of perspective along the Galactic stratum as the ratio of close to wide pairs is not greater there.

The number of visual pairs in which relative orbital motion, although often very slow, has been noted, is now about fifteen hundred; and orbital elements have been derived for several hundred systems, their reliability diminishing in general with increase in period, which varies from a few years to several hundred years. The shapes of the orbits are more eccentric as the periods are longer, ranging from about 0·4 to more than 0·6 on the average; and the periods increase on the whole with spectral type of primary in the order B, A, F, G, K, M, this being probably at least partly due to decrease in the average masses of the stars down the Main Sequence in that order.

The brighter component of a visual binary is found almost invariably to have the larger mass, as would be expected from the mass-luminosity relationship (see pp. 257, 285); and F. C. Leonard in 1922, and the present writer independently, have found that if the primary is a Main Sequence star the secondary is a cooler and redder Main Sequence star, while if the brighter component is a giant the companion is usually hotter and bluer.* This is clearly brought out by the examination of the spectral types, but in visually observing colours it is obscured by the effect of contrast, which tends to make the fainter of two close stars appear bluer than it really is. The spectral relationship provides a reliable criterion as to whether a binary pair has two Main Sequence stars or has a giant primary.

It has recently been shown (1942) by E. T. R. Williams and A. N. Vyssotsky that a substantial proportion of the stars have relatively faint distant companions as well as the closer ones referred to above. In fact these investigators conclude

*It may be a hotter and bluer giant or Main Sequence star, or a cooler and redder star of the Main Sequence.

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that there are probably as many physical companions, at a distance from their primaries at least 1000 times that between the Sun and the Earth, as ordinary close visual companions. Multiple systems are considerably less frequent than pairs, but during the past five or six years a novel kind of member of a stellar system has been discovered by K. Aa. Strand and others by means of measurements of the disturbances of proper motions of certain stars, on photographs of high accuracy. For instance it appears that 61 Cygni has a third component of small mass (only 16 times Jupiter's), and that the binary 70 Ophiuchi has an even smaller attendant (10 times Jupiter's mass). The probable physical characteristics of the former of these has been investigated theoretically by H. N. Russell and he finds it to be a body of planetary type but probably with an internal constitution resembling that of a star and not of a planet, shining by reflected light, its surface temperature being too low for self-luminosity. It appears quite possible that many stars may have similar attendants, which might mean that planetary systems are not uncommon in space.

SPECTROSCOPIC BINARIES

At the present date substantially more than a thousand spectroscopic binaries (see p. 190) have been discovered; and for about 400 of these the secondary is sufficiently luminous to have the lines of its spectrum photographed in addition to those of the primary, thus enabling elements of the orbits, including the ratios of the individual masses of the components, to be computed. The periods of revolution range from a few hours upwards, more than half being shorter than 10 days; and the eccentricities of the orbits are smaller than in the case of the visual binaries, averaging about 0·2 and increasing with length of period, thus continuing the relationship shown by the visual pairs. There is a gap between the longest spectroscopic binary periods and the shortest of the visual systems which seems to be due to observational selection, the displacements of spectral lines in these slower moving pairs being too small for detection so far by the spectroscope while the components are too close for separation visually. Spectroscopic binaries are mostly stars visible to the naked eye, of

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high luminosity and mass, and to this may be due the fact that the large proportion of about a third of all stars examined for duplicity by the spectroscope have been found to be binary pairs. Nearly two-thirds are of B or A type spectrum, and the periods of revolution are generally shorter in these hotter types. Individual masses can only be determined when the angle of inclination of the orbit plane to the line of sight is known (*i.e.*, in eclipsing pairs); but average values corresponding to a mean angle of inclination can be estimated, and these show the reduction in mass with the cooler types of spectrum mentioned earlier. When individual masses of the components are obtainable the more luminous stars are found to be the larger in mass.

ECLIPSING BINARIES

In the case of the eclipsing binaries, of which there are also more than a thousand now known, a larger proportion, having been discovered from the light variation and the nature of the light-curve, are faint and not easy to study spectroscopically. The shape of the light-curve gives the ratios of the diameters of the components to the diameters of their relative orbits round each other; and the orbital eccentricity, exact inclination of orbit plane to the line of sight, shapes and surface brightnesses of the two components can also be computed. When curves of the line-of-sight velocities are obtainable, not only the relative but the actual dimensions, masses and densities can be calculated. Because of this the determination of the types of spectra of eclipsing pairs is of great importance to research on the physical characters of the stars. About half have been found to be A type, a fifth of type B, a seventh or so of type F, and the remainder are G, O, K and M. It is very probable, however, that this predominance of A and B is due to the relative ease of their discovery, because of greater luminosity than in the case of the remaining types, most of which are redder Main Sequence stars. Assuming that the volume of space throughout which eclipsing binaries have been observed is that in which all the bright A type can be seen, we should probably find a much greater number of the fainter G, K and M types, if all these types contained in the volume could be observed.

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In many eclipsing systems, the two stars are so near together that the surfaces facing each other reflect the light of the other component so strongly as to affect the shapes of the light-curve. In such cases the shapes of the stars themselves are found to be ellipsoidal and the amount of light received from the system is affected by the rotation of these ellipsoids which, through tidal interaction, is likely to be in the same direction and period as the orbital revolution. There are indeed several non-eclipsing binaries known with light variation due to their ellipsoidal shapes alone, but so far no single star has been found whose light varies because of rotation of a non-spherical shape.

VARIABLE STARS

Progress in the discovery and study of variable stars has been exceedingly great since the beginning of the century when only about 400 stars that change in brightness were known. At the present time the number is of the order of 20,000, a large proportion having been discovered by photographic methods notably at Harvard College Observatory or its branches. A common classification may be taken to be as follows : Class 1, Eclipsing Pairs ; 2, Cepheid Variables ; 3, Long Period Variables ; 4, Irregular Variables ; 5, Novae, or Temporary Stars, including Supernovae.

ECLIPSING VARIABLES

The Eclipsing Pairs may be sub-divided into three broad types : the Algol kind that remain practically of the same brightness between the minima, except for a small secondary minimum produced by the eclipse of the substantially less luminous secondary by the primary ; the β Lyrae class with marked secondary minima and more rounded light curves with components elongated in shape by their mutual gravitational attraction ; and the W Ursae Majoris stars with light curves conspicuously convex upwards between eclipses composed of two much elongated rather dense dwarf stars revolving almost in contact. The range of periods in the first two kinds is great, about a fifth of a day to a number of years (*e.g.*, ϵ

Aurigae, 27 years); in the third it is from about the same lower limit to a day and a third only.

CEPHEID VARIABLES

The Cepheid Variables have already been mentioned and the relation between their luminosities and length of period (see p. 260).* They are named after δ Cephei, the first discovered and best known of the class. About 1250 are listed with periods from several hours to more than 40 days, and there are also many hundreds of a similar kind in the globular clusters, the "cluster" type, with periods of less than a day for 600 of them. Cepheids of the longer periods have also been found in some of the spiral nebulae and similar external systems.

LONG PERIOD VARIABLES

The Long Period Variables are giant stars of low temperature spectrum; about 1100 of M type, and 200 of S, R or N type all with bright lines in the spectra, are known. The periods range from about 100 to more than 600 days, with a strong concentration in the range 200 to 400 days. As shown by H. Ludendorff in 1924, the periods and amplitudes of variation increase on the average with redness. The variation of visual brightness is about 20 to 1500 times (3 to 8 magnitudes) between maximum and minimum light, averaging about 100 times (5 magnitudes). The variation of total energy radiated, as measured at Mount Wilson by Joy, and Pettit and Nicholson, with a thermo-couple on the 100-inch reflector, is very much less. The periods are not regular; maxima or minima may be some weeks before or after the expected times in certain cases; a few have even shown changes in length of period over a number of years.

*The following gives the relation approximately :—

Period Days.	*Luminosity (visual), Sun=1.*
0·5	100
1·0	175
5·0	700
10·0	1300
50·0	7600

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IRREGULAR VARIABLES

The Irregular Variables are very diverse in character and include some semi-periodic stars such as α Orionis (Betelgeuse) and α Herculis, some that suddenly brighten two or three magnitudes and then fade gradually, and others that remain for long periods at a fairly constant brightness, suddenly fade several magnitudes and vary irregularly until they regain ordinary brightness. In the case of Betelgeuse, the radial velocities, the apparent angular diameter, as measured with the Mount Wilson interferometer (see p. 258), and the light changes, all seem to vary together. All these stars appear to be giants or supergiants; but there are some irregular variables which apparently belong to the Main Sequence; these are usually found associated with obscuring nebulosity.

NOVAE

Of Novae or Temporary Stars there have been more than a hundred recorded, exclusive of a number appearing in systems external to our Milky Way System. Over 70 of these have been observed in the present century. A typical Nova is characterized by a very sudden increase in light usually occupying less than a week, of from about ten thousand to a million-fold, and by a diminution that is much slower, but faster than the average at first. The fall in light is often marked by a series of almost regular subordinate variations, and the final brightness reached, in from about ten to thirty years, is seldom perfectly constant and generally about the same as before the outburst. The spectroscopic phenomena following the outburst appear to be best accounted for as due to shells of gas moving outwards at great velocity (hundreds of miles per second) from the star. The concentration of Novae towards the Milky Way is generally very marked, and especially to the star clouds of Sagittarius (see p. 271). From various methods, including direct parallax measurements for a few, combination of proper motions and radial velocities (from certain fine spectral lines not affected by the large outward displacements), intensities of interstellar lines (see p. 269), and in several cases from measurement of the angular expansion of expelled bright nebulosity, an average luminosity

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of about 100,000 times the Sun's has been derived by K. Lundmark and other workers since the beginning of the century. Recently D. B. McLaughlin has found that the luminosities range from several hundred thousand to several thousand times the Sun's, according to the rate of fall in light from the maximum, the stars that fade quickest being the brightest. A considerable number of faint Novae have been noted in about a dozen of the nearer spiral nebulae, over 150 of them in the Andromeda nebula alone; the similarity of their light curves to those of the Novae seen in our Milky Way regions indicates that they are the same class of star.

SUPERNOVAE

But there was one of these in the Andromeda nebula, appearing in 1885, that attained about 7th magnitude, or several thousand times as bright as the others. This was the forerunner of more than 50 of an even more luminous sort of Novae found in stellar systems external to our Galaxy, to which has been given the name of Supernovae. They are of two classes, one about 100 million times, and the other 10 million times, the Sun's luminosity, with radial velocities of ejected gases something like ten times as great as for ordinary Novae. Several very bright Temporary Stars seen in our Milky Way (those of 1054, 1572 (Tycho's) and 1604 (Kepler's)) appear to have been Supernovae.

F. Zwicky has estimated the average frequency of their appearance in a stellar system as about one in 600 years, or something like a ten thousandth of the frequency of the ordinary Novae in our own Milky Way system, which is from 10 to 20 per annum, allowing for faint unobserved objects, according to S. Bailey's study of Harvard photographs.

STELLAR SUPERFICIES

Until the present century the generally accepted idea of a star's superficies seems to have involved a photosphere or light-emitting layer composed of solid or liquid incandescent particles of substances difficult to volatilize. But when methods of obtaining surface temperatures showed that these

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were too great for anything but matter in the gaseous condition, it became clear that a star's outer layers were intensely heated gases, with a transparent atmosphere above a gaseous photosphere so constituted as to emit a continuous spectrum. From Stéfan's Law (see p. 219) it could be shown that just outside the photospheres of all but the very coolest star the temperature of any matter there would be at least several thousand degrees. Stellar atmospheric surroundings are therefore occupied by material mostly in a gaseous state.

Modern theory of the constitution of stellar atmospheres depends on the nature of the structure of the atom. This theory has been built up during the century by many workers, among whom may be mentioned M. N. Saha, H. N. Russell, R. H. Fowler, E. A. Milne, A. Pannekoek, D. Menzel, C. Gaposchkin, A. Unsöld and M. Minnaert. A new conception of the atom had appeared, and it was developed by the work between 1911 and 1913 of Lord Rutherford and N. Bohr. According to this conception, the atom is not indivisible as had been believed by the early pioneers of atomic theory. It can be broken and the parts taken from it are formed to be always the same for any element, and to be charges of negative electricity, known as "electrons." The complete atom is electrically neutral and has at its centre a nucleus, built up of positive units of electricity, "protons," and neutral units, "neutrons." This nucleus is believed to be less than a billionth of an inch in diameter; it possesses nearly all the mass of the atom, and has a positive electrical charge. Like a miniature solar system it is surrounded by a number of negative electrons moving about it in orbits the largest of which are something like twenty thousand times the nucleus in diameter. The positive charge of the nucleus is exactly balanced by the negative charges of all the electrons.*

One element differs from another in the number of its electrons. This number is called the "atomic number,"; for hydrogen it is one; for helium, two; lithium, three, and

*This is a simplified conception which was at first taken to represent the facts. But it has become clear that the existence of planetary orbital movement does not necessarily follow from the observed phenomena, while estimates of dimensions, depending as they do upon arbitrary assumptions as to distribution of the electric charges, are very uncertain.

so on up to 92 in the case of uranium. At the high temperatures of a star's atmosphere, atoms move about with great velocities, colliding and loading each other's electrons with energy which raises them to higher level orbits temporarily. Leaving these higher orbits for lower ones, the electrons emit pulses or *quanta* of radiation with a wave-length according to the difference in energy between the two levels. When an electron is actually removed from an atom by the thermal agitation, the atom is said to be singly "ionised," and multiply ionised if more than one electron is lost ; and its spectral lines differ* for the various degrees of ionisation. High temperature and the consequent atomic agitation favours ionisation ; greater density, however, provides a better chance of the atoms becoming neutral again by picking up electrons. The proportion of the atoms ionised in a particular stellar atmosphere thus depends on the relation between temperature and density. Being different according to the ionisation of the atoms, the spectral lines, and in consequence the types of spectra, are thus largely dependent on these two physical conditions, of which temperature is the more important.

The relative abundance of the elements in the atmospheres of the stars is considered to be much the same as for the Sun. Study of the Solar spectrum, where lines of nearly three-quarters of the known elements have been found, gives the constitution of the Sun's outer layer as slightly more than 98½ per cent for hydrogen and helium, one per cent for oxygen and magnesium together, and less than half of one per cent for the other elements.

Lines in stellar spectra have been found to be broadened by various factors and in some this is clearly a Doppler effect, due to the motion to and from the observer of the two sides of the star in a rotation about an axis not far from perpendicular to the line of sight. The stars concerned are of O, B, A and F type but practically none of other types, and the rotation is much faster than the Sun's. O. Struve has been one of the most active investigators in this line of work.

*The absorption lines also. The particular wave-length of the continuous photospheric spectrum is absorbed and re-emitted by the atom but in all directions, only part of it coming through to the observer the result being the comparative darkness of an absorption line.

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From a broadening of spectral lines due to the Zeeman effect (see p. 210), Hale and his collaborators found evidence of a general Solar magnetic field which appears, however, to vary in strength. In 1947, H. W. Babcock's observations showed that several rapidly rotating stars have a magnetic field much stronger than the Sun's. The stars investigated are all of the spectral types for which rapid rotation seems common, and to this rapid rotation and the possession of highly ionised atmospheres the strong general magnetic fields have been attributed.

REFERENCES.

[1] *Popular Astronomy*, vol. 22, p. 19 (1914).

[2] For a good description see *Popular Astronomy*, vol. 29, p. 19 (1921).

[3] A description of these methods is given in the author's "Outline of Stellar Astronomy," pp. 151-4 (1947).

[4] *Publications of the Lick Observatory*, vol. 18.

[5] Paper by O. R. Walkey, *Monthly Notices of the Royal Astronomical Society*, vol. 106, p. 274 (1946).

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CHAPTER XVII

TWENTIETH CENTURY (*Continued*)

STELLAR INTERIORS

THE pioneer investigations in the theory of stellar interiors were made by J. Homer Lane (see p. 220), A. Ritter, and Lord Kelvin, in the latter part of the nineteenth century, their work culminating in the exhaustive researches of R. Emden published in 1907. Among other workers in the subject, R. A. Sampson, K. Schwarzschild, A. S. Eddington, J. H. Jeans, S. Rosseland and E. A. Milne have been outstanding.

For the average star the following sketches briefly what may be termed the most generally accepted ideas. As such a star is a mass of intensely hot and highly ionised gas it is possible to calculate the pressure and temperature at any point in it, since the properties of a highly-ionised gas approximate to those of a "perfect" gas,* where a simple relationship exists between temperature, pressure, density and the average weight of its particles. In the case of a star the average particle weight is never far from unity, because of the stripping from all the atoms of nearly all their satellite electrons. For example the average particle weight for hydrogen, where the atom is split into two parts, a proton and an electron, is $\frac{1}{2}$, in helium where the division is into three, the value is 4/3; with carbon it is 12/7 and so on, the highest value being for uranium, 238/93. Because of the high proportion of hydrogen and of helium in a star the average of these particle weights is low; in the Sun, where the proportion of hydrogen is great, the value is usually taken as unity, and in the stars generally, with varying amounts of hydrogen, from 1·3 to 0·6.

Assuming a distribution of internal density, the temperatures and pressures at all points, necessary for equilibrium in the star's interior, can be calculated. This distribution is

*That is to say, with perfectly elastic molecules of infinitely small size and no forces of attraction on each other.

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not easy to determine ; but different models have been assumed for the purpose of calculations of the star's luminosity to check up with the results of observation. Luminosity, which is the rate of escape of radiation through the star's photosphere, partly depends on the opacity of the stellar material and this can be derived on the idea that absorption and re-emission of energy in the star are due to processes involving interactions between the ionised atoms and free electrons, which are more frequent the more particles there are in a given volume, but happen less often when the particles are moving fast (at high temperatures).

The pressure of radiation plays a very important part inside a star. Radiation is known both by theory and from experiment to have mass and momentum, and to exert pressure on anything in its path. In bodies at the temperature which can be produced in a terrestrial laboratory, a negligible fraction of the heat is radiative, almost all of it being molecular agitation. The quantities are more nearly equal at the enormous temperatures in stellar interiors, and the pressure of the radiation substantially assists the elasticity or gas pressure of atomic movement to oppose the inward gravitational pull of the material. Calculation shows that the proportion which radiation pressure bears to the total of the two increases with the mass of the star, so that it appears probable that there is a maximum value of mass beyond which a star cannot normally exist (Eddington).

Using our knowledge of radiation and the properties of gases it is possible to compute the interior distribution of densities, pressures and temperatures for a given mass, luminosity and diameter. An idea of the density gradient inside the star may also be obtained by study of the light curves and radial velocity curves of eclipsing binaries from which rotational movements of the major axes of the orbits and ellipsoidal distortions of the star's shapes can be derived, such features being governed by the interior distribution of mass. A rapid increase of density towards the centre is found, a recent calculation for the Sun (mean density 1·4 times water) giving a central density of more than a hundred times water.

The current ideas for normal stars may be thus summarized. The stars of the Main Sequence are composed of matter that appears to behave as a "perfect" gas with their radiation pressures not large compared with the interior gas pressures; the central temperatures range from about ten million degrees (M type) to about 30 million degrees (B type), the mean temperature throughout the star being about a half or less of these figures; the central densities are from about ten to a hundred or more times that of water, the mean densities being about a fiftieth or less of those at the centre; and the radiation per unit of mass ranges from about a twentieth (M type) to several hundred times (B type) of what it is for the Sun. Ordinary giant stars have similar general properties, but their central densities are considerably smaller, say a tenth as much, and their central temperatures are from less than five million up to ten million degrees, with their radiation per unit of mass more uniform than for the Main Sequence, averaging about 40 times the amount for the Sun.

STELLAR EVOLUTION: EARLY THEORIES

In an earlier chapter (see p. 186) reference is made to the theories of stellar evolution current in the late nineteenth century. Astrophysicists generally then regarded the hottest type white stars as the youngest, and the cooler red stars as the oldest, and it seems likely that it was not thought possible for a diffuse gaseous body to produce the kind of spectra observed. For the Sun, for instance, fifty years ago it was "generally believed that the Solar photosphere is a condensation surface; that, mainly due to the cooling caused by the rapid diminution of pressure experienced by the upward currents with the consequent expansion, certain gases are precipitated in the form of luminous clouds. These clouds form the photosphere, the visible surface of the Sun, and from the enormous effect of the Solar gravity must form almost at the same level all over the whole Sun and at all times. It is also generally believed that carbon, here the most refractory of substances, and the most difficult to volatilize, is, with perhaps silicon, the principle element thus precipitated."[1] Lockyer, however, accepted the ideas of Lane and others (see p. 220),

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and he classified stars in an ascending and descending temperature sequence, but he was practically alone in this view.

About the year 1913 a revolution in ideas took place, and the Giant and Dwarf theory of Hertzsprung and Russell (see p. 256) became generally believed in. This theory was in principle a revival of Lockyer's ideas. By it the stars were supposed to start to be visible as cool red stars of M type with low density and enormous volume. They contracted and rose in temperature passing along the spectral series K, G, F to A, and B—*i.e.*, the opposite of the formerly accepted order. At some stage of the contraction, usually at about A type spectrum, the density became too great for the perfect gas laws to apply, the rise in temperature was checked, and finally the star cooled down again as a liquid or solid would do; in the last stage it returned down the spectral series to type M and to extinction.

But difficulties began to be encountered. Eddington's calculations ten or eleven years later showed that the material of the comparatively dense Main Sequence stars seemed to act as a perfect gas (see p. 282), since the luminosities he derived for them corresponded to an approximation to that state; and he found their luminosities closely related to their masses as was also the case for giant stars.* It soon became evident that although the giant series with its increasing temperatures from M type onwards might present no insuperable difficulties, the descending temperature Main Sequence series did not appear to be explicable as in the Hertzsprung-Russell scheme. The loss of mass which appeared possible by radiation during a star's lifetime was evidently not nearly enough to account for the great diminution of luminosity from the B to the M type.

STELLAR EVOLUTION: PRESENT-DAY THEORY

As for present ideas on the course of stellar evolution it may be stated that there is no very generally accepted hypothesis.

*Luminosity values increase very rapidly with mass. The following are approximate average values, in terms of the Sun :—

Mass.	*Luminosity.*	*Mass.*	*Luminosity.*
32	3300	1	1
8	330	$\frac{3}{4}$	1/6
2	13	$\frac{1}{2}$	1/50

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Most astronomers, however, take the view that a star begins life as an enormous spherical mass of very tenuous gas contracting under gravity. It has been thought possible that the clouds of dust and gas in the dark obscuring nebulae (see pp. 196, 297) provide the material for stars, and L. Spitzer has recently suggested that any condensations in them will tend to become moulded into spherical shapes by the external pressure of radiation from surrounding stars, and that this will be helped by the gravitational attraction of the enclosed matter, with the formation of stars of low temperature and very small density. The interior temperature is still much too low to start up any thermonuclear processes; but when the advancing temperature gets as high as say half a million degrees at the centre, nuclear reactions commence which, by increase of temperature and of gas pressure, prevent further contraction.

As stated when dealing with the source of stellar energy (see p. 224), the first reaction is likely to have been the one in which deuterons are involved. Following the exhaustion of the material for this reaction, contraction and heating up take place until, with central temperatures of from two million to about nine million degrees, the reactions with lithium or beryllium atoms begin, and then those with boron atoms.

As these elements get used up there are more stages of contraction rapidly passed through and the radiation is derived from the energy of gravitational contraction until the central temperature reaches about twenty million degrees when the carbon cycle begins. During the stages before that cycle is active the star shows spectra of M, K, G and F type giant stars in succession, until arrival at central temperatures of ten million to thirty million degrees brings it into the zone of the Main Sequence.

It is thought that the deuteron reaction marks not only the ordinary red giant or super-giant M and K stage, but also probably the Long Period variable; and that the lithium and beryllium processes which occur in the G and F type giants and supergiants are taking place also within the Cepheid type of variable. The processes described entail great increase in generation of energy as they are reached at stages of higher temperature. The consequence may be that in each case

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rather suddenly an expansionary state is reached with accompanying cooling and therefore a temporary reduction of production of energy which creates a somwehat unstable condition. In such instances circumstances appear to be present that may produce pulsating Long Period or Cepheid variable stars (see p. 276). The boron reaction marks the next stage of the F type or A type giants before they reach the Main Sequence.

According to these views, which are representative of those of G. Gamov and others, a star passes through the stages described without much increase in luminosity, until, on reaching the Main Sequence it passes *upwards* in surface temperature to the B or to the O type, according to its mass, the hydrogen atoms being meanwhile used up in the carbon cycle.

When the hydrogen is exhausted there does not appear to be any source but gravitational attraction for the star to use in the production of energy (although there is perhaps the possibility of some other thermonuclear processes, not yet identified, at temperatures over twenty million degrees) and the star drops greatly in luminosity to the white dwarf stage, maybe more than ten thousand times fainter.

Modern theory suggests that a star in its last radiating stage is a small body of extremely great density and the white dwarfs may be in this very late part of the life of a star. Their average densities are of the order of 50,000 or more times that of water with central densities hundreds of times again as great as that. The central temperatures are, however, not thought to be very high as compared with those of Main Sequence stars—probably of the order of ten million degrees. The enormous densities are due to the stripped state of the atoms through ionisation ; nuclei and electrons are jammed tightly together (see p. 259).

Recent theoretical researches have shown that there is a limit to this compression and that at that limit energy will not be available for any radiation. This would appear to make the white dwarf star an intermediate stage between a normal star and an "invisible" one.

There are a number of unexplained difficulties to be removed, and much work is necessary to clear them up and arrive at a satisfactory theory.

ORIGIN OF BINARY SYSTEMS

The origin of binary or multiple systems of stars is also far from a solution. Several methods have been suggested for their formation the chief three of which are :—

1. The chance encounter of two stars, resulting in their revolution about a common centre of gravity (the capture theory).
2. By a previous existence, in an original dust cloud or nebulae, of independent nuclei separated by a distance comparable with those now observed in visual systems, and by subsequent condensation about such nuclei.
3. By division of a single star into two components through centrifugal and possibly occasionally tidal, fission, influenced by radiation pressure.

The first of these does not seem likely to have been frequent, taking into account the great distances which separate the stars in the stellar system, and does not appear to be dynamically practicable without the presence of a third body or a local retarding medium. The second and third may both be valid and may be the normal ones for relatively widely-spaced systems, and for the close spectroscopic binaries, respectively.

Recent researches of G. P. Kuiper suggest a common origin for both, as he finds that the relative frequency of lengths of major axes of orbits seems to be nearly the same for all classes of binary. He also considers that fission is not possible with the accepted model of a star's internal structure; and that even if fission were a method of origin, the distribution of the ratios of masses of the components should be similar for all classes of binaries, which is not the case.

There appear to be difficulties in the way of all attempts at explanation, and it may be that some theory of a catastrophic nature will prove to be the correct one. For instance in 1932, E. A. Milne proposed a hypothesis for the origin of Novae according to which there is a certain stage in the

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evolution of a star when internal instability sets in, resulting in a "collapsed" dense state accompanied by the liberation of much gravitational energy. This, in the case of a rotating star, might be followed by division into two or more components neither of which would necessarily remain collapsed, with consequent birth of a binary or multiple system. Something of the sort may be required to explain the existence of binary pairs like Sirius, composed of a normal star and a white dwarf.

CAUSE OF STELLAR VARIABILITY

The cause of stellar intrinsic variability, *i.e.*, change in brightness other than that due to eclipse by a companion or obscuration by interstellar clouds, has been the subject of much speculation. Two of such speculations which appear to be possibilities are : a periodic occurrence on a large scale of spots such as those seen on the Sun, and formation and dissipation of obscuring veils in the outer regions of a star owing to the accumulation of opaque material of same kind. So far, however, there does not appear to be any strong spectroscopic or other evidence for such types of variation.

The work of J. Stebbins and others with the photo-electric photometer has shown that a sensible proportion of the stars are probably slightly variable. This applies particularly to the diffuse red supergiant or giant type ; these have been found by Stebbins and Huffer as likely to be inconstant in light, the larger, more luminous, and redder they are. This suggests an instability of physical conditions in such cases such as has been noted in the variability of the diameters of the red supergiants, Betelgeuse and Antares, revealed by the Mount Wilson interferometer. The stars of the Main Sequence are much more constant in light.

In the case of the Cepheids, simultaneous changes of brightness, spectral type and colour show that the physical cause is connected with variation in their effective temperatures, however this may be effected. The forms of their light curves make it evident that rotation of the star, with some parts of its surface less luminous or obscured, cannot be the explanation, and also rule out eclipse of one component of a binary star by the other. All theories involving binary

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systems seem effectually prohibited as no spectral lines of a secondary have ever been even suspected, and also because calculation of the probable dimensions of any possible system gets diameters of orbits actually smaller than those that the primary stars themselves should have. These points against the binary theory were put forward by H. Shapley in 1914.

The large amount, and the quickness, of the changes in light, call for a periodical transformation of heat energy to some other form and back again, little being expended in radiation in the short interval. The form of energy suggested is gravitational; and a theory of pulsation of the star, first suggested by H. C. Plummer about 1913, and given strong support by Shapley and Eddington, whereby a periodical expansion takes place under the opposing forces of gravitation and gaseous elasticity, has met with general acceptance. The pulsation theory has been the subject of many theoretical investigations by Eddington and others. What seems to be its most promising form has recently been advanced by M. Schwarzschild. This proposes that there is a periodic wave of compression of the star's material moving up from below its photosphere into higher atmospheric regions; this differs from the pulsation theory in its original form that supposed the star to be expanding and contracting with all its parts moving outwards and inwards simultaneously. The theory of Schwarzshild altered this by introducing the idea of an outer region where waves of compression move outwards; the central part pulsates as one body but there are compressional waves in the outer parts passing outwards to, and through, the photosphere into the regions of the star's atmosphere.

Various features of the relation between the form and phases of the curves of light variation and those of the line-of-sight velocities of the photosphere, the near side moving towards the observer when the star is expanding and from him when it is contracting, receive what seem consistent explanations. This revision of the pulsation theory thus appears to provide at least a good foundation for a more satisfactory theory of the observed phenomena of Cepheids than is found in any other proposal. It also seems to give a probable explanation of the variation in the Long Period

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Stars, a phenomena that appears to be essentially of the same sort, although there the very low densities and temperatures may exaggerate and make more erratic the effects of the pulsation, which are possibly also affected by formation of cloudy veils in these stars' upper atmosphere as has been suggested by P. Merrill. In the Long Period stars, however, it has been found that the curve of line-of-sight velocities to be used is that for the bright emission lines; these seem to originate below the level of origin of the absorption lines in the spectrum, masses of luminous gases rising through the photospheric layer to disappear at an upper level (Merrill, 1946).

Irregular Variables are mostly giant stars of large dimensions and small density. A suggestion sometimes made that their changes in light are caused by interstellar clouds drifting in front of the star, is perhaps the correct explanation for a number of faint variables found in the constellation of Orion. But this hypothesis does not suit for the giant irregular of the usual kind, as they have changes in spectra that mean an intrinsic cause. The formation of enormous spots or of obscuring veils has, as already stated, been proposed as an explanation.

CAUSE OF NOVA OUTBURSTS

The cause of the outburst of light in a Nova, some early theories of which are mentioned in a former chapter (see p. 194), is now generally considered to be a sudden production, or release, of sub-atomic energy somewhere below the surface of a star that may have developed a zone of instability; the equilibrium of this zone is easily disturbed, with a sudden outburst of radiation.

The result is a violent expansion of the star's outer layers at speeds beyond the control of its gravitation; the layers swell but keep on radiating, the enlargement of the radiating surface causing an enormous increase in the star's apparent brightness. This expanding shell becomes thinner and more transparent and is excited to shine even more strongly by the very short wave high energy radiation from the star's inner regions exposed by the loss of the surface material. The spectral phenomena observed are in the main due to this shell.

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It produces absorption lines displaced towards the violet where it comes between the star and the observer, and bright broad lines, or bands, from the other parts which have motions in all directions with respect to the observer.

D. B. McLaughlin and others have recently advanced the idea that Novae may be the results of repeated outbursts of a star. Several stars of a Nova-like type are known that repeat, the interval being proportional to the amount of the change in light; and it is thought possible that the Novae which are of much greater range between their pre-Nova brightness and maximum light, may also repeat at intervals of several thousand years. It has also been pointed out that if the phenomenon is not a recurring one, peculiar to a particular type of star, then most, if not all stars, must pass through a Nova stage judging by the frequency of the phenomenon.

As there is a great difference between the luminosity of a Nova and of a Supernova, and much greater velocities of expansion are observed in the latter, it has been thought that the causes of the outbursts must be essentially different. It has been suggested that the old hypothesis of collision between stars (see p. 194) may be suitable for the Supernovae. Their frequency seems to be of the order of a ten thousandth of that of a normal Nova, but although F. Zwicky has found that they appear on the average about only once in 600 years in a particular stellar system like our Milky Way (see p. 278) that interval is probably too short for collisions of the stars of any system like our Galaxy, which are on the average very widely spread apart.

GALACTIC STAR CLUSTERS

Since the general adoption of photographic methods, study of the clusters of stars has received special attention. There are two classes of stellar cluster, the "Galactic" or "Open" kind, and the "Globular" variety. Of the former more than 300 have been listed ranging from the loose aggregation like the Pleiades to small groups revealed only by the telescope, and ranging in stellar population from those with hundreds of members down to small numbers barely distinguishable from chance groupings. Ability to estimate their distances

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and dimensions has been a consequence of the discovery of the giant and dwarf classifications of the stars, as when the spectral types and stellar magnitudes of the individual stars of the clusters are once known, a knowledge of their approximate real luminosities follows from the giant and dwarf grouping shown. By these luminosities and the measured stellar magnitudes for the stars, their distances are obtainable, just as the distance of a light of known candle-power can be derived from its apparent brightness; and the diameters for the clusters corresponding to their angular diameters follow. Early workers in this field were Shapley, N. Rasmuson and G. Raab, but the most important systematic investigation was that by R. J. Trumpler published in 1930.

The general distribution on the sky of galactic clusters shows them to belong to our Milky Way system, but owing to the inadequacy of the data, due probably to the effects of obscuration of light, the positions in space of those studied in detail are found to be confined to a region six or seven thousand years in radius round the Sun, those clusters which presumably exist at greater distances, particularly towards the centre of the Galaxy, being as yet unavailable for close research. The further advance necessary into space may be found possible by photographs using light of longer wavelengths than the usual "actinic" rays, which is less absorbed by interstellar dust; greater distances could then be investigated and this might help to bring out structural features of the main Galaxy so far not detectable by the study of the known clusters.

Their overall diameters have been found to range from about fifteen to sixty years, the majority being between thirty and fifty light years. And there appears to be a connection of diameter with concentration of the stars and also with poorness or richness of numbers of stars; the diameters are less with greater concentration but increase with richness of population. The spacing of the contained stars is considerably closer together than that of the stars in the Sun's neighbourhood and an observer situated in the centre of one of them would see many stars shining even more brilliantly than Sirius, our brightest fixed star.

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GLOBULAR STAR CLUSTERS

A great part of the knowledge we have of the globular clusters has been a consequence of the work of H. Shapley since 1914 when he first studied them by means of photographs taken with the large reflectors at Mount Wilson, and later since his appointment in 1921 as Director of Harvard College Observatory, while J. Scheiner, H. Ludendorff, von Zeipel, Plummer, Eddington, and Jeans have also produced much important theoretical and practical results. The methods employed for finding the distances have been various. In order of approximate importance they are: the period-luminosity relationship of Cepheids in the clusters; the magnitudes of the contained brightest stars, which have a fairly constant luminosity; angular diameters of the clusters' main parts; and the integrated stellar magnitudes of the clusters, their total luminosities averaging about 150,000 times the Sun's.

The numbers of contained stars are of the order of scores, if not hundreds, of thousands; and all spectral types appear to be present. But the stars so far studied are mostly giants or bright Main Sequence members; long exposure photographs with the largest telescopes of the future will settle to what extent the remainder of the Main Sequence are represented. Globular clusters are usually strongly concentrated towards the centre, but detailed study shows that there are degrees of concentration, some being much looser than the average. Their actual dimensions are much greater than for galactic clusters, the densest parts being from thirty to a hundred light years in diameter, with the overall sizes three or four times as great. The density of stellar distribution is very high, perhaps more than a thousand times that in the Sun's vicinity towards their centres, but the average distances apart of the stars are nevertheless not small, being probably a substantial fraction of a light year.

The space distribution is very striking. Their distances from the Sun are immense* (about 25,000 to over 180,000 light years) and they are almost equally distributed on each side

*The revision in ideas entailed is brought out by reference to J. E. Gore's estimate in 1894 of about 150 light years for one of the brightest.

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of the plane of the Milky Way, occupying a large flattened ellipsoidal volume about 150,000 light years in diameter by about 120,000 in thickness with its centre towards the Galactic centre region of Sagittarius and about 30,000 light years from us.

Of these beautiful objects there are more than 90 catalogued ranging in size from that known as 47 Toucani, nearly a degree in apparent diameter as measured on photographs and visible to the naked eye as a hazy fifth magnitude star, down to a very distant one about five minutes of arc in diameter and of the brightness of an eleventh magnitude star. The finest globular cluster visible in northern latitudes is Messier 13 in the constellation Hercules, diameter about a third of a degree and just visible to unassisted vision.

THE NEBULAE

The view to which the elder Herschel appears to have inclined (see p. 119) that there are two general classes of nebulae, one of a diffuse type found more often in or near Milky Way areas, and the other usually small white nebulae more numerous away from that zone, has gained general credence during the century. In 1922, E. Hubble of Mount Wilson, suggested the definite division into two classes, Galactic and Non-Galactic, the first composed of the diffuse and irregular type and the "planetary" nebulae, and the other the spirals and ellipsoidal types.

PLANETARY NEBULAE

There are about 335 planetary nebulae known. They are round or oval masses of nebulosity sometimes consisting of concentric rings and usually having a central star; the diameters are mostly less than a minute of arc. The central stars are very hot (25,000 to 100,000 degrees) and therefore much brighter photographically than visually. The distances of these nebulae have not been reliably measured directly, but from consideration of their proper motions and radial velocities it appears that they are very distant, more than a thousand to about fifty thousand light years away, situated relatively closely to the Galactic plane, and of the order of

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ten thousand times the distance between the Earth and the Sun in diameter. It has been suggested that they are the relics of Novae, but there are strong reasons for doubting this ; for one thing their number seems to be much too small, in view of the frequency of Novae.

DIFFUSE NEBULAE: CAUSE OF LUMINOSITY

In 1912 V. M. Slipher of the Lowell Observatory found that some of the diffuse nebulae have dark line spectra, and photometric measures by E. Hertzsprung in 1913 supported Slipher's suggestion that these nebulae shine by light reflected from their associated stars. It was noted also that diffuse nebulae and planetaries generally have bright temperature B or O type stars connected with them. This suggested that the source of the luminosity of the nebulae is the radiation from these stars acting on the nebulous clouds of gas and dust. Adopting this hypothesis, Hubble in 1922 related the brightness and angular extents of the diffuse nebulosities to the apparent magnitudes of these stars, and he found conclusive evidence that the intensity of the illumination varies inversely as the square of the distance from the stars, as should be the case, and that the nebulae reflect, or absorb and re-emit, the radiations falling on them.

The Pleiades nebulosity is a prominent example of the case where the luminosity is reflected light, and in some of such nebulae, with continuous spectra and dark lines, the colour of the nebulosity has been found to be about the same as that of the exciting star. Dimensions of the diffuse nebulae, the distances of which have been determined by various methods, have proved to be very great. Trumpler found diameters ranging from twenty to seventy light years for some which are connected with Galactic clusters the distances of which are approximately known. For the planetaries it is considered that the ultra-violet radiation of the very hot central stars, after absorption by the atoms of the gases, causes the luminosity ; but in this case the relation of distance from the exciting star to the nebulosity is not the same as for the diffuse nebulae.

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SPECTRA OF NEBULAE

The spectra of the gaseous nebulae, first observed by Huggins in 1864 (see p. 186) were noted to be fairly similar. The bright lines seen were later found to be of hydrogen, helium and (occasionally) singly-ionised carbon and doubly-ionised nitrogen ; but the most conspicuous lines were those attributed to a hypothetical element "nebulium," and there are a number of less prominent lines. To investigators it seemed likely that these are not due to undiscovered elements, but probably produced by ionised known elements subjected to physical conditions not produceable in terrestrial laboratories ; and that some cause, such as a density much lower than can be produced in our laboratories, must be the reason for the unidentified radiations. After a considerable amount of unsuccessful speculation by others, I. S. Bowen showed in 1927, from considerations of physical theory based on much laboratory experiment, that the extremely low densities of the gases, prevalent in nebulae, and the consequent very prolonged intervals between atomic collisions, provide the conditions for radiations of the wave-lengths concerned from ionised atoms which have lost one or more of their electrons. For example, the following lines are due to oxygen ; a red line, the two green "nebulium" lines, a blue one, an ultra-violet one and possibly others in that region ; while two in the violet are produced by neon.

VARIABLE NEBULAE

Several nebulae vary in shape and brightness. Two of the best known are in the constellations Corona Australis and Monoceros. They both resemble comets in shape with a nucleus at their heads, consisting of an irregularly variable star. The spectra of both these nebulae are continuous, with faint bright lines superposed, and somewhat like those of their attached stars.

OBSCURING CLOUDS

Reference has been made (see p. 196) to photographs taken in 1894 by Wolf and in 1895 by Barnard, showing dark markings on the sky believed at the time to be starless vacuities

in their direction in space. Such starless regions have been familiar to astronomers since the sky surveys of W. Herschel who, there is some reason to believe, suspected them to be due to some other cause than mere absence of stars.[2] But the first astronomer definitely to recognise these dark spaces as the effect of obscuring clouds was probably A. Secchi of the Vatican Observatory, Rome, in 1877.

In 1902 Wolf photographed a remarkable nebula in Cygnus which is encircled by a dark space forming the end of a dark channel among the stars of the area; he conjectured that the dark appearances might be due to an obscuring mass in space, and he remarked that he considered similar features exist for all extended nebulae. These observations were confirmed by W. S. Franks in 1905. Independently of these, Barnard's work led him in 1905 to suspect that in the great nebulous area in Scorpio "certain outlying whorls of the nebulosity have become dark and that they are the cause of the obliteration of the small stars near." By 1919 he was definitely of the opinion that the dark sky markings are for the most part not really starless regions but due to obscurations, between us and the stars, which are seen more readily in Milky Way areas owing to the denser stellar background there. He published a list of 182 of the markings; and in 1925, K. Lundmark and P. J. Melotte discovered as many as 1550, covering in the aggregate about two per cent of the area of the whole sky, by counts on photographic plates. Most, and also the largest, of these are near or in the zone of the Milky Way.

Sometimes, as in the Pleiades, Orion and Scorpio-Ophiuchus regions, the clouds merge into luminous nebulosity near certain stars, with obvious physical connection. The distances of such stars being approximately known, it is found for example, that dark clouds in Taurus, Ophiuchus and Orion, are respectively, about 500, 500 and 650 light years away; and in some cases the distances of the clouds are estimated by counts of the stars down to a particular magnitude, deficiences in the numbers, inside the area of the cloud, at particular magnitudes (the average distances of which are approximately known) indicating the distance of the cloud. In many the dimensions are very great. The series of clouds which produce

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the bifurcation of the Milky Way from Cygnus to Centaurus are many hundred of light years in extent, while the "Coal Sack" near the Southern Cross is probably about 30 light years in diameter.

It is considered that the obscuration by these clouds is produced chiefly by dust. Fine dust, having a great surface per unit of mass, is much more effective in stopping light than larger bodies, H. N. Russell remarking that even less than a tenth of an ounce per square foot whatever its extension from us, would be sufficient for complete opacity. That the clouds cannot be gaseous is shown by the absence of the motions of the neighbouring stars that would be produced by the enormous mass of gas necessary for the obscuring effects. It has been suggested that the dust clouds may be accumulations of matter expelled from the stars; but this does not look probable judging by the fact that the material of Solar prominences seems mostly to fall back into the Sun, and it appears more likely that they are original cosmic material from which the stars themselves are born (see p. 286).

THE LOCAL SYSTEM

Many years ago B. A. Gould pointed to a belt of bright naked-eye stars distributed in a great circle inclined at an angle to the Milky Way, crossing it in Cassiopeia, and near the southern Cross; and Sir John Herschel had previously drawn attention to the southern part of this stream. The statistical work of C. V. L. Charlier in 1916 was taken to indicate the existence of a local system in the Milky Way stratum, of which this great circle of bright stars was a manifestation, about 2000 light years in diameter by 700 light years thickness with its centre 300 light years away in the direction of Argo. By a study of the distribution of the B type stars brighter than seventh magnitude, Shapley in 1919 considered that he had confirmed the existence of this system, to which a considerable fraction of the brighter A type stars appears also to belong. It was also later found by Hubble that the Galactic gaseous nebulae and dark clouds seem to be concentrated in such a local system and in the more distant Milky Way clouds, the spaces between being apparently devoid of such objects.

Investigations of the past few years have produced results which do not all favour the existence of this Local System. These are based on counts of stars that give different conclusions according to the amount of absorption of light in space assumed in the calculation. B. J. Bok has found a result in favour and others have supported it; and more recently Bok and MacRae have obtained indications that the Sun is a member of a system of elongated shape which perhaps lies along a spiral arm in our Galaxy. But J. H. Oort, using different methods, in which counts of the numbers of external system visible at different Galactic latitudes were employed as a means of estimating the light absorption in our Milky Way stratum, has obtained figures which would mean that our Sun is situated in a "local void," perhaps between two arms of a spiral Galactic system. It is considered, therefore, by some astronomers, that the conception of a clearly marked Local System may have to be abandoned. It should be noted in this connection that the shearing effect of the different speeds of rotation of the stars round the Galactic centre would appear to entail that the existence of such a system could be only temporary, although on the other hand, the condensations on the arms of many spiral external galaxies (which also rotate) may be such local systems there.

THE GALACTIC MAIN SYSTEM

That the main centre of concentration of the globular clusters on the sky is in the direction of the dense star clouds in Sagittarius was remarked by E. Hertzsprung in 1912, and in 1917 C. D. Perrine pointed out that the centre of their occupied volume was similarly placed in space. In 1918, Shapley, noting that the greatest frequence of faint Novae and of high luminosity distant stars was in Sagittarius, took this to be further evidence that the centre of our Galactic system itself is at that part of space, at the distance he had found for the centre of the system of globular clusters. His first conclusions as to the shape and dimensions of the system were that it is a flattened disc about 300,000 light years in diameter and perhaps 10,000 light years in thickness, with the centre at 65,000 light years from the Sun in the direction

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of Sagittarius, surrounded by a less-flattened roughly ellipsoidal group of globular clusters, nine-tenths of which are contained in a thickness of 100,000 light years, the disc structure 10,000 light years thick being composed of star clouds and bright and dark nebulosities.

These conclusions were based chiefly on photometry of the magnitudes and real luminosities of different types of stars. Statistical methods, using star counts, parallaxes and proper motions, and radial velocities, had also been used to find the form and size of the Galaxy, the most comprehensive of these statistical investigations being those by Kapteyn published in 1923. His results were considerably different from those of Shapley, as he found a diameter of about 60,000 light years and 12,000 light years thickness, for a lenticular system, the Sun being near the centre. Kapteyn's results were only intended by him to be a preliminary outline giving a generalized view, taking no account of the clustered nature of the Galaxy nor of irregularities in its general outlines. The discovery a few years later of an appreciable amount of interstellar light absorption (see p. 268) indicated that this generalized scheme is far from the reality. It also showed that the dimensions of the Shapley system were considerably too large, a further exaggeration being due to an assumption of rather too bright luminosities for the Cepheids used in the measurement of distances.

The present ideas of the dimensions of the Milky Way system make the diameter 100,000 to 120,000 light years with the Sun's distance from its centre about 30,000 light years. The estimates of its total mass based on the dynamics of its rotation are about one to two hundred thousand million times that of the Sun, perhaps half of this made up of stars and the other half of dust and gas. Comparison of the estimates of size previously current (see p. 196) shows the great expansion in our ideas of the size of our system, which is largely due to the work of Shapley and Kapteyn.

RADIATIONS FROM SPACE

The radiations from Milky Way regions of the sky, referred to earlier (see p. 202) were first noted by Jansky in America

in 1933. During the past few years a number of workers, including Reber in the States, and Hey, Phillips, Parsons, and Moxon in this country, have confirmed Jansky's main discovery that the radiations come generally from the Milky Way, and have also studied in greater detail the characteristics of the two principal sources, the stronger in Sagittarius, the other in Cygnus. The origin of these radiations is unknown. But Greenstein, Henyey and Keenan in the States have concluded that if the radiation of the kind observed from each star of the Milky Way system is of a strength equal to that noted from the Sun at times of sunspot activity (see p. 203), the amount from all the stars would be very much too small for the observed results. It has been calculated, however, that the radiation might be produced by interstellar clouds of atoms and electrons, if the effective temperature of such clouds corresponding to the energies of movements of these particles, can be taken to be about a hundred thousand degrees.* With some further development there is a possibility here of obtaining ideas of Galactic shape and structure from the strength of these radiations in various directions, at any rate as regards the thickness of material between us and the further edges of the Galaxy.

The radiations of the preceding paragraph are of a different nature altogether from the Cosmic Rays that were for a time suspected to be very high energy short wave radiations, and thought by some to be possibly the result of the annihilation of matter through collision of a proton and electron. These Cosmic Rays are now thought to be mainly composed of protons of very great speed of movement; their origin has been attributed by some scientists to the explosive phenomena of Supernovae (see pp. 278, 292), but this is highly speculative.

THE NON-GALACTIC NEBULAE

Most of the non-galactic nebulae have a spiral or related form of structure such as was first seen by Lord Rosse in 1845

*It may be said that the temperature of an interstellar cloud near a star, if measured in terms of atomic motion, would be of the order of thousands of degrees normally. The temperature in interstellar space itself, however, if measured by the equilibrium temperature of a small meteoric particle some distance from a star in space, is probably only two or three degrees above absolute zero.

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with his 6-foot reflector in the Canes Venatici nebula Messier 51 (see p. 149). The existence of this type of nebula in enormous numbers was not known until photography had been developed as the essential tool for their discovery and study, by the work of Common, Roberts and Keeler (see p. 195). Wolf was also one of the earliest to photograph them systematically, and he was first to discover a cluster of the objects when, in 1901, he found more than a hundred spaced closely together on the sky in Coma Berenices, and later another group in Virgo. Many other groups of the kind have since been noted.

Most astronomers of the early part of the present century considered that these non-galactic objects are not star systems like our Milky Way at great distances from it. In fact the spirals were being regarded at that time by many astronomers as *planetary* systems in the making, instead of stellar systems many thousands of times as large. But by 1914, Eddington, Gill, H. D. Curtis, and others had adopted the view that they are external systems, although for some time after that date, reported discoveries by van Maanen of measurable motions in the arms of some of the larger spirals, made their status as "external universes" appear incredible owing to the impossibly great real velocities which these measurements would represent. The extraordinary high luminosity of the brighter Novae in the Andromeda and other spirals (the Supernovae of later days), necessary with the distances corresponding to what was required if these formations were distant Galaxies, were also against the idea.

EXTERNAL UNIVERSES

In 1922-4 Hubble succeeded in taking photographs showing two of the largest spirals partially resolved into stars, and detected a number of Cepheid variables. The distances of these variables could be derived from the Period-Luminosity Relation (see p. 276) and this settled the matter in favour of the external universe hypothesis, showing also that there must have been some systematic errors in the published accounts of internal motions. Confirmation of Hubble's results (about

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900,000 light years* for the Andromeda nebula and for Messier 33) was provided by the work of others such as E. Öpik and K. Lundmark, the observed stellar magnitudes of the normal Novae of these external systems giving luminosities of the same order as those found for Galactic Novae. And in 1923, Shapley who had previously been against the larger distances, estimated the remoteness of an irregular stellar system in Sagittarius, from the brightness of its most luminous stars, as of the order of a million light years.

DETERMINATION OF DISTANCES

Once the status of these objects had been demonstrated, systematic determination of their distances and dimensions was undertaken. The lines on which this was done may be briefly described as follows. The brightest stars in the ten nearby galaxies (the distances of which had been fairly accurately obtained from Cepheids and by other methods) could be closely studied and were found to be of rather uniform luminosity, roughly 50,000 times that of the Sun. This information could be applied for other galaxies somewhat further away, in which a few of the individual brightest stars could be detected in photographs, and their distances estimated. The luminosity of the brightest stars could also be used for members of clusters of nebulae, and the distances of these clusters rather more securely computed, that for the Virgo cluster (the nearest) being about 7 million light years. Measurements of the luminosities of the brighter galaxies in all such clusters also showed a rough uniformity from one cluster to another of about 100 million times the Sun's. This provided a method of estimating the distances of faint galaxies too far away for the separate stars in them to be photographed. Distances of clusters as far away as 100 million light years or more could thus be estimated.

CLASSIFICATION

A scheme of classification due to Hubble is now generally adopted. It involves a series of three broad types; elliptical,

*Later reduced to 700,000 light years allowing for the effect of light absorption in space.

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spiral, and spirals with a luminous bar crossing the nucleus and its surrounding disc of nebulous light. The elliptical class is divided into sub-classes of varying amounts of ellipticity of shape; the spirals, into those with their arms closely disposed and others with them more openly spread and distinct, and the barred spirals are divided similarly. There is also a relatively infrequent irregular type, the Magellanic Clouds being its largest specimens. In the elliptical class the shape is really ellipsoidal or spheroidal, with varying degrees of oblateness, and not in any case a torpedo-shaped body, which would be gravitationally unstable.

DIMENSIONS

According to Hubble the average dimensions of the main bodies of the various types are: elliptical, 2000 to 5000 light years; spirals, 6000 to 10,000 light years; and irregulars, 6000 light years. These are diameters over the longer axes of form; they do not correspond to overall dimensions, which may be several times as great. There is a large range in size, although the majority are close to the average. For example, the Andromeda nebula, Messier 31, is about 40,000 light years in length, and 9000 in breadth over its brighter parts; but if the outlying haze of faint stars is included the dimensions are 60,000 by 54,000 light years, comparable with those of our own Galaxy. Both are evidently giant systems. At the other extreme we find the elliptic companion nebula to Messier 31, Messier 32, 3000 light years in diameter.

The masses of several of the larger spirals have been estimated by various methods, the values ranging, in terms of the Sun's mass, from thousands of millions to a hundred thousand millions. Periods of rotation of millions of years have been found from line-of-sight velocities spectrographically measured at points away from their centres; and from the observed inclinations of their spectral lines, the rotation periods of the nuclear regions of thirty galaxies have recently been found by Mayall to range from 1½ million to more than 200 million years.

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STELLAR POPULATIONS

Following the resolution into stars of the two companion galaxies of M31 by means of photographs taken in red light with the 100-inch Mt. Wilson reflector in 1944, W. Baade[3] has suggested that there are two types of Stellar Populations. One is found in the arms of spiral galaxies but not in their central regions, the other resembles what is found in these central regions, in ellipsoidal systems like the two companions of M31, and in globular clusters. Bright giant and supergiant stars are common in the former, Population I, but Population II has no stars brighter than a normal giant, while I has a higher proportion of binary stars but less novae or supernovae. Our Galaxy has both types, I being apparently commoner near the Sun. Further research on this hypothesis is being carried out with the most powerful instrumental means.

DISTRIBUTION OF GALAXIES IN SPACE

Systematic surveys of the whole sky have been carried out by Harvard, Lick and Mount Wilson Observatories, from which information of the distribution of the galaxies throughout space has been sought. The distribution found from these researches is markedly non-uniform from the small-scale aspect. The galaxies are found singly and in groups of increasing numbers up to great clusters containing hundreds or even thousands. But in terms of great space volumes, when samples of very large numbers are concerned, there is smoothing out of the clustering tendency and the distribution comes nearer to a large-scale uniformity.

THE "RED-SHIFTS"

One of the most remarkable features of the galaxies is found in the displacements of their spectral lines. The first critical measurements of these were made by V. M. Slipher in 1912 at the Lowell Observatory. By the year 1925 he had measured displacements in the lines of 41, and those for two others had been measured elsewhere. These shifts were towards the red end of the spectrum for all but one or two, and were found to be much greater than those for any other

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celestial object, star or nebula. As they were towards the red they could be regarded as indicating recessive velocities. When the amounts so found were related by Hubble to the distances of the galaxies, he derived a relationship which, treating the shifts as indications of recessive velocity, gave an increase of about a hundred miles per second per million light years of distance. It was then seen that, assuming that this "velocity-distance" relation continued for still more remote objects than those whose distances were known, the shifts for such objects could be used as measures of distance, or as checks on distances otherwise found. By 1940 more than 200 red-shifts for galaxies had been measured, many of them by Humason at Mount Wilson, the greatest of which corresponded, on the velocity interpretation, to 26,000 miles per second for a galaxy in a cluster believed to be about 240 million light years distant.

The only cause known so far for such red-shifts is recessive motion, and consequently it is generally thought that the explanation is really such a motion indicating a recession of every galaxy from all the others and therefore an expansion of the material universe. All astronomers do not accept this explanation, however; but everyone is, nevertheless, aware that some new physical process will have to be found to explain the phenomenon, if the expansion idea is not correct.

In the opinion of Hubble the conclusions as to distribution of the galaxies obtainable with the most powerful equipment so far available, appear to favour the conception of a stationary universe, but do not definitely rule out the possibility of one that is expanding. Judgment has meantime to be held up until further information is obtained. The new 200-inch aperture retlector at Mount Palomar in California should provide the required data. Owing to the great space-penetrating power (twice that of the 100-inch) the effects of the red-shifts of the most distant objects open to its study will probably be large enough to allow of a definite decision as to their cause, whether velocity of recession or some new principle such as a gravitational drag on light by the matter which it passes in space (as suggested by F. Zwicky), or loss of energy by the radiation on its way to us from the enormous distances entailed (as suggested by W. D. MacMillan).

THE COSMIC TIME SCALE

One accompaniment of a recessional movement of the galaxies and an expansion of the Universe would have been a central congestion of matter about two or three thousand million years ago ; and a date for the origin of the stars and other celestial objects is apparently suggested. This extent of time might thus be taken as the age of the stars. It will be noted that it is of the same order as the estimates of the age of the Earth and Solar System (see p. 222).

There are two cosmic time scales discussed by astronomers —a few thousand millions and millions of millions (billions) of years. The shorter scale is on the whole the more favoured one, and has in its support the effect of the expansion of the Universe (if there is an expansion), and the duration of star clusters and binary star systems as calculated by S. Chandrasekhar and B. J. Bok. And it is also contended that a supposed equipartition of energy among the stars (see p. 263), previously thought to be in agreement with the longer time scale, is vitiated by the marked deviation from general equipartition in a number of types of stars, and also by the fact that, in all arguments relating to the matter, local solar motion is used, *i.e.*, the solar movement with respect to the average for the stars in the Sun's vicinity, which, according to the well-established theory of galactic rotation, has only limited significance.

But there are some who are still inclined to favour the longer scale that was generally believed in twenty or so years ago. For example, work on the distribution of the contents and constitution of clusters of galaxies by F. Zwicky is thought by him to demonstrate that those clusters could not have been formed in the time of the shorter scale, and to indicate that the Universe is stationary and not in expansion, the red-shifts being due to some cause other than recession.

It is evident that practically all the most important problems facing astronomers, such as the reason for the red-shifts, stellar origin and evolution, the source of stellar energy, and the true cosmic time scale, are as yet far from having unquestioned solutions. It is hoped that much of the necessary

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information and some approach towards satisfactory theories will be the result of the use of the powerful new methods and equipment now available and of the revival of co-operation in research on an international basis.

REFERENCES.

[1] E. W. Maunder, *Journal of the British Astronomical Association*, vol. 7, p. 117 (1896).

[2] See *Journal of the British Astronomical Association*, vol. 45, p. 247 (1935).

[3] W. Baade, *Astrophysical Journal* (Sept., 1944).

Pure Mathematical Physics

SOME BOOKS FOR REFERENCE

HISTORICAL

"A History of Physical Astronomy," Grant, 1852.
"A History of Astronomy," Berry, 1898.
"The Royal Observatory, Greenwich," Maunder, 1900.
"History of Astronomy during the Nineteenth Century," Clerke, 1902.
"History of the Planetary Systems from Thales to Kepler," Dreyer, 1906.
"A History of Astronomy," Bryant, 1907.
"A History of Greek Astronomy," Heath, 1913.
"History of The Royal Astronomical Society," Dreyer and others, 1923.
"A Source Book in Astronomy," Shapley and Howarth, 1929.
"A Hundred Years of Astronomy," Waterfield, 1938.

BIOGRAPHY

"Tycho Brahe," Dreyer, 1890.
"Great Astronomers," Ball, 1895.
"The Herschels and Modern Astronomy," Clerke, 1895.
"Galileo : His Life and Work," Fahie, 1903.
"Reminiscences of an Astronomer," Newcomb, 1903.
"David Gill, Man and Astronomer," Forbes, 1916.
"Makers of Astronomy," Macpherson, 1933.
"An Astronomer's Life," Frost, 1933.
"The Herschel Chronicle," Lubbock, 1933.
"Isaac Newton : a Biography," More, 1934.
"Biography of Percival Lowell," Lowell, 1935.
"Hevelius, Flamsteed and Halley," MacPike, 1937.
"Copernicus, the Founder of Modern Astronomy," Armitage, 1938.

TEXT BOOKS

"Astronomy," Russell, Dugan, Stewart, vol. i, 1945 ; vol. ii, 1927.
"Astronomy," Moulton, 1931.
"General Astronomy," Spencer Jones, 1934.

SOLAR SYSTEM

"Earth, Moon and Planets," Whipple, 1942.
"The Moon," Goodacre, 1929.
"Our Sun," Menzel, 1949.
"Mars and its Canals," Lowell, 1906.
"The Solar System and its Origin," Russell, 1935.
"Comets," Olivier, 1930.

Pure Mathematical Physics

SIDEREAL

"Atoms, Stars and Nebulae," Goldberg and Aller, 1944.
"From Atoms to Stars," Davidson, 1947.
"An Outline of Stellar Astronomy," Doig, 1947.
"The Binary Stars," Aitken, 1935.
"The Story of Variable Stars," Campbell and Jacchia, 1942.
"The Milky Way," Bok and Bok, 1945.
"The Realm of the Nebulae," Hubble, 1936.
"Galaxies," Shapley, 1945.

INSTRUMENTS

"The Telescope," Bell, 1922.
"Telescopes and Accessories," Dimitroff and Baker, 1945.

GENERAL AND PICTORIAL

"Splendour of the Heavens," Phillips and Others, 1923.

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